Incentive Mechanisms to Enforce Sustainable Forest Exploitation*

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June 2, 2004

Abstract

This paper addresses the global and urgent environmental issue of tropical deforestation. In our approach, foreign transfers from northern countries to the forestry or developing countries are proposed as an incentive mechanism to guarantee a sustainable management of forests in those developing countries. We consider the two players, the North (or donor community) and the South (the forestry country) to have different utilities for forest conservation. The South, whose revenue function involves a trade-off between forest exploitation and agricultural activities. The North, who represents a set of countries, aspiring to ensure a sustainable exploitation of the tropical forests. The objective of this paper is to determine incentive strategies, conditioning the funds' transfers directly by the South's actions regarding forests exploitation. These strategies can be used by the North to indirectly force the South to choose an optimal deforestation policy which is sustainable over time.

*Research supported by NSERC-Canada. Research completed when the second author was visiting professor at GERAD-HEC Montréal. The second author's research was also partially supported by MCYT under project BEC2002-0236 and by JCYL under project VA051/03, confinanced by FEDER funds.

1 Introduction

The global rates of tropical forest destruction are raising the general concern. According to the Foods and Agriculture Organization's (FAO) estimates, 53000 square miles of tropical forests were destroyed each year during the eighties. A report of the World Resources Institute (see, Matthews (2001)) confirms that the deforestation rates did not slow but on the contrary continue to be rapid.

The adverse consequences of tropical deforestation on biodiversity conservation and climate change are considerable. In fact, the rate of carbon dioxide released in the atmosphere due to the tropical deforestation is approximately 1.6 billion metric tons per year. This constitutes a significant contribution compared to a rate of 6 billion metric tons of carbon emitted from fossil fuel burning; one of the main factors of global warming (Earth observatory, 2001). On the other hand, serious scientific estimates indicate that, in average, 137 species forms of life (plants and animals) are driven into extinction every day due to habitat destruction (Wilson, 1992).

The main causes of tropical deforestation seem to be the conversion of forested land to agricultural use and, at a lower level, the forestry activities (e.g. Amelung and Diehl, 1992; Barbier et al., 1991; Barbier and Burgess, 1997; Kaimowitz and Angelsen, 1999; Southgate et al., 1991; and Southgate, 1990).

It is easily understood that since a country gets revenue from agriculture and forestry activities, the temptation is high to follow a laisser-faire policy when it comes to deforestation. In fact, scholars are pointing out that even if the costs of forest preservation are small compared to the large non-economic benefits from doing so, at a domestic level these benefits remain much smaller than the global ones (e.g. Montgomery, 2002; Chomitz and Kumari, 1998; Cline, 1992).

Barret (1994), Von Amsberg (1994), Van Soest (1998) and Van Soest and Lensink (2000) argue that in some instances the allocation of forest lands to alternative use may enhance the domestic country's welfare, while decreasing the welfare from the global perspective. This confers to the deforestation its international externality dimension, and implies the need of global sources to finance the forest conservation.

Indeed, given the nature of this problem, an efficient solution should bring an alternative source of revenue to the domestic or forestry countries to help them solve their economic issues leading to the over exploitation of forests.

Following this idea, differential games and optimal control theory have been applied to prove that financial transfers from developed community (or the North) to developing countries (or the South), may improve both the rainforest conservation and the welfare of the domestic country.

Barbier and Rauscher (1994), using optimal control theory, consider lump-sum aid donations as indirect instrument to conserve the forest by reducing the necessity to exploit it.

Lump-sum transfers have been, however, criticized as being a passive instrument to combat deforestation. A more active way to use transfers would be to make the amount of payment conditional to the effort deployed by the recipient country to conserve the forest.

In this perspective, a transfer that consists in paying a fixed price per unit of land conserved was proposed by Stähler (1996), using an optimal control approach and by Mohr (1996) in a bargaining game framework. These authors raise the point that this kind of dependency between the transfers and the forest size may, however, has adverse effects in the long run. As the international community's willingness to pay is higher when the forested land becomes smaller, the forestry country can use his "market power" to arise the per-unit compensation through a strategic deforestation behavior. Mohr (1996) explained that, in this case, the credibility problem about whether the donor community is indeed "hard nosed" about the fixed compensation can afford an incentive for the recipient country to increase his deforestation rate.

Consequently, a transfer payment that penalizes an excessive deforestation rate in addition to compensating for each unit of forested land conserved may be advantageous, especially if we consider that the ecosystem biodiversity is, to a large extent, irreversible.

Such a transfer payment, which makes the recipient countries directly confronted to the results of their land use decisions, was first used in a contract approach by Van Soest and Lensink (2000), and in a differential game approach by Martín-Herrán et al. (2002) and Fredj et al. (2004).

In all these models, the developed community's main concern is forest preservation, whereas the forestry country's objective is the maximization of the total revenues he can extract from forestry and agricultural activities. In other words, the problem here is that essentially resource managers do not internalize the loss of biodiversity or the loss of an important carbon sink in their objective function. What this means is that the benefit of the tropical forests in terms of cure from cancer and other illnesses and the mitigation of climate change that represent the concern of developed countries are ignored by the domestic forestry countries.

Consequently and as it has been proven in the above studies, it should be in the interest of both developed community and forestry countries to implement the agreed transfer program. Nevertheless, there is no guarantee that both players would stick to their respective engagements as dictated by the agreed policy. One of the players can have the advantage and hence the incentive to deviate from the initial agreement if he knows that the other player will stick at his engagement. The strategies implied by the agreement are then no longer equilibrium.

This calls for the use of incentive mechanism strategies to counter this eventual problem. The externality problem in several environmental issues have led to the use of incentive mechanism design, e.g. Jørgensen and Zaccour (2001) use incentive equilibrium strategies in the pollution control problem. Originally the idea of incentive equilibrium has been developed and applied to resource management problems by Ehtämo and Hämäläinen (1986, 1989, 1993).

This paper is concerned with the design of incentive strategies mechanisms that can be used by a donor community so that the forestry countries found it optimal to choose in equilibrium a deforestation policy which is sustainable in the long run.

In this work, we assume that the northern community has a real concern about the forest preservation. Therefore, she can not put her future credibility in jeopardy by retracting at an intermediate date from what she announced at the initial instant of the game. Especially that we expect this environmental agreement to be renewed at the end of the horizon or later on. We, then, exclude the possibility of deviation of this player.

Unilateral deviation remains, however, possible for the forestry country as it can be more beneficial for him to receive the compensating transfer without making any effort of forest conservation. To avoid this problem, the developed community can use an incentive mechanism to force the forestry country to stick to the agreed strategy.

The objective of the northern community is that, while maximizing his stream of revenues, the forestry country chooses a deforestation rate that is sustainable in the long run. The official definition of sustainable development stated in the Word Commission on Environment and Development's report (1987) is "development that meets the needs of the present without compromising the ability of future generations to meet their own needs".

To ensure the participation of the forestry country in the program, he should ensure at least the same actualized revenues as what he can have without the developed community's intervention, for the period covered by the agreement. Furthermore, to guarantee the implementation of the agreement once the forestry country accepts to receive the transfer, the developed community has to make the transfer dependent directly on the forestry country's actions regarding the forest exploitation. The objective from doing so is that the forestry country finds it optimal to choose in the short run equilibrium the deforestation rate that leads to sustainable forest stock in the long run. This shows in a deforestation rate equal to the forest's natural regeneration, which is typical for renewable resource case (see, for example, Beltratti, 1995). Our results show that this is possible and can be implemented using an incentive transfer mechanism where the amount of transfers is linear in the forest stock and linear-quadratic in the deforestation rate.

The main contributions of the paper are the following. First, we solve for both the short-run and long-run equilibria. This allows us to calculate the sustainable forest exploitation in the long-run and to estimate the loss in the total welfare the forestry country can bare from a more sustainable exploitation.

Second, we define a transfer function which is composed of a lump-sum amount, that guarantees the participation of the recipient country while compensating him for the total loss due to a better forest conservation, and of a conditional part which represents the incentive mechanism ensuring that the forestry country will respect its engagement of a sustainable forest exploitation. To our knowledge this kind of incentive mechanism has never been used previously in the sustainable forest management literature.

In contrast with other previous studies in our paper we do not need to model explicitly the developed community's utility function which palliates one of the main criticisms addressed in this literature. Moreover, we consider a different dynamics of the forest stock as we allow the possibility of regeneration of the resource.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 characterizes the optimal control solutions for the forestry country without the intervention of the northern community, under the finite and infinite planning horizons. Section 4 deals with the issue of distribution over time of the total payment flowing from *North* to *South* and with the design of an incentive mechanism that guarantees the application of the sustainable forest exploitation at each time t. Section 5 provides a numerical illustration of our results. Finally, section 6 states the concluding remarks.

2 The model

The original model is settled for the forestry country by Ehui et al. (1990) as an allocation problem of forested land between forest activities and agricultural use. The objective is to maximize the present value of a utility index $U(\cdot)$, which measures society's satisfaction. The utility index is concave increasing in the aggregate benefit (π), measuring the societal benefit from agricultural and forestry. This later depends mainly on the forest size, the deforestation rate and the earth's productivity.

A detailed and sophisticated specification of this model was proposed by Van Soest and Lensink (2000), where the utility index is presented as the total net revenues from agricultural and forest use.

The revenues from forest activities are measured each time t as the wood production $q(t)$ times the prevailed price of timber $P(t)$. The timber can be extracted using clearfelling or selective logging methods. Under the first method the land is completely deforested and converted to agricultural use. Assuming that there are n commercially valuable stems per unit of land, the quantity of timber supplied each period of time t, is then equal to n times the deforestation rate $D(t)$. Under the selective logging method, only a fraction γ of timber is extracted and the quantity supplied each period of time t, using this method is equal to γ times n multiplied by the size of the forest $F(t)$. The total timber production can be described as follows[1]:

$$q\left(t\right) = nD\left(t\right) + n\gamma F(t). \tag{1}$$

For convenience n is normalized to unity such that the timber price $P\left(t\right)$ represents the value of all commercially valuable timber per unit of land. The (inverse) demand function is assumed, following a long tradition in economics literature, linear and given by

$$P\left(t\right) = \bar{P} - \theta q\left(t\right), \tag{2}$$

[1] In the original specification of the model by Van Soest and Lensink (2000), the timber supply under the selective method is equal to $n\gamma\left(F(t) - D(t)\right)$. As we are working with a dynamic model we found it more convenient to consider the size of the land not converted to agriculture use as equal to the size of the forest by this time.

where $\bar{P}$ is the maximal market price obtained when $D(t)$ tends toward zero and θ is a positive parameter.

The agricultural net revenue depends on the size of the land under cultivation $F_0 - F(t)$, where F_0 is the initial size of the forest stock; on the agricultural price which is fixed to $\bar{P}_A$; and on land productivity $Z(t)$. This later depends positively on the deforestation rate since burning off the forest cover releases nutrients that increase the soil productivity in the short run. It is decreasing on the cumulative deforestation as, in the long run, soil productivity falls because of nutrient depletion and decreases soil productivity because the farness of forest cover causes the deceleration of the soil formation:

$$Z(t) = \bar{Z} + \alpha D(t) - \beta [F_0 - F(t)]. \tag{3}$$

Combining the revenues from forest exploitation and agriculture activities the total revenue function of the *South* is thus

$$R(t) = P(t)\, q(t) + \bar{P}_A Z(t) [F_0 - F(t)]. \tag{4}$$

This function shows clearly that there is a trade-off between deforestation and conservation of the rainforest.

We suppose that the rainforest evolves according to the following differential equation typical in a renewable resource context[2],

$$\dot{F}(t) = -D(t) + rF(t); \quad F(0) = F_0, \tag{5}$$

where r denotes the natural regeneration rate of the rainforest.

The forestry country choosing the deforestation rate aims at maximizing its stream of revenues actualized at rate ρ:

$$\int_0^T e^{-\rho t} R(t)\, dt + e^{-\rho T} \Phi(F(T)),$$
$$\text{subject to } (1) - (5).$$

Function Φ is the salvage value function. The time horizon $[0, T]$ may be bounded or unbounded depending on whether T is finite or infinite, $T = +\infty$. In the infinite horizon there is no salvage value function.

As we can notice this dynamic optimization problem belongs to the well known linear-quadratic class, since the motion equation of the state variable, the forest stock, is linear and the objective function is quadratic in both the state and control variables. The total revenue function of the *South* can be expressed as:

$$R(t) = -a_1 F(t)^2 + a_2 F(t) - a_3 D(t)^2 + a_4 D(t) - a_5 D(t) F(t) + a_6,$$

where

$$a_1 = \beta \bar{P}_A + \theta\gamma^2, \;\; a_2 = \gamma\bar{P} + \bar{P}_A \left(2\beta F_0 - \bar{Z}\right), \;\; a_3 = \theta,$$
$$a_4 = \bar{P} + \alpha\bar{P}_A F_0, \;\; a_5 = 2\theta\gamma + \alpha\bar{P}_A, \;\; a_6 = \bar{P}_A F_0 \left(\bar{Z} - \beta F_0\right). \tag{6}$$

[2] In the model specified by Van Soest and Lensink (2000), the forest dynamics is the typical one of a non renewable resource with $\dot{F}(t) = -D(t)$.

This functional form shows more clearly how the forest stock and the deforestation rate affect the instantaneous revenues. $a_1 > 0$ and $a_5 > 0$ reflect the decrease in the forest marginal revenues when the forest stock (respectively the deforestation rate) increases. $a_3 > 0$ and $a_5 > 0$ reflect the decrease in the marginal revenues of deforestation when the deforestation rate (respectively the forest stock) increases.

3 Forest exploitation under finite and infinite planning horizons

In this section we derive optimal solutions for the forestry country dynamic maximization problem regarding the forest exploitation in the context of finite and infinite time horizons.

We expect that the optimization problem in the short-run (finite horizon) leads to an over extraction as the forestry country does not take into account the long-run impacts of his actions. For this reason we consider as a counterpart the optimization problem with an infinite horizon, which leads in the long-run equilibrium to a sustainable exploitation with $\dot{F}(t) = 0$, implying that the deforestation rate at equilibrium in the long-run will be equal to the natural regeneration of the forest.

Our objective is to induce the forestry country choosing a sustainable deforestation rate while optimizing in the short-run (or finite horizon). In a first step we derive the solutions of the optimization problems with finite and infinite time horizons. Afterwards in the next section, we will design the necessary incentive strategies that can be used by a second part, which could be the northern community or the developed countries as addressed in the previous literature, that will enforce the sustainable forest exploitation in the forestry country.

3.1 Infinite Horizon $(T = \infty)$

The optimization problem in the infinite horizon planning period is given by

$$\max_{\{D(t)\}} \int_0^\infty e^{-\rho t} R(t)\, dt,$$
$$\text{subject to } (1) - (5),$$

The following proposition provides the optimal solution to the *South* problem when an infinite planning horizon is considered. The superscript s stands for *sustainable* solution.

Proposition 1 *Assuming an interior solution, the optimal control, state and costate variables in the infinite horizon scenario which lead to a forest stock path* *converging in the long-run to the steady-state, F^*, are given by:*

$$F^s(t) = F^* + (F_0 - F^*) e^{(\rho-\Delta)\frac{t}{2}}, \quad (7)$$
$$\lambda^s(t) = \lambda^* + (F_0 - F^*) (a_3(\rho - 2r - \Delta) - a_5) e^{(\rho-\Delta)\frac{t}{2}},$$
$$D^s(t) = rF^* - \frac{1}{2}(\rho - \Delta - 2r)(F_0 - F^*) e^{(\rho-\Delta)\frac{t}{2}}, \quad (8)$$

where F^* and λ^* represent, respectively, the long-run equilibrium or steady-state of the forest size and its shadow value, given by the following expressions:

$$F^* = \frac{a_2 + a_4(r-\rho)}{2a_1 - 2a_3 r(\rho - r) - a_5(\rho - 2r)}, \quad (9)$$
$$\lambda^* = \frac{-a_2(a_5 + 2ra_3) + a_4(ra_5 + 2a_1)}{2a_1 - 2a_3 r(\rho - r) - a_5(\rho - 2r)}, \quad (10)$$

and $\Delta = \sqrt{\frac{4a_1 + (\rho-2r)[(\rho-2r)a_3 - 2a_5]}{a_3}}$.

The maximized net revenue per period of time is given by

$$R^s(t) = A_1 e^{(\rho-\Delta)t} + A_2 e^{(\rho-\Delta)\frac{t}{2}} + A_3,$$

where constants A_i for $i = 1, 2, 3$ are given in the Appendix.

Proof. The current value Hamiltonian associated with the problem described above is[3]

$$H(D, F, \lambda) = R(D, F) + \lambda\dot{F}$$
$$= -a_1F^2 + a_2F - a_3D^2 + a_4D - a_5DF + a_6 + \lambda(-D + rF),$$

where λ is the costate variable associated with the forest stock F.

An interior solution of this problem must satisfy the first order optimal conditions given by

$$H_D = -2a_3D - a_5F - \lambda + a_4 = 0, \quad (11)$$
$$\dot{F} = H_\lambda = -D + rF; \quad F(0) = F_0, \quad (12)$$
$$\dot{\lambda} = \rho\lambda - H_F = 2a_1F + a_5D + (\rho - r)\lambda - a_2, \quad (13)$$

together with the transversality condition $\lim_{t\to\infty} e^{-\rho t}\lambda(t) F(t) = 0$.

Since $H_{DD} = -2a_3 < 0$ the above conditions are also sufficient for this maximization problem.

Replacing the parameters a_3, a_4 and a_5 by their original values in terms of the model's parameters as defined in (6), and taking into account equation (1) after few manipulations we can rewrite condition (11) as:

$$\lambda = \bar{P} - 2\theta q + \alpha\bar{P}_A(F_0 - F).$$

[3]For the rest of the paper, time arguments are omitted in order to simplify notation, when no confusion can arise from doing so.

This expression means that the optimal deforestation rate at each time t should be such that the marginal cost of deforestation (measured by the current shadow value of the forest) equals the net marginal benefit (the sum of the marginal income from selling the timber $\bar{P} - 2\theta q$ and the additional agriculture revenue $\alpha \bar{P}_A (F_0 - F)$).

The second condition (12) simply describes the dynamics of the forest, decreasing at the deforestation rate D, reduced by the natural regeneration of the forest.

Finally if we replace in (13) the parameters a_i $(i = 1, 2, 5)$ by their original values given in (6), after some arrangements we can rewrite the time evolution of the costate variable as

$$\dot{\lambda} = (\rho - r)\lambda - \gamma\left[\bar{P} - 2\theta q\right] + \bar{P}_A\left[\bar{Z} + \alpha D - 2\beta(F_0 - F)\right].$$

This condition indicates that the shadow value of the forest increases or decreases at rate $\rho - r$, depending on whether $\rho - r$ is positive or negative, reduced by the net benefits of conserving an additional unit of land. This latter is equal to the direct marginal revenues of logging it minus the opportunity cost that is equal to the indirect marginal agriculture revenue that could be earned from conserving this unit of land which indirectly enhances agricultural yield through the productivity effect.

From equation (11) we can also extract the expression of D as a function of the state variable F and its shadow value λ:

$$D = -\frac{1}{2a_3}(\lambda - a_4 + a_5 F). \tag{14}$$

Replacing this latter in (12) and (13) we get the following system of differential equations:

$$\begin{aligned}
\dot{F} &= \left(\frac{a_5}{2a_3} + r\right)F + \frac{1}{2a_3}\lambda - \frac{a_4}{2a_3}; \quad F(0) = F_0, \\
\dot{\lambda} &= \left(2a_1 - \frac{1}{2a_3}a_5^2\right)F + \left(\rho - r - \frac{a_5}{2a_3}\right)\lambda + \frac{a_4 a_5}{2a_3} - a_2.
\end{aligned}$$

Solving this dynamic system taking into account the transversality condition $\lim_{t\to\infty} e^{-\rho t}\lambda(t) F(t) = 0$, we have:

$$\begin{aligned}
F^s(t) &= F^* + (F_0 - F^*)\, e^{(\rho - \Delta)\frac{t}{2}}, \\
\lambda^s(t) &= \lambda^* + (F_0 - F^*)\left(a_3(\rho - 2r - \Delta) - a_5\right) e^{(\rho - \Delta)\frac{t}{2}},
\end{aligned}$$

where F^* and λ^* represent respectively the long run equilibrium or steady state of the forest size and its shadow value which expressions are given in the statement of the proposition.

Substituting F and λ by their optimal values F^s and λ^s in (14) we then get the optimal path of the deforestation rate:

$$D^s(t) = -\frac{1}{2a_3}(\lambda^* + a_5 F^* - a_4) + \frac{4a_1 - a_5(\Delta - 2r + \rho)}{2\left(a_3(\Delta - 2r + \rho) - a_5\right)}(F_0 - F^*)\, e^{(\rho - \Delta)\frac{t}{2}},$$

which after some manipulations can be rewritten as in (8).

In order to have the optimal forest stock path converging to the steady-state F^* we need to impose that $\rho - \Delta < 0$. This condition is satisfied if and only if the following inequality applies:

$$2a_1 - 2a_3 r(\rho - r) - a_5(\rho - 2r) > 0. \tag{15}$$

This later condition guarantees that the term inside the square root in Δ is always positive. Moreover, under this condition the steady-state values of the forest size and its shadow value are positive if the following conditions are satisfied:

$$a_2 + a_4(r - \rho) > 0, \quad a_4(2a_1 + ra_5) - a_2(a_5 + 2ra_3) > 0. \tag{16}$$

Replacing, the deforestation and the forest stock by its optimal paths, D^s and F^s, in the objective function, the net revenues from forest exploitation and agriculture use can be written as

$$R^s(t) = A_1 e^{(\rho-\Delta)t} + A_2 e^{(\rho-\Delta)\frac{t}{2}} + A_3,$$

where constants A_i for $i = 1, 2, 3$ are given in the Appendix. ■

From now on we assume that $\rho - r < 0$, which guarantees the fulfillment of (15) and the first inequality in (16). The values of the model's parameters are considered such that the second inequality in (16) is satisfied.

The asymptotically stable trajectory for the forest stock increases or decreases towards its steady-state depending on the initial size of the forest. Under condition $\rho - r < 0$, it is easy to see that the convergence of the forest shadow value to its steady-state presents the opposite behavior. That is, if the forest stock increases its shadow value decreases and vice versa.

The steady-state of the forest stock equation (9) can also be written as

$$\begin{aligned} F^* &= \frac{\gamma\bar{P} + \left(\bar{P} + \alpha F_0 P_A\right)(r - \rho) + \bar{P}_A\left(2\beta F_0 - \bar{Z}\right)}{2\beta\bar{P}_A + 2\theta\gamma^2 - 2r\theta(\rho - r) - (\rho - 2r)\left(2\theta\gamma + \alpha\bar{P}_A\right)} \\ &= F_0 - \frac{\bar{P}_A\left(\bar{Z} + \alpha r F_0\right) - (r - \rho + \gamma)\left(\bar{P} - 2(r + \gamma)\theta F_0\right)}{2\beta\bar{P}_A + 2\theta\gamma^2 - 2r\theta(\rho - r) - (\rho - 2r)\left(2\theta\gamma + \alpha\bar{P}_A\right)}. \end{aligned}$$

If $\rho - r < 0$, the denominator of the second term in the right hand side is positive. The forest size will be decreasing ($F^* < F_0$) if and only if the numerator is also positive. The comparative statics shows that a higher forestry revenues induced by a relative increase in the timber price leads to a higher steady-state of the forest stock. This tallies with the respective result in Ehui et al. (1990). An increase in the land productivity has however a negative effect on the steady-state level of the forest size. The effect of a relative increase in agriculture price is however undetermined and so is the effect of the discount rate.

3.2 Finite Horizon

The optimization problem in the short run or finite horizon planning period differs from the previous one, as it dictates the use of a salvage value to avoid the complete depletion of the forest, which implies a new transversality condition. The optimization problem to be solved is given by

$$\max_{\{D(t)\}} \int_0^T e^{-\rho t} R(t)\, dt + \phi F(T),$$
$$\text{subject to } (1)-(5),$$

where the salvage value function Φ has been considered linear for simplicity. That is, $\phi F(T)$ is the salvage value at the end of the horizon and ϕ the marginal value of the forest at the terminal time T.

The following proposition provides the optimal solution to this problem where the superscript T stands for bounded finite horizon scenario, with terminal time T.

Proposition 2 *Assuming an interior solution, the optimal control, state and costate variables in the short-run equilibrium or finite horizon scenario are given by*

$$F^T(t) = F^* + \frac{a_3(\Delta+\rho-2r)-a_5}{a_5^2-4a_1a_3} C_1 e^{(\rho-\Delta)\frac{t}{2}} + \frac{(\rho-2r-\Delta)a_3-a_5}{a_5^2-4a_1a_3} C_2 e^{(\rho+\Delta)\frac{t}{2}}, \quad (17)$$

$$\lambda^T(t) = \lambda^* + C_1 e^{(\rho-\Delta)\frac{t}{2}} + C_2 e^{(\rho+\Delta)\frac{t}{2}}, \quad (18)$$

$$D^T(t) = \frac{1}{2a_3}(a_4-\lambda^*-a_5F^*) + \frac{[(\Delta+\rho-2r)a_5-4a_1]C_1}{2(a_5^2-4a_1a_3)} e^{(\rho-\Delta)\frac{t}{2}} + \quad (19)$$
$$\frac{[4a_1-(\rho-2r-\Delta)a_5]C_2}{2(a_5^2-4a_1a_3)} e^{(\rho+\Delta)\frac{t}{2}}.$$

The maximized net revenue per period t is given by

$$R^T(t) = B_1e^{t\rho} + B_2e^{t(\rho+\Delta)} + B_3e^{(\rho+\Delta)\frac{t}{2}} + B_4e^{(\rho-\Delta)t} + B_5e^{(\rho-\Delta)\frac{t}{2}} + B_6.$$

Constants C_1, C_2 and $B_i, i = 1, \ldots, 6$ are given in the Appendix.

Proof. The first order optimal conditions are given by (11)-(13) together with the transversality condition $\lambda(T) = \phi$.

Following the same reasoning than in the infinite horizon scenario the optimal solutions described in the proposition above can be derived.

Inserting the optimal time paths for the control and state variables, D^T and F^T, in the objective function gives the optimal revenue R^T. ■

The results in the above proposition are intuitive. Indeed, it is readily seen from the optimal conditions that the deforestation policy satisfies the familiar rule of marginal revenue from deforestation must equal its marginal cost, as in the infinite maximization problem. The interpretation of the results stays the same as in this later scenario, except that in the finite problem the forest stock path does not converge to the long-run equilibrium where the forest exploitation is sustainable.

The objective in the next section is to induce the forestry country to choose the deforestation policy leading to a sustainable forest exploitation along the time horizon $[0,T]$. That is, using incentive strategies the donor community or $North$ wants to guarantee that the $South$'s optimal deforestation policy is the one obtained as the solution of the optimization problem when an infinite time horizon is considered.

4 Total payment and incentive strategies

In this section we, first, focus on the computation of the difference between the $South$'s welfare for the finite and infinite horizon scenarios. The donor community has to compensate the forestry country for his loss of welfare when using sustainable strategies. How to distribute the total payment over time is the issue addressed in the first subsection. The second one is devoted to the design of an incentive mechanism which guarantees a sustainable forest exploitation.

4.1 Distribution over time of the total payment

To encourage the domestic country to adopt a sustainable forest exploitation rather than using the strategies obtained as solution of the finite horizon optimization problem, the international community is willing to pay the opportunity cost of doing so. This later is evaluated as the difference between the total net revenues for the infinite and finite horizon optimization problem from the initial date to the terminal one T, actualized at rate ρ. That is,

$$J^T - J^s = \int_0^T e^{-\rho t}\left(R^T(t) - R^s(t)\right)dt + e^{-\rho T}\phi\left(F^T(T) - F^s(T)\right).$$

The last term in this expression assumes that the $South$ uses the same salvage value function to evaluate the forest stock at at the end of the horizon T.

Next proposition characterizes the value of this difference.

Proposition 3 *The difference between the forestry country's welfare for the finite and infinite planning time horizons is given by:*

$$\begin{aligned} J^T - J^s = {} & TB_1 + \frac{B_2}{\Delta}\left(e^{\Delta T} - 1\right) + \frac{2B_3}{\Delta - \rho}\left(e^{-(\rho-\Delta)\frac{T}{2}} - 1\right) - \frac{M_1}{\Delta}\left(e^{-\Delta T} - 1\right) - \\ & \frac{2M_2}{\Delta+\rho}\left(e^{-(\rho+\Delta)\frac{T}{2}} - 1\right) + \phi\left[\frac{(\rho - 2r - \Delta)a_3 - a_5}{a_5^2 - 4a_1a_3}C_2 e^{-(\rho-\Delta)\frac{T}{2}} + \right. \\ & \left.\left(\frac{\left[a_3\left(\Delta + \rho - 2r\right) - a_5\right]C_1}{a_5^2 - 4a_1a_3} - (F_0 - F^*)\right)e^{-(\rho+\Delta)\frac{T}{2}}\right], \end{aligned} \quad (20)$$

where constants M_1 and M_2 are given in the Appendix.

Proof. The forestry country's welfare for the period $[0,T]$, under the optimal policies of forest exploitation associated to the finite and infinite horizon scenarios are respectively

$$J^T = \int_0^T e^{-\rho t} R^T(t)\, dt + e^{-\rho T}\phi F^T(T), \quad J^s = \int_0^T e^{-\rho t} R^s(t)\, dt + e^{-\rho T}\phi F^s(T).$$

Therefore, the difference is given by:

$$J^T - J^s = \int_0^T e^{-\rho t}\left(R^T(t) - R^s(t)\right) dt + e^{-\rho T}\phi\left(F^T(T) - F^s(T)\right).$$

After some easy computations, the following expression has been derived:

$$R^T(t) - R^s(t) = B_1 e^{t\rho} + B_2 e^{t(\rho+\Delta)} + B_3 e^{(\rho+\Delta)\frac{t}{2}} + M_1 e^{(\rho-\Delta)t} + M_2 e^{(\rho-\Delta)\frac{t}{2}}, \quad (21)$$

where constants $B_i, i = 1, \ldots 3$ and $M_j, j = 1,2$ are given in the Appendix.

The difference between the size of the forest stock at time T under the finite and infinite time horizon scenarios is:

$$\begin{aligned} F^T(T) - F^s(T) \quad = \quad & \left(\frac{\left[a_3(\Delta + \rho - 2r) - a_5\right] C_1}{a_5^2 - 4a_1a_3} - (F_0 - F^*)\right) e^{(\rho-\Delta)\frac{T}{2}} + \\ & \frac{\left[a_3(\rho - 2r - \Delta) - a_5\right] C_2}{a_5^2 - 4a_1a_3} e^{(\rho+\Delta)\frac{T}{2}}. \end{aligned}$$

Thus, the integration of the difference in (21) over the time interval $[0,T]$ and the addition of the term $e^{-\rho T}\phi\left(F^T(T) - F^s(T)\right)$ leads to the expression in (20). ∎

The total amount $J^T - J^s$ is the minimum quantity the international community has to pay to the forestry country to compensate the loss of revenues due to a better forest conservation. To distribute this total amount $J^T - J^s$ over time the international community can use different methods to transfer an amount $S(t)$ each period of time t to the forestry country. She can choose different specifications for the function $S(t)$ such that:

$$\int_0^T e^{-\rho t} S(t) dt = J^T - J^s. \quad (22)$$

The first distribution method that we can think about, is to transfer exactly the difference in the net revenue as calculated at this period, reduced by the present value of the average difference in the forest salvage value at the end of the horizon. This means that the transfer at each period of time $t \in [0,T]$ is as follows:

$$S(t) = \left(R^T(t) - R^s(t)\right) + e^{-\rho(T-t)} \frac{\phi\left[F^T(T) - F^s(T)\right]}{T}.$$

It can be easily checked that this specification for the instantaneous transfer function satisfies (22).

However the problem that can arise from this distribution method is that the instantaneous amount transferred could be negative for some of the periods during which the difference in revenues that the forestry country can earn by following the optimal deforestation associated with the finite horizon scenario (the first term in the right hand side of the transfer expression) is inferior than the loss in the average salvage value attributed to the forest at the terminal date T (the second term in the right hand side of the transfer expression).

To avoid this problem we propose as a second method to distribute the total welfare loss uniformly over time. According to this method, and taking into account that condition (22) has to be satisfied, the domestic country will receive each period of time $t \in [0, T]$ a fixed amount given by:

$$\bar{S} = \frac{\rho}{1 - e^{-\rho T}} \left(J^T - J^s \right).$$

By accepting to compensate the loss of revenue of the forestry country, the international community encourages him to adopt the sustainable strategies in the short-run without bearing any cost from doing so. Its objective is to concretize the application of these strategies.

However, this compensation does not guarantee that the domestic country will not exploit unsustainably the forest, in the sense that as long as his optimal strategies are different from the sustainable ones the forestry country will be tempted to continue maximizing his revenue using his optimal strategies at the same time he will take advantage of the received transfer.

Therefore, the international community has to bring forward incentive strategies to enforce the application of the sustainable strategies. This issue is addressed in the next subsection.

4.2 The design of incentive strategies

To achieve the goal of sustainable forest exploitation, the international community has to control the $South$'s action, or in other words she has to give him the incentive to follow the sustainable paths for forest size and deforestation rate.

To determine the incentive program that allows the implementation of such deforestation policy, we have to examine the new optimization problem of the forestry country with the transfer received. We want to guarantee that as solution of this optimization problem the forestry country will choose as new optimal deforestation rate the one that will ensure a sustainable forest exploitation. In other words, the optimal path of the forest stock associated to this problem should be given by F^s as described in (7).

This objective cannot be reached unless the forestry country is not sure of the amount of transfer he will receive since it is dependent on his action, mainly on the deforestation rate and the observable forest size at each period of time.

In other words, the international community will use the transfer as an incentive mechanism to guarantee a sustainable forest exploitation. The transfer received by the forestry country at each period of time will be of the form:

$$S^I(t) = \bar{S} + \max\{0, S(D, F)\}, \tag{23}$$

where superscript I stands for *Incentive* mechanism.

The first term in the right hand side is the lump-sum transfer that guarantees that the welfare of the forestry country is not worst of when the transfer mechanism is implemented compared to the case where she managed the forest without any intervention from the international community. The second term is the non-negative amount of transfer that depends on the efforts employed by the forestry country to preserve the forest and respect his engagements to exploit it in a sustainable way.

Following the previous literature (see, e.g. Van Soest & Lensink (2000) and Fredj et al. (2004)) we assume that S is an increasing function of F and decreasing of D. To characterize the exact form that should take the conditional part of the transfers $S(D,F)$, we have to address the new maximization problem of the forestry country while receiving a transfer $S(t)$ as described in equation (23).

The new net revenue function is equal to the old one plus the transfer received each period of time t:

$$\begin{aligned} R^I(D,F) &= R(D,F) + \bar{S} + S(D,F) \\ &= -a_1F^2 + a_2F - a_3D^2 + a_4D - a_5DF + S(D,F) + \bar{S} + a_6, \end{aligned}$$

Next proposition determines the two point boundary problem that must be satisfied by any optimal solution to the optimization problem when the incentive mechanism is implemented.

Proposition 4 *Assuming interior solutions, the optimal control, state and costate variables of the optimization problem when the forestry country receives a transfer function as in (23) satisfy:*

$$D^I = \frac{1}{2a_3}\left(\frac{\partial S}{\partial D}(D^I,F^I) - a_5F^I - \lambda^I + a_4\right), \quad (24)$$

$$\dot{F^I} = -D^I + rF^I; \quad F^I(0) = F_0, \quad (25)$$

$$\dot{\lambda^I} = a_5D^I + 2a_1F^I + (\rho - r)\lambda^I - \frac{\partial S}{\partial F}(D^I,F^I) - a_2; \quad \lambda^I(T) = \phi. \quad (26)$$

Proof. The present value of the Hamiltonian associated with the new optimization problem is given by:

$$H^I = -a_1F^2 + a_2F - a_3D^2 + a_4D - a_5DF + S(D,F) + \lambda^I(-D + rF) + \bar{S} + a_6,$$

where λ^I denotes the new costate variable associated with the forest stock.

Assuming interior solution the necessary conditions of the Maximum Principle of Pontryagin that guarantee the maximization of the new actualized stream of revenues are given by (24)-(26). ■

The incentive mechanism which guarantees the implementation of the sustainable exploitation path is such that F^s, the forest stock path for the infinite horizon optimization problem, given in (7), is the solution of the system (24)-(26).

The dynamic system (25)-(26) can be expressed as a linear system, once the expression of D in (24) has been replaced, if and only if the conditional transfer $S(F, D)$ is quadratic.

The linear dynamic system for variables F and λ can be expressed in matrix form as follows:

$$\begin{bmatrix} \dot{F} \\ \dot{\lambda} \end{bmatrix} = A^I(t) \begin{bmatrix} F \\ \lambda \end{bmatrix} + B^I(t), \tag{27}$$

with boundary conditions

$$F(0) = F_0 \quad \text{and} \quad \lambda(T) = \phi, \tag{28}$$

where both the matrix $A^I(t)$ and the vector $B^I(t)$ depend on the specification of the conditional transfer $S(D, F)$. The conditional transfer function has to be defined appropriately in order to have as optimal path for the state variable the sustainable solution. In other words, we impose that $F^I(t) = F^s(t) = F^* + (F_0 - F^*)\, e^{(\rho-\Delta)\frac{t}{2}}$, $\forall t \in [0, T]$, where $F^I(t)$ denotes the forest stock path obtained as a solution of system (27).

There may exist many transfer functions that can lead to forest stock paths satisfying this condition and that force the South to choose at equilibrium a sustainable deforestation rate leading to the sustainable steady state of the forest size in the long-run. Among the possible specifications for the transfer mechanism we focus on a transfer function linear in the state variable F and quadratic in the control variable D, as suggested by Van Soest and Lensink (2000):

$$S(D, F) = v(t)\, F(t) - w(t)\, D(t) - \frac{z(t)}{2} D^2(t). \tag{29}$$

Functions v, w and z are assumed to be positive to incorporate the hypothesis that the transfer function depends positively on the forest stock and negatively on the deforestation rate. Let us notice however that this function is different from the one proposed in Van Soest and Lensink (2000) as the coefficient v, w, and z are not necessarily constants.

4.2.1 Linear quadratic incentive function

Proposition 5 *There exists at least a solution for $v(t)$, $w(t)$ and $z(t)$ for the incentive transfer mechanism given in (29) that forces the forestry country to choose in the short-run equilibrium a deforestation rate leading to a sustainable forest path.*

Proof. The linear quadratic incentive function given in (29) implies that $\frac{dS}{dF} = v(t)$ and $\frac{dS}{dD} = -w(t) - z(t)\, D(t)$.

Expression (24) can then be rewritten as:

$$D^I(t) = -\frac{a_5 F^I(t) + \lambda^I(t) + w(t) - a_4}{2a_3 + z(t)}.$$

Replacing D^I by its new expression in equations (25) and (26) and using the matrix notation in (27) we have:

$$A^I(t) = \begin{bmatrix} \frac{a_5}{2a_3+z(t)} + r & \frac{1}{2a_3+z(t)} \\ 2a_1 - \frac{a_5^2}{2a_3+z(t)} & \rho - r - \frac{a_5}{2a_3+z(t)} \end{bmatrix},$$

$$B^I(t) = \begin{bmatrix} \frac{w(t)-a_4}{2a_3+z(t)} \\ -v(t) - a_2 - \frac{a_5(w(t)-a_4)}{2a_3+z(t)} \end{bmatrix}.$$

Let us note that both matrix $A^I(t)$ and vector $B^I(t)$ are different from the case without transfers. The matrix $A^I(t)$ only depends on the coefficient function $z(t)$, while the vector $B^I(t)$ depends on all the coefficient functions, $v(t), w(t)$ and $z(t)$.

Our purpose here is to find the right expressions of $v(t), w(t)$ and $z(t)$ interfering in $A^I(t)$ and $B^I(t)$ such that $F^s(t) = F^* + (F_0 - F^*)\, e^{(\rho-\Delta)\frac{t}{2}}$ satisfies the system (27) and the boundary conditions given in (28). Imposing this result, implies that the following expressions must be satisfied:

$$\frac{(\rho-\Delta)(F_0-F^*)}{2} e^{(\rho-\Delta)\frac{t}{2}} = \left(\frac{a_5}{2a_3+z(t)}+r\right)\left(F^*+(F_0-F^*)e^{(\rho-\Delta)\frac{t}{2}}\right)+\frac{\lambda(t)}{2a_3+z(t)}+ \frac{w(t)-a_4}{2a_3+z(t)}, \tag{30}$$

$$\dot{\lambda}(t) = \left(2a_1 - \frac{a_5^2}{2a_3+z(t)}\right)\left(F^*+(F_0-F^*)\, e^{(\rho-\Delta)\frac{t}{2}}\right)+\left(\rho-r-\frac{a_5}{2a_3+z(t)}\right)\lambda(t)- v(t)-\frac{(w(t)-a_4)\, a_5}{2a_3+z(t)}-a_2. \tag{31}$$

From (30) in the system above we can obtain the expression of $w(t)$ as a function of $\lambda(t)$ and $v(t)$:

$$w(t) = \frac{((2a_3+z(t))(\rho-\Delta-2r)-2a_5)(F_0-F^*)}{2} e^{(\rho-\Delta)\frac{t}{2}} - ((2a_3+z(t))\, r + a_5)\, F^* + a_4 - \lambda(t). \tag{32}$$

Substituting this expression of $w(t)$ in equation (31), after some manipulations we find the following expression for the dynamics of the shadow price:

$$\dot{\lambda}(t) = (\rho - r)\lambda(t) - v(t) + F^*(2a_1 + ra_5) - a_2 + \frac{(4a_1 + (2r+\Delta-\rho)\, a_5)(F_0 - F^*)}{2} e^{(\rho-\Delta)\frac{t}{2}}. \tag{33}$$

Let us note that this differential equation does not depend on functions $w(t)$ and $z(t)$ but only depends on $v(t)$, the rewarding coefficient related to the forest stock in the transfer function.

Fixing an expression for $v(t)$ equation (33), together with the final condition $\lambda(T) = \phi$ allows us to solve for the optimal path of the new shadow value, denoted by $\lambda^I(t)$. Replacing $\lambda(t)$ in equation (32) by $\lambda^I(t)$, we obtain a new expression for $w(t)$ which depends on $z(t)$. Let us notice that we do need to propose an explicit form of $z(t)$ to find the necessary form of $w(t)$, in addition to the proposed form of $v(t)$. In fact for each possible choice of $v(t)$, we obtain infinite possibilities for $w(t)$, depending on the functional form of $z(t)$.

The new path of the deforestation rate involved by the incentive mechanism is then

$$D^I = -\frac{a_5 F^s(t) + \lambda^I(t) + w(t) - a_4}{2a_3 + z(t)}, \tag{34}$$

where $F^I(t)$ has been replaced by the sustainable optimal path $F^s(t)$.

Given that the implementation of the transfer mechanism leads the forestry country follows a sustainable exploitation of the forest, $F^I(t) = F^s(t), \forall t \in [0, T]$, and according to the dynamics of the forest stock described in equation (5), it is straightforward to deduce that the only possibility to achieve the result is when the deforestation path coincides with the sustainable deforestation rate, that is, $D^I(t) = D^s(t), \forall t \in [0, T]$. Comparing the optimal paths of these deforestation rates as described in equations (8) and (34), we conclude that this objective can be met if and only if

$$2a_3 \left[\lambda^I(t) + w(t)\right] = \left[2a_3 + z(t)\right] \lambda^s(t) + z(t) \left[a_5 F^s(t) - a_4\right].$$

This relation between the incentive and sustainable equilibria regarding the shadow value of the forest, implies that the coefficient functions $v(t), w(t)$ and $z(t)$ related to the forest size and the deforestation rate in the transfer function make the sustainable exploitation of the forest possible acting on the shadow value of the forest. ■

Remark 6 *The choice of the positive functions $v(t)$ and $z(t)$ must be such that the corresponding function $w(t)$ given in (32) attains positive values along the whole time horizon $[0, T]$.*

Remark 7 *Function $v(t)$ can be chosen constant through the time interval $[0, T]$, but in this case the functions $w(t)$ and $z(t)$ cannot be, in general, simultaneously constant in order to guarantee that the incentive mechanism attains its objective. In the particular case where $w(t) = \bar{w}, z(t) = \bar{z}\ \forall t \in [0, T]$, the transfer mechanism can induce the forestry country to follow a sustainable exploitation of the forest if and only if the following relationship between $\bar{w}$ and $\bar{z}$ holds:*

$$\bar{w} = \frac{\left[(2a_3 + \bar{z})(\rho - \Delta - 2r) - 2a_5\right](F_0 - F^*)}{2} e^{(\rho - \Delta)\frac{T}{2}} - \left[(2a_3 + \bar{z})\, r + a_5\right] F^* + a_4 - \phi.$$

4.2.2 Implementability of the incentive strategies

Until now we have focused on the characterization of the transfer mechanism which guarantees that the forestry country applies a sustainable forest exploitation. The forestry country receives at least a lump sum transfer which ensures that his welfare is not worst when the transfer mechanism is implemented than when it is not (see equation (23)). Then, the *South* does not have any incentive to deviate from the optimal policy prescribed by the solution of the new optimization problem when the transfer mechanism applies. Therefore, to ensure the implementability of this incentive mechanism we need to determine the *North*'s minimum budget required to apply these incentive strategies.

The total amount of transfers which flow from *North* to *South* when the linear quadratic incentive function has been implemented is given by:

$$\int_0^T e^{-\rho t} S^I(t)\,dt = \int_0^T e^{-\rho t}\left[\bar{S} + S(F^s(t), D^s(t))\right]\,dt =$$
$$\frac{\bar{S}}{\rho}(1 - e^{-\rho T}) + \int_0^T e^{-\rho t}\left[v(t)F^s(t) - w(t)D^s(t) - \frac{z(t)}{2}(D^s(t))^2\right]\,dt.$$

The total amount of transfer depends on the specification of the coefficients functions, $v(t), w(t)$ and $z(t)$ that determine the linear quadratic transfer function. Therefore, the final expression of the amount of transfer can be calculated only once a functional specification for these coefficient functions has been fixed.

The next example shows how the transfer mechanism can be implemented for one of the simplest specifications of the coefficient functions.

Example 8 *Let consider a linear specification for the coefficient function* $v(t) = at + b$, *where* a *and* b *are constants.*

Solving for equation (33) *taking into account the final condition* $\lambda(T) = \phi$, *the optimal path of the forest shadow value is given by:*

$$\lambda^I(t) = \frac{a}{\rho - r}t + He^{-(T-t)(\rho - r)} - \frac{(4a_1 + (2r + \Delta - \rho)a_5)(F_0 - F^*)}{\Delta - 2r + \rho}e^{(\rho - \Delta)\frac{t}{2}} -$$
$$\frac{(2a_1 + ra_5)(\rho - r)F^* - a - (\rho - r)(b + a_2)}{(\rho - r)^2},$$

where

$$H = \phi - \frac{aT}{\rho - r} + \frac{F^*(\rho - r)(2a_1 + ra_5) - (b + a_2)(\rho - r) - a}{(\rho - r)^2} +$$
$$\frac{(F_0 - F^*)(4a_1 + (2r + \Delta - \rho)a_5)e^{(\rho - \Delta)\frac{T}{2}}}{\Delta - 2r + \rho}.$$

Replacing $\lambda(t)$ *by its optimal expression* $\lambda^I(t)$ *in equation (32) we find*

$$w(t) = \frac{(8a_1 - 4a_5(\rho - 2r) - (2a_3 + z(t))(\Delta - 2r + \rho)(2r + \Delta - \rho))(F_0 - F^*)}{2(\Delta - 2r + \rho)}e^{(\rho - \Delta)\frac{t}{2}} -$$
$$\frac{a}{\rho - r}t - He^{(r-\rho)(T-t)}. \quad (35)$$

Finally replacing the expressions of $F^s(t), \lambda^I(t)$ *and* $w(t)$ *in equation (34) we* *have*

$$\begin{aligned} D^I(t) &= \left(r + \frac{rz(t)}{2a_3 + z}\right) F^* - \frac{2a_3 + z(t)}{4a_3 + z(t)} \left(\left(\rho - \Delta - 2r\right)\right) \left(F_0 - F^*\right) e^{(\rho-\Delta)\frac{t}{2}} \\ &= rF^* - \frac{\left(\rho - \Delta - 2r\right)\left(F_0 - F^*\right)}{2} e^{(\rho-\Delta)\frac{t}{2}}. \end{aligned}$$

This expression is equal to $D^s(t)$, $\forall t \in [0,T]$, whatever the functional form of $z(t)$.

To determine the total amount of transfers that the South receives from the North we assume a constant specification for function $z(t) = \bar{z}$, for all t. This amount in this case is given by:

$$\int_0^T e^{-\rho t} S^I(t)\, dt = \int_0^T e^{-\rho t} \left[\bar{S} + (at+b)F^s(t) - w(t)D^s(t) - \frac{\bar{z}}{2}\left(D^s(t)\right)^2\right] dt,$$

where $w(t)$ has to be replaced by its expression in (35). After some easy but tedious computations the above expression can be rewritten as:

$$\begin{aligned} &\frac{1}{\rho}\left[\bar{S}+bF^* - \frac{\bar{z}}{2}(rF^*)^2\right](1-e^{-\rho T}) + \frac{aF^*}{\rho(\rho-r)}(1-(1+\rho T)e^{-\rho T}) + HF^*(e^{(r-\rho)T} - e^{-\rho T}) + \\ &\frac{2a(F_0-F^*)}{(\rho-r)(\rho+\Delta)}\left(1-(1+\frac{\rho+\Delta}{2}T)e^{-\frac{(\rho+\Delta)}{2}T}\right) - H(F_0-F^*)(e^{-\frac{(\rho+\Delta)}{2}T} - e^{(r-\rho)T}) + \\ &\frac{2(F_0-F^*)}{\rho+\Delta}\left[b+\frac{rF^*(8a_1-4a_5(\rho-2r)+2a_3(2r+\Delta-\rho)(\Delta-2r+\rho))}{2(\Delta-2r+\rho)}\right](1-e^{-\frac{(\rho+\Delta)}{2}T}) + \\ &\frac{(\rho-\Delta-2r)(F_0-F^*)^2(16a_1-8a_5(\rho-2r)-(4a_3+\bar{z})(2r+\Delta-\rho)(\Delta-2r+\rho))}{8\Delta(\Delta-2r+\rho)}(1-e^{-\Delta T}). \end{aligned}$$

5 Numerical Illustration

In this section we provide a numerical example for which the optimal time paths of the forest stock, deforestation rate and revenues have been computed for the finite and infinite planning horizon. After that, we show the optimal path of the transfers which allows the donor community to induce the forestry country to follow a sustainable deforestation path.

The value of the parameters, which are mostly inspired from Van Soest and Lensink (2000), are the following:

$\bar{P} = 45000$, $\bar{P}_A = 150$, $\bar{Z} = 60$, $F_0 = 2000$, $\theta = 20$, $\gamma = 0.15$, $\alpha = 0.1$, $\beta = 0.1$, $\rho = 0.1$, $r = 0.2$, $\Phi = 17000, T = 10$.

For these values the steady-states for the forest stock and the shadow price of the forest are:

$$F^* = 1717.1, \quad \text{and} \quad \lambda^* = 25204.$$

The time paths of the forest stock, the deforestation rate, the shadow price and the revenues in the infinite horizon or sustainable scenario are given by:

$$\begin{aligned}
F^s(t) &= 1717.1 + 282.89e^{-0.93t}, \\
D^s(t) &= 343.42 + 318.53e^{-0.93t}, \\
\lambda^s(t) &= 25204 - 18682e^{-0.93t}, \\
R^s(t) &= [-5.15e^{-1.85t} + 7.32e^{-0.93t} + 22.62] \times 10^6.
\end{aligned}$$

For the finite horizon scenario the time paths are:

$$\begin{aligned}
F^T(t) &= 1717.10 + 282.92e^{-0.93t} - 0.02e^{1.03t}, \\
D^T(t) &= 343.42 - 318.55e^{-0.9260t} + 0.0197e^{1.03t}, \\
\lambda^T(t) &= 25204 - 18684e^{-0.93t} - 0.2872e^{1.03t}, \\
R^T(t) &= -0.001e^{0.1t} - 0.01e^{2.05t} + 556.84e^{1.03t} + \\
&\quad [-5.16e^{-1.85t} + 7.32e^{-0.93t} + 22.62] \times 10^6.
\end{aligned}$$

Figures (1) to (4) show these time paths. In all the figures continuous line corresponds to the sustainable scenario, while the dash line denotes the finite horizon scenario.

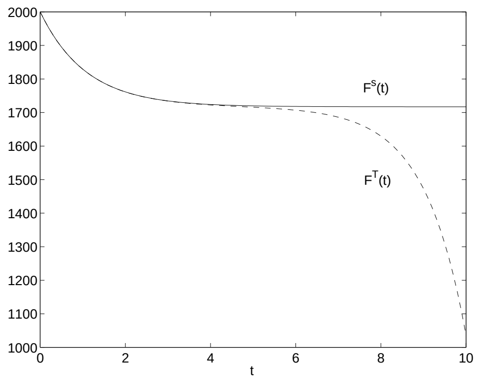

Figure 1: Forest stock time paths

As figures (1) and (2) show, in the sustainable scenario, both the forest stock and the deforestation rate time paths are decreasing functions, and converge towards their steady-state values.

In the finite horizon scenario, as in the sustainable one, the forest stock decreases during the whole time horizon. However, in the finite horizon scenario the forest stock path is a convex function during an initial period of time, after which it becomes concave. That is, for times lower than 3 (see, figure (1), dash line) the forest stock decreases at an increasing rate when t increases. When the time variable is greater than 3, the speed of the forest's stock destruction decreases as t increases.

This behavior is a direct result of the deforestation policy applied. The time path of the deforestation rate is also S-shaped. Hence, it is always increasing function of time; concave for an initial period and convex afterwards (see, figure (2) dash line).

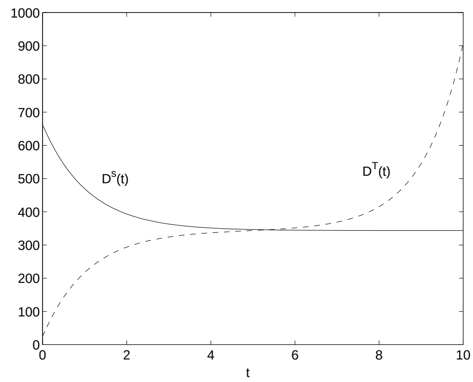

Figure 2: Deforestation time paths

Comparing the time paths for the forest stock under the two different scenarios, we notice that the forest stock remains almost the same in both scenarios until the time is approximately equal to 4. From this point of time to the end of the horizon ($T = 10$), the forest stock is always larger in the sustainable scenario compared to the finite horizon one.

From the comparison of the deforestation rate paths we can conclude that the forestry country starts applying a stronger deforestation policy in the infinite time problem than in the finite one. However, this behavior is inverted from the middle of the time horizon until the end.

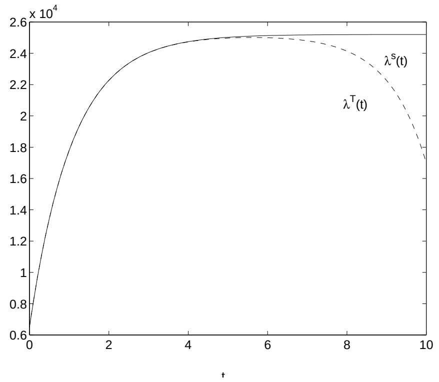

Figure 3: Shadow price time paths

Figure (3) indicates that the shadow price of the forest stock is always increasing in the sustainable scenario. This behavior can be interpreted as the forest stock is decreasing, the value the forestry country gives to an additional unit of forest is increasing. However, the shadow price of the forest stock is U-shaped in the finite horizon scenario. Even though in this scenario the forest stock also decreases along the whole time horizon (see figure (1) dash line), the forestry country when applying a short run policy evaluates less any additional unit of forest during an initial period of time, after which the evaluation is reversed. From the comparison of the time paths of the shadow prices we derive exactly the same behavior as when comparing the forest stock paths for both scenarios.

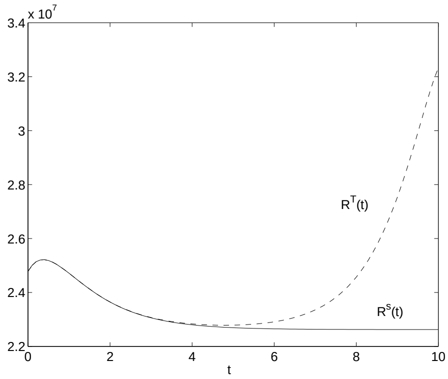

Figure 4: Revenues time paths

The instantaneous revenues in the sustainable scenario increase during an initial period of time, after which they decrease towards a stationary value. On the other hand, in the finite time horizon scenario they present an inverted U-shape, except for a very short initial time period (see, Figure (4)).

As this figure shows the revenues from time 5 are much greater in the finite horizon scenario than in the sustainable one. The difference between the time paths of the instantaneous revenues makes it more profitable for the forestry country to follow the deforestation policy prescribed by the solution of the finite horizon optimization problem than to apply the sustainable policy.

As we have already noted in Section 4, to encourage the forestry country to adopt a sustainable forest exploitation, the international community has to pay the difference between the total net revenues for the infinite and finite horizon problems from time 0 until T actualized at rate ρ. This difference amounts $J^T - J^s = 8.88 \times 10^5$ for this example.

To achieve the goal of sustainable forest exploitation the international community will use the transfer as an incentive mechanism to force the forestry country following the sustainable paths for forest size and deforestation rate.

The transfer received by the forestry country at each period of time is defined in (23), where the function $S(D, F)$ is assumed to be linear in the forest stock and linear and quadratic in the deforestation rate, as established in (29).

As in Example **??**, we consider a linear specification for function $v(t) = at+b$, where the coefficients a and b are fixed at 50 and 750, respectively. We also consider a constant specification for the function $z(t) = 5$ for all t. Under these assumptions the time path for the transfer function is given by:

$$S^I(t) = [16.81 - 0.86t - (1.45t + 3.35)e^{-0.93t} + 2.54e^{-1.85t} - 2.41e^{0.1(10-t)}] \times 10^5.$$

Figure (5) shows the time path of the incentive mechanism transfers. The amount of transfers flowing from the $North$ to the $South$ decreases during a first period of time. A second period of increasing transfers follows, after which the amount of transfers starts to decrease again for the rest of the time horizon.

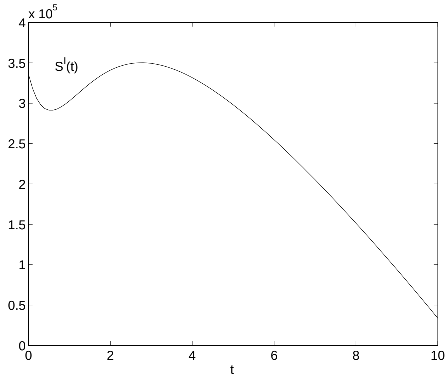

Figure 5: Transfers time path

The total amount of transfers the forestry country receives for the whole planning period $[0, 10]$ is 1.03×10^7. Therefore, the international community has to have at least this budget at her disposal in order to implement this transfer mechanism.

6 Conclusion

Using a detailed model showing the trade-off between the agriculture and forestry use of forests (deforestation and agriculture productivity), we showed that the international community can encourage the forestry countries to participate in a program aiming at a better forest conservation while compensating him for the revenue loss he can bare from doing so. We also showed that using incentive transfer mechanism, the donor community or North can enforce a sustainable forest exploitation in the short-run. We proved that a linear-quadratic specification of the transfer mechanism exists and is a possible solution to this issue.

7 Appendix

- **Constants** $A_i, i = 1,2,3$ **in Proposition 1**.

$$
\begin{aligned}
A_1 &= -\frac{(F_0-F^*)^2\left(4a_1+(2a_5+a_3\,(2r+\Delta-\rho))\,(2r+\Delta-\rho)\right)}{4},\\
A_2 &= \frac{(F_0-F^*)\left(F^*\left(a_5^2-4a_1a_3\right)+\lambda^*\left(a_5+a_3\,(2r+\Delta-\rho)\right)+2a_2a_3-a_4a_5\right)}{2a_3},\\
A_3 &= \frac{(a_4-a_5F^*-\lambda^*)\,(a_4-a_5F^*+\lambda^*)+4a_3\left(-a_1(F^*)^2+a_2F^*+a_6\right)}{4a_3}.
\end{aligned}
$$

- **Constants** $C_i, i = 1,2$ **and** $B_j, j = 1,\ldots,6$ **in Proposition 2**.

$$C_1 = \frac{(\phi-\lambda^*)\left((2r-\rho+\Delta)a_3+a_5\right)-\left(a_5^2-4a_1a_3\right)(F^*-F_0)\,e^{(\Delta+\rho)\frac{T}{2}}}{\left(a_3\,(\Delta+2r-\rho)+a_5\right)e^{(\rho-\Delta)\frac{T}{2}}+\left(a_3\,(\Delta-2r+\rho)-a_5\right)e^{(\Delta+\rho)\frac{T}{2}}},$$

$$C_2 = \frac{\left(a_3\,(\Delta-2r+\rho)-a_5\right)(\phi-\lambda^*)+\left(a_5^2-4a_1a_3\right)(F^*-F_0)\,e^{(\rho-\Delta)\frac{T}{2}}}{\left(a_3\,(\Delta+2r-\rho)+a_5\right)e^{(\rho-\Delta)\frac{T}{2}}+\left(a_3\,(\Delta-2r+\rho)-a_5\right)e^{(\Delta+\rho)\frac{T}{2}}},$$

$$B_1 = -\frac{C_2\left(a_3a_5^2\Delta^2C_2+2C_1\left(a_5^2-4a_1a_3\right)\left(-4a_1+2a_5\,(-2r+\rho)+a_3\left(\Delta^2-(2r-\rho)^2\right)\right)\right)}{4\left(a_5^2-4a_1a_3\right)^2},$$

$$B_2 = \frac{C_2^2\left(a_3a_5^2\Delta^2+\left(a_5^2-4a_1a_3\right)\left(2\,(a_5+2a_1)+a_3\,(2r+\Delta-\rho)\right)(2r+\Delta-\rho)\right)}{4\left(a_5^2-4a_1a_3\right)^2},$$

$$B_3 = -\frac{C_2\left(\left(a_5^2-4a_1a_3\right)\lambda^*+\left(2a_2a_3-a_4a_5+\left(a_5^2-4a_1a_3\right)F^*\right)\left(a_5+a_3\,(\Delta+2r-\rho)\right)\right)}{2a_3\left(a_5^2-4a_1a_3\right)},$$

$$B_4 = \frac{C_1^2\left(4a_1+(2a_5+a_3\,(2r-\Delta-\rho))\,(2r-\Delta-\rho)\right)}{4\left(a_5^2-4a_1a_3\right)},$$

$$B_5 = -\frac{C_1\left(\left(a_5^2-4a_1a_3\right)\lambda^*+\left(2a_2a_3-a_4a_5+\left(a_5^2-4a_1a_3\right)F^*\right)\left(a_5+a_3\,(2r-\rho-\Delta)\right)\right)}{2a_3\left(a_5^2-4a_1a_3\right)},$$

$$B_6 = A_3.$$

- **Constants** $M_i, i = 1,2$ **in Proposition 3**.

$$M_1 = \frac{C_1^2\left(4a_1 + \left(2a_5 + a_3\left(2r - \Delta - \rho\right)\right)\left(2r - \Delta - \rho\right)\right)}{4\left(a_5^2 - 4a_1a_3\right)} + \frac{\left(F_0 - F^*\right)^2\left(4a_1 + \left(2a_5 + a_3\left(2r + \Delta - \rho\right)\right)\left(2r + \Delta - \rho\right)\right)}{4},$$

$$M_2 = -\frac{C_1\left(\left(a_5^2 - 4a_1a_3\right)\lambda^* + \left(2a_2a_3 - a_4a_5 - \left(4a_1a_3 - a_5^2\right)F^*\right)\left(a_5 + a_3\left(2r - \rho - \Delta\right)\right)\right)}{2a_3\left(a_5^2 - 4a_1a_3\right)} - \frac{\left(F_0 - F^*\right)\left(F^*\left(a_5^2 - 4a_1a_3\right) + \lambda^*\left(a_5 + a_3\left(2r + \Delta - \rho\right)\right) + 2a_2a_3 - a_4a_5\right)}{2a_3}.$$

References

[1] Amelung, T. and M. Diehl (1992), “Deforestation of Tropical Forests: Economic Causes and Impact on Development”, *Kieler Studien, Institut fur Weltwirtschaft an der Universitat Kiel*, No. 241, J.C.B. Mohr Tubingen.

[2] Barbier, E. B., J. C. Burgess and A. Markandya (1991), “The Economics of Tropical Deforestation”, *Ambio*, vol. 20, 55-58.

[3] Barbier, E. B. and J. C. Burgess (1997), “The Economics of Forest Land Use Options”, *Land Economics*, vol. 73, 174-195.

[4] Barbier, E.B. and M. Rauscher (1994), “Trade, Tropical Deforestation and Policy Interventions”, *Environmental and Resource Economics*, vol. 4, 75-90.

[5] Barrett, S. (1994), “The Biodiversity Supergame”, *Environmental and Resource Economics*, vol. 4, 111-122.

[6] Beltratti, A. (1995), “Consumption of Renewable Evironmental Assets, International Coordination and Time Preference, in *Annals of the ISDG*, vol. 2, pp. 47-64.

[7] Chomitz K. M. and K. Kumari (1998), “The Domestic Benefits of Tropical Forests: A Critical Review”, *The World Bank Research Observer*, vol. 13 (1), 13-35.

[8] Cline, W.R. (1992), *The Economics of Global Warming*, Institute for International Economics, Washington D.C.

[9] Earth observatory (2001), Tropical deforestation, http://earthobservatory.nasa.gov/cgi-bin/texis/webinator/printall?/Library/Deforestation/index.html.

[10] Ehtamo, H. and R. P. Hämäläinen (1986), “On affine incentives for dynamic decision problems”. In T. Basar (Ed.), Dynamic games and applications in economics. Berlin: Springer (pp. 47-63).

[11] Ehtamo, H. and R. P. Hämäläinen (1989), "Incentive strategies and equilibria for dynamic games with delayed information", *Journal of Optimization Theory and Applications,* vol. 63 (3), 355-370.

[12] Ehtamo, H. and R. P. Hämäläinen (1993), "A cooperative incentive equilibrium for a resource management problem", *Journal of Economic Dynamics and Control,* vol. 17, 659-678.

[13] Ehui, S.K., Th.W. Hertel and P.V. Preckel (1990), "Forest Resource Depletion, Soil Dynamics, and Agricultural Development in the Tropics", *Journal of Economic and Environmental Management*, vol. 18 (2), 136-154.

[14] Earth observatory (2001), Tropical deforestation, http://earthobservatory.nasa.gov/cgi-bin/texis/webinator/printall?/Library/Deforestation/index.html.

[15] Fredj, K., G. Martín-Herrán and G. Zaccour (2004), "Slowing deforestation pace through subsidies: a differential game", *Automatica*, vol. 40 (2), 301-309.

[16] Jørgensen, S. and G. Zaccour (2001), "Incentive equilibrium strategies and welfare allocation in a dynamic game of pollution control", Automatica 37 (1), 29-36.

[17] Kaimowitz, D. and A. Angelsen (1999), "Rethinking the Causes of Deforestation: Lessons from Economic Models", *The World Bank Research Observer*, vol. 14 (1), 73-98.

[18] Martín-Herrán, G., P. Cartigny, E. Motte and M. Tidball (2002), "Deforestation and Foreign Transfers: A Differential Games Approach", Proceedings of the Tenth International Symposium on Dynamic Games and Applications, L.A. Petrosjan and N.A. Zenkevich (eds.), St. Petersburg (Rusia), vol. II, 512-525.

[19] Matthews, E. (2001), "Understanding the Forest Resources Assessment", *World Resource Institute Study*

[20] Mohr, E. (1996), "Sustainable Development and International Distribution: Theory and Application to Rainforests", *Review of International Economics*, vol. 4 (2), 152-171.

[21] Montgomery, A. C (2002), "Ranking the Benefits of Biodiversity: An Exploration of Relative Values", *Journal of Environmental Management*, vol. 65, 313-326.

[22] Southgate, D. (1990), "The Causes of Land Degradation Along 'Spontaneous' Expanding Agricultural Frontiers", *Land Economics*, vol. 66, 93-101.

[23] Southgate, D., R. Sierra and L. Brown (1991), "The Causes of Tropical Deforestation in Ecuador: A Statistical Analysis", *World Development*, vol. 19, 1145-1151.

[24] Stähler, F. (1996), "On International Compensations for Environmental Stocks", *Environmental and Resource Economics*, vol. 8, 1-13.

[25] Van Soest, D. (1998), *Tropical Deforestation: An Economic Perspective*, LAbyrint publication, The Netherlands.

[26] Van Soest, D. and R. Lensink (2000), "Foreign Transfers and Tropical Deforestation: What Terms of Conditionality?", *American Journal of Agricultural Economics*, vol. 82, 389-399.

[27] Von Amsberg, J. (1994), "Economic Parameters of Deforestation", *World Bank Policy Research Working*, Paper no. 1350, World Bank Policy Research Department, World Bank, Washington D.C.

[28] Wilson, E. O. (1992), *The diversity of life*, Harvard University Press, Cambridge, MA.

[29] World Commission on Environment and Development (1987), *Our Common Future*, Oxford University Press, Oxford.