Environmental and Trade Policies in a Two-Sector Endogenous Growth Model with Pollution
(Preliminary Version)
by

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Abstract

In this paper, the interactions between environmental policy and trade policy as well as their effect on growth in economies which care about the environment are examined, in the context of a two-sector endogenous growth model, where an infinitely-lived representative agent accumulates two types of capital, physical capital, and human capital, which are used to produce two goods. We study (and compare) the competitive equilibrium of both the small open economy and the closed economy, focusing on factor intensity conditions in both sectors and the behavior of relative prices of final goods. As expected, we show that the dynamic behavior of the equilibrium path depends heavily upon factor intensity conditions. We examine the impact of environmental policy on prices and growth. Also, we determine the optimal environmental policy and study how it does change when the economy opens to the international market. Also, we examine how it does interact with trade policy.

Keywords: pollution; endogenous growth; small open economy; closed economy; optimal pollution tax;

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1 Introduction

There is already a large literature studying the effect of pollution, as an externality from production that decreases utility, on the optimal rate of growth. When the negative effect of pollution on welfare is taken into account, the optimal rate of growth always decreases, and it may even happen that optimal sustainable growth is not feasible, as shown in Stokey [21]. Therefore, environmental concerns will eventually limit growth.

While the link between growth and the environment is important, trade may also change environmental outcomes through different ways. Trade may encourage a relocation of polluting industries from countries with strict environmental policy to those with less stringent policy, which in turn may impact on global pollution or on environmental policy because of concerns of international competitiveness. In this context, it is our purpose to examine the interactions between environmental policy and trade policy as well as their effect on growth in economies which care about the environment.[1]

Previous studies on growth and the environment have either assumed that there is a infinitely lived representative agent and that the political system acts in its interest, or have considered instead that preferences are heterogeneous and that there may exist intragenerational conflicts of interest, as in John and Pecchenino [14]. In this case, there may be room for lobbying in favor of some outcomes, and, consequently, the optimal policy may not be implemented. In a static context, we should mention Fredriksson [11], Fredriksson [12], Fredriksson and Svensson [13], Damania, Fredriksson and List [9], among others.[2]

In this paper, we investigate the dynamic behavior of a two-sector endogenous growth model, where an infinitely-lived representative agent accumulates two types of capital, physical capital and human capital, which are used to produce two goods, and where the social planner represents the interests of society. These two goods are produced under different constant returns to scale technologies. Moreover, it is assumed that one of the sectors (sector 1 or T) generates pollution as a by-product of production. A tax on output that pol-

[1] See Copeland and Taylor [6].

[2] This will be considered in future research.

lutes is levied, which is taken as given by the agent. The revenues generated by this tax represent real transfers to the agent from the government.

In a first step, we assume that this economy is a small open economy, while in a second step we consider the case of a closed economy. We study the competitive equilibrium in these economies, taking the pollution externality as given, focusing on factor intensity conditions in both sectors and the behavior of relative prices of final goods. As expected, we show that the dynamic behavior of the equilibrium path depends heavily upon factor intensity conditions. We examine the impact of environmental policy on prices and growth. Also, we determine the optimal environmental policy and study how it does change when the economy opens to the international market. Also, we examine how it does interact with trade policy. We consider both non separable and separable preferences, as they reflect different views of society with respect to environmental preservation.

We show that the optimal pollution tax in a small open economy with perfect capital mobility and non separable preferences is zero in the long-run. In contrast, in a closed economy, a positive optimal pollution tax exists. With separability, reflecting more conservative preferences towards environment, we show that the optimal pollution tax would imply the equality between the rate on traded bonds and the discount rate. In a closed economy, the optimal tax is the highest possible or the one that eliminates growth. With non separable preferences, the optimal tax is compatible with positive growth of the economy. Finally, in a second-best world, we consider the case of a small open economy with a tariff on imports. In this case, the second-best optimal pollution tax does not vanish asymptotically, but, instead, should be equated to the tariff rate.

In the context of the endogenous growth literature, our results are related to those in Turnovsky [22], and Sampaolesi [20]. While Turnovsky [22] develops an endogenous growth model for a small open economy with perfect capital mobility but without externalities, in the case of Sampaolesi [20], the small open economy is completely specialized in the production of a pollution-intensive good, abstracting from the possiblity of lending or borrowing from the outside world. In this case, the optimal environmental tax reduces growth, as expected.

The optimal tax will be positive whenever the pre-tax competitive path lies above the efficient path. Finally, our results can also be related to the literature on optimal taxation in representative-agent models. In particular, Judd [17], Chamley [5], Jones, Manuelli and Rossi [15], [16], show that the optimal tax rate on capital income is zero asymptotically whenever the planner solves a unconstrained problem, that is, without fixed factors, restrictions on tax rates or regimes, that can be due to political or other types of constraints. Moreover, these results also hold in the context of a (small) open economy, as shown in Correia [7], [8].

The remainder of the paper is organized as follows. Section 2 presents the two-sector endogenous growth model for a small open economy and a closed economy and examines the corresponding competitive equilibria with non separable preferences. Section 3 discusses the social planner's problem with non separable preferences. Section 4 considers the case of separable preferences. Section 5 studies the optimal pollution tax in a second-best context characterized by a tariff on imports, and, finally, Section 6 concludes the paper. Technical details are presented in the appendix.

2 Competitive Equilibrium

2.1 Small Open Economy

The economy is characterized by a constant population, normalized to one. The representative agent accumulates two types of capital for rental at the competitively determined rental rate. The first is physical capital, K, which is traded, and the second is human capital, H, which is nontraded. Neither of these capital goods is subject to depreciation for simplicity.

These two forms of capital are used by the agent to produce a tradable good, Y_T, taken to be the numéraire, and a nontradable good, Y_N. We assume linearly homogeneous production functions:

$$Y_T = aK_T^{\alpha} H_T^{1-\alpha}; \quad 0 < \alpha < 1 \tag{1}$$

$$Y_N = bK_N^{\beta} H_N^{1-\beta}; \quad 0 < \beta < 1 \tag{2}$$

where K_T, H_T, K_N and H_N denote the allocation of the respective capital good to the production of the traded good and nontraded good, respectively. Both forms of capital are costlessly and instantaneously mobile across the two sectors, with the sectoral allocations being constrained by:

$$K_T + K_N = K \qquad H_T + H_N = H \tag{3}$$

The fact that the two production functions are linearly homogeneous in the two production factors, K and H, is critical for an equilibrium with steady endogenous growth to exist.

Let p denote the relative price of nontraded good in terms of traded good. Production of good Y_T generates local pollution.[3] σY_T is the total damage from pollution, where $0 < \sigma < 1$ is an exogenously given damage coefficient of pollution per unit of production of good Y_T:

$$P = \sigma Y_T \tag{4}$$

The government is restricted to one environmental policy instrument, a pollution tax, $\tau \in T \subset R_+$, levied per unit of pollution produced in the tradable sector, in order to internalize the environmental externality.[4] Total revenue from pollution taxes equals $R(\tau)$:

$$R(\tau) = \tau\sigma Y_T \tag{5}$$

Also, it is assumed that the revenues generated by this tax represent real transfers to the agent from the government, denoted by $T = \tau\sigma Y_T$.

We assume that the accumulation of traded capital

$$\dot{K} = I \tag{6}$$

involves costs represented by:

$$\Gamma(I, K) = I\left(p^* + h\frac{I}{2K}\right) \tag{7}$$

where p^* represents the exogenous world price of new traded capital and h represents adjustment costs.[5] In order to undertake gross investment of I units of

[3] We build on Turnovsky[22] and introduce a pollution externality and environmental policy.
[4] See Stokey [21], among others.
[5] Since there are no trade frictions at this point, $p_T = p_T^* = p^*$.

capital, Ip^* units of output must be set aside to be installed as capital, together with $h.\frac{I^2}{2K}$ units which are used during installation. Thus, gross investment at rate I has an opportunity cost of $I\left(p^* + h\frac{I}{2K}\right)$ units of output. Total installation cost $h\frac{I^2}{2K}$ is nonnegative, with a minimum value of zero when gross investment is zero. The linear homogeneity of this function is also necessary in order for a steady-state equilibrium growth path to be sustained.

In addition to the two types of capital, the agent also accumulates net foreign bonds, B, that pay an exogenously given world interest rate, r. Thus, the agent´s budget constraint is given by:

$$\dot{B} = (1 - \tau\sigma)\, Y_T + pY_N + rB + T - C_T - pC_N - \Gamma(I, K) - p\dot{H} \tag{8}$$

where T denotes the exogenous lump-sum transfer from the government, and C_T and C_N are the agent´s consumption of the traded and nontraded goods, respectively. C_T and investment in physical capital are perfect substitutes, as well as C_N and investment in human capital. In other words, C_T and I come from a single output stream of goods, Y_T, and the same holds for C_N and $\dot{H}$, which come both from Y_N.

The representative agent´s decision problem consists of choosing C_T, C_N, K_T, K_N, H_T, H_N, I, $\dot{H}$ to maximize the present value of the stream of utility, as follows:[6]

$$U = \int_0^{\infty} \frac{1}{\gamma} \left(C_T^{\theta} C_N^{1-\theta} Q^{\mu}\right)^{\gamma} e^{-\rho t} dt, \qquad 0 \leq \theta \leq 1 \tag{9}$$

subject to the constraints (1), (2), (3), (6), (7), (8) and the initial stocks of assets K_0, H_0 and B_0.

The environment affects society´s welfare, but not the production technology. The effect on welfare enters directly in the utility function. Individual utility depends on consumption of traded and nontraded goods as well as on the flow of environmental services, Q, measured in quality units. This flow is given by

$$Q = 1/P \tag{10}$$

where P is the flow of pollution defined in (4), a production externality with a negative impact on the flow of environmental services provided. As $0 < \sigma < 1$

[6] The utility function of the representative agent is assumed to be of the constant elasticity of marginal utility type. The intertemporal elasticity of substitution is given by $\frac{1}{1-\gamma}$.

the flow of pollution is less than proportional to total production of the tradable good. This implies that the quality of the environment is a decreasing function of the physical and human capital stocks. Even ignoring regenerating processes that can stop the deterioration of the environment, equation (10) need not to lead to catastrophe in finite time, as long as Q approaches zero asymptotically (that is, $\lim\limits_{K_T \to \infty} Q = 0$; $\lim\limits_{H_T \to \infty} Q = 0$).

Each agent perceives P as exogenous, although, in the aggregate, it varies over time. Therefore, the problem represents the competitive equilibrium for this economy. Although P is taken as given, not only the pollution tax affects the competitive outcome, but also the externality does.

For the utility function to be increasing and strictly concave in C_T, C_N and Q it is assumed that $\mu > 0$, $\gamma < 1$, $\mu\gamma < 1$ and $\gamma(1+\mu) < 1$. The value of γ depends on the relationship between consumption and the flow of environmental services. The sign of the cross derivatives $\frac{\partial^2 U}{\partial C_T \partial Q} = \gamma\theta\mu C_T^{\theta\gamma-1} C_N^{(1-\theta)\gamma} Q^{\mu\gamma-1}$ and $\frac{\partial^2 U}{\partial C_N \partial Q} = \gamma(1-\theta)\mu C_T^{\theta\gamma} C_N^{(1-\theta)\gamma-1} Q^{\mu\gamma-1}$ depends on wether γ is smaller or larger than 0. If $\gamma > 0$ both cross derivatives are positive. The marginal utility of consumption is an increasing function of environmental quality, so the two goods are substitutes. If $\gamma < 0$ both cross derivatives are negative. The marginal utility of consumption is a decreasing function of environmental quality, so the two goods are complements.[7]

After substituting some of the constraints, and eliminating K_N, H_N, the current value Hamiltonian for this optimization problem is:

$$\begin{aligned} H = \frac{1}{\gamma}\left(C_T^{\theta} C_N^{1-\theta} P^{-\mu}\right)^{\gamma} + \acute{q} I \qquad & + \lambda[(1-\tau\sigma)\, a K_T^{\alpha} H_T^{1-\alpha} + pb(K-K_T)^{\beta}(H-H_T)^{1-\beta} \\ + rB \qquad & + T - C_T - pC_N - I\left(p^{*} + h\frac{I}{2K}\right) - p\dot{H}] + \acute{u}\dot{H} \end{aligned} \tag{11}$$

where $\acute{q}$ is the shadow value of the traded capital stock, λ is the shadow value of wealth held in the form of internationally traded bonds and $\acute{u}$ is the shadow value of the nontraded capital stock. Let $q = \acute{q}/\lambda$ be the market value of traded capital in terms of the price of foreign bonds.

[7] Thus, from (10), consumption and pollution are complements if $\gamma > 0$ and substitutes if $\gamma < 0$.

The first-order conditions with respect to the decision variables are given by:

$$\theta C_T^{\theta\gamma-1} C_N^{\gamma(1-\theta)} P^{-\mu\gamma} = \lambda \tag{12}$$

$$(1-\theta) C_T^{\theta\gamma} C_N^{\gamma(1-\theta)-1} P^{-\mu\gamma} = \lambda p \tag{13}$$

$$(1-\tau\sigma) a\alpha K_T^{\alpha-1} H_T^{1-\alpha} = pb\beta(K-K_T)^{\beta-1}(H-H_T)^{1-\beta} = r_K \tag{14}$$

$$(1-\tau\sigma) a (1-\alpha) K_T^{\alpha} H_T^{-\alpha} = pb(1-\beta)(K-K_T)^{\beta}(H-H_T)^{-\beta} = r_H \tag{15}$$

$$p^* + h\frac{I}{K} = q \tag{16}$$

$$\frac{u'}{\lambda} = p \tag{17}$$

Equations (12) and (13) relate the marginal utility of the two consumption goods to the shadow value of wealth held in the form of internationally traded bonds. Equations (14) and (15) equal the marginal returns to traded and non-traded capital (r_K and r_H, respectively) across the two sectors of production. r_K and r_H are both valued in terms of traded output. By inspection, we observe that in the presence of a pollution tax the marginal returns of both traded and nontraded capital are lower than in the absence of the tax. Equation (16) equals the marginal cost of an additional unit of investment in traded capital to the market value of capital. The marginal cost of investment includes the price of new traded capital and the marginal installation cost hI/K. This equation can be solved to yield the expression for the rate of accumulation of traded capital:

$$\frac{I}{K} = \frac{\dot{K}}{K} = \frac{q-p^*}{h} = \phi(t). \tag{18}$$

Therefore, starting from an initial level of K_0 the stock of capital at time t is $K_t = K_0 exp\left\{\int_0^t \phi(s)\,ds\right\}$. Equation (17) equals the value of nontraded capital in terms of the price of foreign bonds to the relative price of the nontraded good.

The optimality conditions with respect to the traded bond B, the traded capital K and the nontraded capital H, lead to the following arbitrage conditions:

$$\rho - \frac{\dot{\lambda}}{\lambda} = r \tag{19}$$

$$\frac{\dot{q}}{q} + \frac{r_K}{q} + \frac{(q-p^*)^2}{2hq} = r \tag{20}$$

$$\frac{\dot{p}}{p} + \frac{r_H}{p} = r \tag{21}$$

Equation (19) equates the the marginal return on consumption to the fixed rate of return on holding a foreign bond. Since both ρ and r are constants, it implies a constant growth rate of the marginal utility λ. Equation (20) equates the rate of return on the foreign bond to rate of return on traded capital taking into account the installation costs of capital. Finally, equation (21) equates the total rate of return on nontraded capital to the rate of return on the traded bond.

Moreover, the following transversality conditions must hold:

$$\lim_{t\to\infty} \lambda B e^{-\rho t} = 0; \qquad \lim_{t\to\infty} q\acute{} K e^{-\rho t} = 0; \qquad \lim_{t\to\infty} u\acute{} H e^{-\rho t} = 0 \tag{22}$$

2.1.1 Determination of equilibrium

Define aggregate consumption $C = C_T + pC_N$, expressed in terms of the traded good as numéraire. Together with the first-order conditions (12) and (13) we have we have the demand functions for C_T and C_N:

$$C_T = \theta C \qquad pC_N = (1-\theta)\, C \tag{23}$$

and

$$\frac{\dot{C}_T}{C_T} = \frac{\dot{C}}{C}; \qquad \frac{\dot{p}}{p} + \frac{\dot{C}_N}{C_N} = \frac{\dot{C}}{C} \tag{24}$$

Combining the time derivative of (12) with (19) and (24) implies that aggregate consumption grows at:

$$\frac{\dot{C}}{C} = \frac{r - \rho - \gamma\,(1-\theta)\,\frac{\dot{p}}{p} - \mu\gamma\frac{\dot{P}}{P}}{1-\gamma} = \Phi(t) \tag{25}$$

So, the growth rate on consumption depends on the rate of inflation of the relative price, $\frac{\dot{p}}{p}$, as well as on the growth rate of pollution, $\frac{\dot{P}}{P}$. This last term indicates the negative effect of a decrease in environmental quality on the steady-state consumption path. Intuitively, a decrease in Q decreases the marginal utility of consumption, decreasing the incentive to consume at all times.

Static allocation conditions

In this first stage, we express the sectoral capital intensities and marginal products of traded capital and nontraded capital in terms of the relative price of nontraded to traded goods. We also express the absolute levels of the allocation of the two types of capital in terms of the aggregate stocks K and H.

Let $\omega = \frac{K_T}{H_T}$ denote the traded to nontraded capital ratio in the traded sector. Dividing equation (14) by (15) we get the relationship between the capital intensities in the two sectors:

$$\frac{K_N}{H_N} = \left(\frac{1-\alpha}{1-\beta}\right)\left(\frac{\beta}{\alpha}\right)\omega \tag{26}$$

From (26), it is clear that the relationship between the two capital intensities is the same as in the case without tax, as the tax is levied on traded output, affecting simetrically both inputs.

Substituting this equation in (14) yields:

$$\omega = \delta\left(\frac{1}{1-\tau\sigma}\right)^{\frac{1}{\alpha-\beta}} p^{\frac{1}{\alpha-\beta}}; \qquad \text{where } \delta = \left[\left(\frac{b}{a}\right)\left(\frac{\beta}{\alpha}\right)^{\beta}\left(\frac{1-\beta}{1-\alpha}\right)^{1-\beta}\right]^{\frac{1}{\alpha-\beta}} \tag{27}$$

Equations (26) and (27) yield the relationships between the sectoral capital intensities and the relative price of nontraded to traded output. Together with (14) and (15), we get the following expressions for the marginal products of the two types of capital in terms of the relative price of nontraded to traded goods:

$$r_K(p) = a\alpha(1-\tau\sigma)\,\omega^{\alpha-1} = a\alpha\delta^{\alpha-1}(1-\tau\sigma)^{\frac{1-\beta}{\alpha-\beta}} p^{\frac{\alpha-1}{\alpha-\beta}} \tag{28}$$

$$r_H(p) = a(1-\alpha)(1-\tau\sigma)\,\omega^{\alpha} = a(1-\alpha)\,\delta^{\alpha}(1-\tau\sigma)^{\frac{-\beta}{\alpha-\beta}} p^{\frac{\alpha}{\alpha-\beta}} \tag{29}$$

Equation (26) together with the resource constraints of K and H (equations (3)), determines the following expressions for the levels of the capital stocks instantaneously employed in the two sectors:

$$K_T = \frac{\beta(1-\alpha)\,\omega H - \alpha(1-\beta)K}{\beta-\alpha}; \qquad H_T = \frac{\beta(1-\alpha)\,\omega H - \alpha(1-\beta)K}{\omega(\beta-\alpha)} \tag{30}$$

$$K_N = \frac{\beta(1-\alpha)(K-\omega H)}{\beta-\alpha}; \qquad H_N = \frac{\alpha(1-\beta)(K-\omega H)}{\omega(\beta-\alpha)}. \tag{31}$$

These expressions can be written in terms of the relative price p by substituting for ω from (27). As it becomes clear from (30) and (31), the equilibrium sectoral allocation of capital depends upon the relative sectoral capital intensity $(\beta-\alpha)$.

In order to the sectoral capital allocations K_i and H_i to be nonnegative, the sectoral and aggregate capital intensities must satisfy the following conditions:

$$\text{If } \beta > \alpha: \quad \frac{r_H}{r_K}\left(\frac{\beta}{1-\beta}\right) = \frac{K_N}{H_N} > \frac{K}{H} > \frac{K_T}{H_T} = \frac{r_H}{r_K}\left(\frac{\alpha}{1-\alpha}\right) \tag{32}$$

$$\text{If } \beta < \alpha: \quad \frac{r_H}{r_K}\left(\frac{\beta}{1-\beta}\right) = \frac{K_N}{H_N} < \frac{K}{H} < \frac{K_T}{H_T} = \frac{r_H}{r_K}\left(\frac{\alpha}{1-\alpha}\right) \tag{33}$$

Price dynamics

Combining the expressions for r_K and r_H, given by (28) and (29), with the arbitrage conditions on the two types of capital, (20) and (21), we get the dynamics of the relative price, p, and of the price of traded capital, q, as follows:

$$\dot{p} = rp - a(1-\alpha)\delta^{\alpha}(1-\tau\sigma)^{\frac{-\beta}{\alpha-\beta}} p^{\frac{\alpha}{\alpha-\beta}} \tag{34}$$

$$\dot{q} = rq - a\alpha\delta^{\alpha-1}(1-\tau\sigma)^{\frac{1-\beta}{\alpha-\beta}} p^{\frac{\alpha-1}{\alpha-\beta}} - \frac{(q-p^*)^2}{2h} \tag{35}$$

These equations are recursive: the relative price of the two goods evolves autonomously according to (34), determining the evolution of the market price of traded capital. Also, they emphasize the importance of adjustment costs. In the absence of such costs $h \to 0$, $q \to p^*$(from (16)) and (35) reduces to a static equation determining p.

In order for the traded capital stock to follow a path of steady growth, the stationary solution of the system of differential equations, (34) and (35), ($\dot{p} = \dot{q} = 0$), must have at least one real solution. Thus, the steady-state relative price of the nontraded good is:

$$\tilde{p} = \left[\frac{r}{a(1-\alpha)\delta^{\alpha}}\right]^{\frac{\alpha-\beta}{\beta}}(1-\tau\sigma), \tag{36}$$

Therefore, with tax, the relative price of the nontraded good in the steady-sate is lower than without tax. For $\tilde{p}$, the steady-state rate of return on nontraded capital $r_H(\tilde{p})/\tilde{p}$ just matches the return on traded bonds, r.

The corresponding steady-state solution of q, $\tilde{q}$, is the solution to the following quadratic equation:

$$r\tilde{q} - r_K(\tilde{p}) - \frac{(\tilde{q}-p^*)^2}{2h} = 0 \tag{37}$$

This equation must have real roots so that the stock of traded capital ultimately converges to a balanced growth path. This will be so if and only if:

$$r_K(\tilde{p}) \leq r\left(p^* + \frac{hr}{2}\right) \tag{38}$$

The real roots for (37) are two: $\tilde{q}_1 = (p^* + rh) - \sqrt{(p^* + rh)^2 - \left(p^{*2} + 2hr_K(\tilde{p})\right)}$ (smaller) and $\tilde{q}_2 = (p^* + rh) + \sqrt{(p^* + rh)^2 - \left(p^{*2} + 2hr_K(\tilde{p})\right)}$ (larger). Thus, there are two potential steady-state equilibrium growth rates for traded capital.

Figure 1 illustrates the phase diagram for the price dynamics (34) and (35), assuming that (38) holds, so that a steady-state growth path for traded capital exists. It is assumed that $\alpha > \beta$, or that the tradable sector is relatively intensive in traded capital, as we are considering that the production of the tradable good generates pollution.

Insert Fig. 1

In Fig. 1, the equilibrium point B, which corresponds to the smaller equilibrium $\tilde{q}_1$, is a saddlepoint, with the stable saddlepath being the negatively sloped locus LM. The equilibrium A, which corresponds to the larger equilibrium value, $\tilde{q}_2$, is a locally stable node. But any time path which converges to A violates the transversality condition for traded capital.[8] Thus, the only solutions for p and q which are consistent with both the transversality condition and a steady growth path for K lie on the stable saddlepath LM.

Proposition 1 summarizes the behavior of prices.

Proposition 1 *(i) If $\beta > \alpha$, so that the nontraded sector is relatively intensive in traded capital, the only solutions for p and q which are consistent with the transversality condition on traded capital are $p = \tilde{p}$, given by (36), and $q = \tilde{q}_1$, the (unstable) steady-state solution given by the negative root to (37). In this case there are no transitional dynamics in either the relative price of nontraded*

[8] From the transversality condition on traded capital we have that $\lim_{t\to\infty} q\acute{}Ke^{-\rho t} = \lim_{t\to\infty} q\lambda Ke^{-\rho t}$. Solving equations (18) and (19) we get $K_t = K_0 e^{\int_0^t \left(\frac{q-p^*}{h}\right)ds}$ and $\lambda_t = \lambda_0 e^{(\rho - r)t}$. Combining these expressions implies $\lim_{t\to\infty} q\acute{}Ke^{-\rho t} = \lim_{t\to\infty} q\lambda_0 K_0 e^{\int_0^t \left(\frac{q-p^*}{h} - r\right)ds}$. Substituting the solution for the larger root $\tilde{q}_2$ into this expression, this limit diverges, violating the transversality condition on the capital stock. For the smaller root $\tilde{q}_1$ the transversality condition holds.

to traded goods or the market price of traded capital. In response to any shock, these prices immediately jump to their respective new steady-state values. (ii) If $\alpha > \beta$, so that the traded sector is relatively intensive in traded capital , the only solutions for p and q which are consistent with the transversality condition on traded capital are that p and q lie on the stable saddlepath LM, ultimately converging to $p = \tilde{p}$, given by (36), and $q = \tilde{q}_1$, the (unstable) steady-state solution given by the negative root to (37). In this case, a shock to the economy will generate transitional adjustment paths in both p and q.

The significant feature of Proposition 1 is that it indicates that the dynamic behavior of asset prices, p_t and q_t, is intimately tied to the production structure of the economy, as reflected by the relative sectoral capital intensities. In case (ii), $\alpha > \beta$, as $r_K(\tilde{p})$ with the tax is lower, the smaller equilibrium $\tilde{q}_1$ is lower than in the case without the tax. Therefore, both the price of the installed capital and the relative price of the nontraded good are lower in the steady-state after the tax is imposed.

The solution to the local dynamics of the linearized approximation to the dynamic system represented by (34) and (35) is

$$p_t - \tilde{p} = [p_0 - \tilde{p}]\, e^{\frac{\beta r}{\beta - \alpha} t} \tag{39}$$

$$q_t - \tilde{q} = \left[\frac{\frac{(1-\alpha)}{(\beta-\alpha)} \frac{r_K(\tilde{p})}{\tilde{p}}}{\frac{(p^* + hr) - \tilde{q}}{h} - \frac{\beta r}{\beta - \alpha}} \right] (p_t - \tilde{p}) \tag{40}$$

where $\tilde{q}$ is the smaller root $\tilde{q}_1$. Since $p^* + hr > \tilde{q}$ and $\alpha > \beta$, (40) is a negatively sloped locus, being a linear approximation to LM in Fig. 1. Accordingly, the two asset prices move in opposite directions during the transition. The slope of (40) is negatively affected by the tax. Thus, a higher tax implies a lower slope of the stable saddlepath LM, in absolute value.

The behavior of the sectoral capital intensities ω, K_N/H_N and of the marginal products $r_K(p)$, $r_H(p)$, will mirror that of p. If $\alpha > \beta$, ω, K_N/H_N, r_K, and r_H will vary through time in response to the evolution of the relative price p.

Asset dynamics

In order to derive the dynamics of asset accumulation, it is useful to begin with the equilibrium sectoral outputs in terms of the aggregate stocks of capital,

K and H. These are derived from the production functions by using the optimality conditions (14) and (15) together with (28) and (29), and the equilibrium sectoral allocations in K_T, H_T, K_N, H_N, as determined in (30) and (31). The following relationships hold:

$$Y_T = \frac{-r_K(1-\beta)K + r_H\beta H}{(1-\tau\sigma)(\beta-\alpha)}; \qquad Y_N = \frac{r_K(1-\alpha)K - r_H\alpha H}{p(\beta-\alpha)} \tag{41}$$

From (14) and (15), we conclude that Y_T and Y_N with tax are the same without tax. However, the value of the nontraded good in terms of the traded one, pY_N, is lower in the presence of a tax, as well as the value of total production, $Y_T + pY_N$.

Define aggregate wealth in terms of the traded bonds as numéraire, as

$$W = qK + pH + B \tag{42}$$

Differentiating this expression with respect to time and noting: (i) the accumulation equations (6) and (8); (ii) the production functions (1), (2), together with (41); (iii) the definition of aggregate consumption C; (iv) the optimality condition for investment (18); and (v) the arbitrage conditions (20) and (21), the following relationship describing the rate of aggregate wealth accumulation is obtained:

$$\dot{W}(t) = rW(t) + T(t) - C(t) \tag{43}$$

where $T = \sigma\tau Y_T$. The wealth accumulation depends on C, which evolves according to (25), which in turn depends upon the rate of inflation of the relative price, as well as on T. From the previous section we know that $\dot{p}/p$ depends upon the relative sectoral capital intensities. Thus, the entire profile of asset accumulation depends upon whether $\beta \gtrless \alpha$. We will focus on $\alpha > \beta$.

$\alpha > \beta$: *Traded sector relatively intensive in traded capital*

From Proposition 1, the system eventually converges to the relative price of the nontraded good $p = \tilde{p}$, given by (36), and to the price of the installed capital $q = \tilde{q}_1$, the steady-state solution given by the negative root to (37). With p constant over time, q is also constant, so that, at the steady-state, traded capital grows as follows:

$$\frac{\dot{K}}{K} = \frac{\tilde{q} - p^*}{h} = \tilde{\phi}; \qquad \text{that is} \qquad K_t = K_0 e^{\tilde{\phi} t} \tag{44}$$

where $\tilde{q}$ is the smaller solution to (37). Since $\tilde{q}$ depends on the pollution growth rate, $\tilde{\phi}$ will be different. In particular, we can show that in the case $\alpha > \beta$, $\tilde{\phi}$ is lower with the tax than without it, since the smaller equilibrium $\tilde{q}_1$ is now lower (see Figure 1). In this case, $\tilde{\phi}$ is given by:

$$\tilde{\phi} = r - \frac{\sqrt[2]{2rhp^* + r^2h^2 - 2h\alpha a^{\frac{\beta-\alpha+1}{\beta}}\delta^{\frac{(\beta-\alpha)(\alpha-1)}{\beta}}(1-\alpha)^{\frac{1-\alpha}{\beta}} r^{\frac{\alpha-1}{\beta}}(1-\tau\sigma)}}{h} \quad (45)$$

where $\frac{\partial \tilde{\phi}}{\partial \tau} < 0$, as expected (see Bovenberg and Smulders [4], Elbasha and Roe [10], and Reis [19], among others).

In the steady-state p remains at its steady-state level $\tilde{p}$ ($\dot{p} = 0$) so that consumption grows at the steady-state growth rate :

$$\frac{\dot{C}}{C} = \frac{r-\rho}{1-\gamma} - \frac{\mu\gamma\frac{\dot{P}}{P}}{1-\gamma} = \tilde{\psi} - \frac{\mu\gamma\frac{\dot{P}}{P}}{1-\gamma} = \tilde{\psi} - \frac{\mu\gamma\tilde{\phi}}{1-\gamma} = \tilde{\Phi} \quad \text{that is} \quad C_t = C_0 e^{\tilde{\Phi}t} \quad (46)$$

since we can show that in the steady-state pollution grows at the same rate as capital. Without pollution, the growth rate of consumption in the steady-state is given by $\tilde{\psi}$, which is determined by parameters that are constant. However, in the presence of pollution, the growth rate of consumption is also determined by the growth rate of capital, and therefore, by the productive structure of the economy.

The pollution tax implies a lower growth rate on capital and a higher growth rate on consumption, in the steady-state. Thus, there may exist a tax level for which these two growth rates are the same.

While without the tax consumption and aggregate wealth grow at the same rate in the steady-state, implying that $\frac{\dot{W}}{W} = \frac{\dot{C}}{C} = \tilde{\psi}$, we can now show that with tax and lump-sum transfers they may grow at different rates. To this end, we still need to determine the evolution of T. From (41), and the fact that $\dot{K}/K = \dot{H}/H = \tilde{\phi}$,[9] we can show that T grows at the rate $\tilde{\phi}$, the same growth rate of Y_T, implying that

$$T_t = T_0 e^{\tilde{\phi}t}$$

[9] We will show later that this has to be the case.

Substituting C_t and T_t in (43) and solving, we get[10]

$$W_t = -\frac{T_0 e^{\tilde{\phi} t}}{r - \tilde{\phi}} + \frac{C_0 e^{\tilde{\Phi} t}}{r - \tilde{\Phi}} \tag{47}$$

Thus, the consumption-wealth ratio is:

$$\frac{C}{W} = \frac{C_0 e^{\tilde{\Phi} t}}{-\frac{T_0 e^{\tilde{\phi} t}}{r-\tilde{\phi}} + \frac{C_0 e^{\tilde{\Phi} t}}{r-\tilde{\Phi}}}. \tag{48}$$

As long as $\tilde{\phi} > \tilde{\Phi}$,[11] or $\tilde{\phi} > \tilde{\psi} \frac{1-\gamma}{1-\gamma(1-\mu)}$

$$\lim_{t \to \infty} \frac{C}{W} = \frac{C_0 e^{\tilde{\Phi} t}}{-\frac{T_0 e^{\tilde{\phi} t}}{r-\tilde{\phi}} + \frac{C_0 e^{\tilde{\Phi} t}}{r-\tilde{\Phi}}} = 0. \tag{49}$$

Therefore, aggregate wealth and consumption grow at different rates. Asymptotically, we observe that $\frac{\dot{W}}{W} = r + (\tilde{\phi} - r) = \tilde{\phi}$.

Alternatively, if $\tilde{\phi} = \tilde{\Phi}$, we obtain a constant equilibrium consumption-wealth ratio

$$\frac{C}{W} = \frac{r - \tilde{\Phi}}{\left(1 - \frac{T_0}{C_0}\right)}. \tag{50}$$

Therefore, aggregate wealth and consumption grow at the same rate in the steady-state, as when there is no pollution tax.

As mentioned before, for $\alpha > \beta$, there is an evolution of the relative price p and of the market value of traded capital q along the stable saddlepath LM. Since this path is negatively sloped, the two asset prices move in opposite directions in this phase of transition.

Consumption grows according to equation (25), the solution to which is $C_t = C_0 exp\left\{\int_0^t \Phi(s)\, ds\right\}$. However, we cannot determine now how T evolves outside the steady-state, since it depends on the evolution of H. Therefore, we cannot solve (43) for W in this case.

The growth rate of traded capital is given by (18) and is also time varying, reflecting the evolution of q along the stable locus. As q approaches its steady state, the growth rate of traded capital approaches the steady-state rate given in (44).

[10] From the transversality condition on H: $r > \tilde{\Phi}$ and $r > \tilde{\phi}$, as it will be shown later.

[11] We will show later that $\tilde{\phi} \geq \tilde{\Phi}$.

We need to check whether and under what conditions the transversality conditions for H and B hold. If the transversality condition for H is met, then the transversality condition for bonds will also be met, since $B = W - qK - pH$.

Since the relative price of nontraded capital is time varying (p_t), it is convenient to focus on the rate of accumulation of nontraded capital in value terms, which is given by:

$$\frac{d(pH)}{dt} = \left[r + \frac{r_H(p)\beta}{p(\alpha-\beta)}\right](pH) - \frac{r_K(p)(1-\alpha)K}{(\alpha-\beta)} - (1-\theta)C \tag{51}$$

In the neighborhood of the steady state, this equation can be approximated by:

$$\frac{d(pH)}{dt} = \left[\frac{r\alpha}{(\alpha-\beta)}\right](pH) - \frac{\tilde{r}_K(1-\alpha)K}{(\alpha-\beta)} - (1-\theta)C \tag{52}$$

The time path of p_tH_t around $\tilde{p}$ and $\tilde{q}$ is given by:

$$\begin{aligned} p_tH_t \quad = \quad & \left[p_0H_0 + \frac{(1-\alpha)\tilde{r}_K}{(\alpha-\beta)\left[\tilde{\phi} - \alpha r/(\alpha-\beta)\right]}K_0 + \frac{(1-\theta)}{\left[\tilde{\Phi} - \alpha r/(\alpha-\beta)\right]}C_0\right] e^{\frac{\alpha r}{\alpha-\beta}t} \\ & - \frac{(1-\alpha)\tilde{r}_K}{(\alpha-\beta)\left[\tilde{\phi} - \alpha r/(\alpha-\beta)\right]}K_0e^{\tilde{\phi}t} - \frac{(1-\theta)}{\left[\tilde{\Phi} - \alpha r/(\alpha-\beta)\right]}C_0e^{\tilde{\Phi}t} \end{aligned} \tag{53}$$

From the transversality condition on H, (22), $\lim_{t\to\infty} \lambda pHe^{-\rho t} = 0$. Substituting for $\lambda_t = \lambda_0 e^{(\rho-r)t}$, we obtain $\lim_{t\to\infty} \lambda_0 pHe^{-rt} = 0$, which reduces to $\lim_{t\to\infty} pHe^{-rt} = 0$. Applying this to the three exponential terms in (53), the following conditions must hold: (i) $\alpha r/(\alpha-\beta) - r < 0$, (ii) $r > \tilde{\phi}$ and (iii) $r > \tilde{\Phi}$. Condition (i) is never met for $\alpha > \beta$. In this case it is required that the term in the first parentheses in (53) is zero. Condition (ii) is satisfied by the smaller root $\tilde{q}_1$, that is, by the transversality condition on traded capital (see footnote 8), while (iii) has to be met.

Therefore, it is required that the following holds:

$$p_0H_0 + \frac{(1-\alpha)\tilde{r}_K}{(\alpha-\beta)\left[\tilde{\phi} - \alpha r/(\alpha-\beta)\right]}K_0 + \frac{(1-\theta)}{\left[\tilde{\Phi} - \alpha r/(\alpha-\beta)\right]}C_0 = 0 \tag{54}$$

and $r > \tilde{\Phi}$.

The conditions imposed guarantee that $\lim_{t\to\infty} \lambda We^{-\rho t} = \lim_{t\to\infty} \acute{q}Ke^{-\rho t} = 0$ are met. This implies $\lim_{t\to\infty} \lambda[pH + B]e^{-\rho t} = 0$. The transversality condition on nontraded capital and the solution for $\lambda_t = \lambda_0 e^{(\rho - r)t}$, imply that $\lim_{t \to \infty} Be^{-rt} = 0$ and this condition is equivalent to the national intertemporal budget constraint. So, equation (54) imposes conditions on the initial relative price of the two goods, p_0, and on the initial price of traded capital, q_0, according to which the resulting path of net exports so generated is consistent with the intertemporal solvency for the economy.

Along its transitional adjustment path, nontraded capital H_t is constrained, however, by the requirement that the steady-state K/H ratio must lie within the limits defined in (33) and for this to occur the growth rate of H must converge to the growth rate of K. If $\theta = 1$ so that the agent does not consume the nontraded good (which is a reasonable assumption if H is interpreted as human capital), the convergence of the growth rate of nontraded capital to the growth rate of traded capital is assured ($\dot{K}/K = \dot{H}/H = \tilde{\phi}$).

If $\theta < 1$ so that the agent consumes some of the nontraded good, we must impose an additional condition $\tilde{\phi} \geq \tilde{\Phi}$, so that convergence of growth rates occurs. If $\tilde{\Phi} > \tilde{\phi}$, the K/H ratio would converge to zero, violating (33).

Thus, assuming $\theta = 1$ or $\tilde{\phi} > \tilde{\Phi}$, nontraded capital grows at the same rate as traded capital $\tilde{\phi}$, and the ratio of traded to nontraded capital will converge to a balanced growth path along which is given by

$$\frac{K}{H} = \frac{\left[-\tilde{\phi}\left(\alpha - \beta\right) + \alpha r\right]}{\left(1 - \alpha\right)} \frac{\tilde{p}}{r_K\left(\tilde{p}\right)} \tag{55}$$

implying that $-\tilde{\phi}\left(\alpha - \beta\right) + \alpha r > 0$, or $\tilde{\phi} < \frac{\alpha r}{(\alpha - \beta)}$, for the nonnegativity of (55).

Using the fact that in the steady state $\tilde{r}_H = \tilde{p}r$ and that $r > \tilde{\phi}$, we can show that the steady state ratio K/H (55) satisfies the inequalities in (33), as long as the common equilibrium growth rate of capital $\tilde{\phi} \geq 0$. If the growth rate is strictly positive then K/H lies within the limits of the inequality. If the economy is stationary ($\tilde{\phi} = 0$), then $K/H \to K_T/H_T$ and the equilibrium output of the nontraded good reduces to zero. The economy is fully specialized in the production of the traded commodity. This is because with either no nontraded consumption ($\theta = 1$) or declining consumption ($\tilde{\phi} = 0 > \tilde{\Phi}$) and a fixed stock of nontraded capital, asymptotically, there is no demand for additional output of the nontraded good. The case where $\tilde{\phi} < 0$, so that the economy is declining, drives K/H beyond the boundary of (33): $K/H > K_T/H_T$.

Since $\tilde{p}$ and $r_K(\tilde{p})$ are both identically affected by the presence of the pollution tax, the ratio K/H along the balanced growth path is now higher since $\tilde{\phi}$ is lower. If $\beta > \alpha$, traded capital decreases relatively to the nontraded one along the balanced growh path.

If $\tilde{\phi} = \tilde{\Phi}$, K/H will also converge to a balanced growth path and the corresponding expression can be obtained.

In summary, with a tax on pollution affecting symetrically both traded and nontraded capital in the traded sector, the steady-state is now characterized by a lower relative price of the nontraded good and a lower price of the installed capital. The transitional adjustment path will be characterized by the asset prices moving in opposite directions, reflecting the Stolper-Samuelson theorem in trade theory.

In the steady-state equilibrium, the real rate of return on nontraded capital measured in terms of the numéraire, $\frac{\tilde{r}_H}{\tilde{p}}$, must equal the foreign interest rate. As this rate is fixed, $\tilde{r}_H$ has to decrease in the same proportion as $\tilde{p}$. Therefore, the relative capital intensity ratio in the nontraded sector, K_N/H_N, must remain constant. Since capital can freely move between the two sectors, $K_T/H_T = \omega$ is also constant. This implies that $\tilde{r}_K$ has to decrease in the same proportion as $\tilde{p}$. Consequently, the lower return to traded capital reduces the market price of installed capital, $\tilde{q}$. Hence, the growth rate of traded and nontraded capital, $\tilde{\phi}$, decreases.

Finally, the steady-state growth rate of consumption is now affected, since it also depends on the growth rate of capital. This is in contrast to the case in which the representative agent does not care about pollution and no pollution tax is levied.

In the steady-state pollution increases at the rate $\tilde{\phi}$, for a given tax rate. In the presence of a pollution tax, the competitive equilibrium is characterized by a lower growth rate on capital (traded and nontraded), and a higher growth rate for consumption. When consumption grows at a lower rate than capital, it is possible that the change in the disutility caused by pollution more than compensates the change in utility generated by the increase in consumption.

The larger the tax, the smaller will be $\tilde{p}$ and $\tilde{q}$, implying a lower $\tilde{\phi}$. This results from (45), as $\frac{\partial \tilde{\phi}}{\partial \tau} < 0$. Therefore, eventually there will be a tax rate for which $\tilde{\phi} = \tilde{\Phi}$. This solution would be similar to the one for a closed economy, as we will show later.

2.2 Closed Economy

In this section, we study the competitive equilibrium for the closed economy case, keeping pollution exogenous as before. The representative agent produces two types of goods, good 1, Y_1, and good 2, Y_2, using physical capital, K, and human capital, H, that can be allocated to both sectors. In contrast to section 2.1, the accumulation of K does not involve adjustment costs.[12] Production of Y_1 generates pollution (see (4), where T corresponds to sector 1), and, thus, the pollution tax is levied on sector 1. Also, p denotes the relative price of good 2 in terms of good 1.

The representative agent´s problem consists of choosing C_1, C_2, K_1, K_2, H_1, H_2 to maximize the present value of the stream of utility:

$$U = \int_0^\infty \frac{1}{\gamma}(C_1^\theta C_2^{1-\theta} Q^\mu)^\gamma e^{-\rho t} dt \tag{56}$$

subject to the constraints (1), (2), (3), (10), the budget constraint

$$\dot{K} = (1 - \tau\sigma) Y_1 + pY_2 + T - C_1 - pC_2 - p\dot{H} \tag{57}$$

and the initial stocks of assets K_0, H_0.

The current value Hamiltonian for this optimization problem is:

$$\begin{aligned} H \;=\; & \frac{1}{\gamma}\left(C_1^\theta C_2^{1-\theta} P^{-\mu}\right)^\gamma + q\'[(1-\tau\sigma)\, aK_1^\alpha H_1^{1-\alpha} + pbK_2^\beta H_2^{1-\beta} + \\ & +T - C_1 - pC_2 - p\dot{H}] + u\'\dot{H} \end{aligned} \tag{58}$$

The first-order necessary conditions for an optimal solution are:

$$\theta C_1^{\theta\gamma - 1} C_2^{\gamma(1-\theta)} P^{-\mu\gamma} = q\' \tag{59}$$

$$(1-\theta)\, C_1^{\theta\gamma} C_2^{\gamma(1-\theta)-1} P^{-\mu\gamma} = q\'p \tag{60}$$

[12] See Appendix A for the case of the closed economy with adjustment costs.

$$(1-\tau\sigma)\,a\alpha K_1^{\alpha-1}H_1^{1-\alpha}=pb\beta K_2^{\beta-1}H_2^{1-\beta}=r_K \tag{61}$$

$$(1-\tau\sigma)\,a\,(1-\alpha)\,K_1^{\alpha}H_1^{-\alpha}=pb\,(1-\beta)\,K_2^{\beta}H_2^{-\beta}=r_H \tag{62}$$

The optimality conditions with respect to K and H are given by:

$$\frac{\dot{\acute{q}}}{\acute{q}}=\rho-r_K \tag{63}$$

$$\frac{\dot{\acute{u}}}{\acute{u}}=\rho-\frac{r_H}{p} \tag{64}$$

where p is also the shadow price of human capital in units of physical capital ($p=\acute{u}/\acute{q}$).

The following transversality conditions must hold:

$$\lim_{t\to\infty}\acute{q}Ke^{-\rho t}=0; \qquad \lim_{t\to\infty}\acute{u}He^{-\rho t}=0 \tag{65}$$

2.2.1 Determination of equilibrium

Defining aggregate consumption as before, the same demand functions for C_1 and C_2 are obtained (see (23) and (24)).

Combining the time derivative of (59) with (63) and (24) implies that aggregate consumption grows at:

$$\frac{\dot{C}}{C}=\frac{r_K-\rho-\gamma\,(1-\theta)\,\frac{\dot{p}}{p}-\mu\gamma\frac{\dot{P}}{P}}{1-\gamma}=\Phi(t) \tag{66}$$

As in the small open economy, the growth rate of consumption depends on the rate of inflation of the relative price and on the growth rate of pollution. However, r_K is not exogenously determined.

Dividing equation (61) by (62) we get the same relationship between the capital intensities in the two sectors as before (see (26)). Substituting this equation in (61) yields, as in the small open economy:

$$\omega=\delta\left(\frac{1}{1-\tau\sigma}\right)^{\frac{1}{\alpha-\beta}}p^{\frac{1}{\alpha-\beta}} \tag{67}$$

or,

$$p=\Delta(1-\tau\sigma)\omega^{\alpha-\beta} \tag{68}$$

where $\Delta = \left[\left(\frac{a}{b}\right) \left(\frac{\alpha}{\beta}\right)^{\beta} \left(\frac{1-\alpha}{1-\beta}\right)^{1-\beta} \right]$. Therefore, the expressions for r_K and r_H in terms of p are the same as before (see (28), (29)). Likewise, similar expressions are obtained for given absolute levels of the capital stocks instantaneously employed in the two sectors (see (30), (31)). Also, the same conditions for the nonnegativity of the sectoral capital allocations K_i and H_i hold (see (32), (33)).

Price dynamics

Combining the expressions for r_K and r_H, given by (28) and (29), with the optimality conditions on K and H, (63) and (64), and since $p = u\acute{}/q\acute{}$, the dynamics of the relative price p is given by:

$$\frac{\dot{p}}{p} = (1-\tau\sigma)^{\frac{1-\beta}{\alpha-\beta}} a\alpha\delta^{\alpha-1} p^{\frac{\alpha-1}{\alpha-\beta}} - (1-\tau\sigma)^{\frac{\beta}{\beta-\alpha}} a(1-\alpha)\delta^{\alpha} p^{\frac{\beta}{\alpha-\beta}} \tag{69}$$

The key feature here is that the growth rate of p depends only on p, and not on any other variables.

In the steady-state $\dot{p}/p = 0$. Thus, $\tilde{r}_K = \tilde{r}_H/\tilde{p}$. The steady-state relative price of good 2 is:

$$\tilde{p} = (1-\tau\sigma)^{\frac{1}{\beta-\alpha+1}} \left(\frac{\alpha}{1-\alpha}\frac{1}{\delta}\right)^{\frac{\alpha-\beta}{\beta-\alpha+1}} \tag{70}$$

Therefore, the tax decreases the relative price in the steady-state.

The form of equation (69) has immediate implications to the nature of the dynamics. This relation is a differential equation in the single variable p. The equation is stable if $\alpha > \beta$ $(\partial(\dot{p}/p)/\partial p < 0)$ and unstable if $\beta > \alpha$ $(\partial(\dot{p}/p)/\partial p > 0)$. Thus, if $\alpha > \beta$ - the case we regard as relevant - p converges monotonically to its steady-state value.

Proposition 2 summarizes the behavior of prices.

Proposition 2 *(i) If $\beta > \alpha$, any departure of p from its steady-state value would be magnified over time. p equals its steady-state value at all points in time, $p = \tilde{p}$, given by (70). In this case, there is no transitional dynamics in the relative price of good 2. In response to any shock, the price immediately jumps its new steady-state value. (ii) If $\alpha > \beta$, p converges monotonically to its steady-state value $p = \tilde{p}$, given by (70). In this case, a shock to the economy will generate transitional adjustment path.*

Similarly to the open economy, the dynamic behavior of p_t is tied to the production structure of the economy. Also, the price dynamics will be reflected on the behavior of the sectoral capital intensities and on the marginal products $r_K(p)$ and $r_H(p)$. From equation (67), the monotonic convergence of p when $\alpha > \beta$ implies that $\omega = K_1/H_1$ will also converge monotonically to its steady-state value. Given the relationship between ω and K_2/H_2, also the sectoral capital intensity in sector 2 converges monotonically to its steady-state value. ω determines the marginal product of physical capital and the marginal product of human capital in the production of goods. Therefore, r_K and r_H will also converge monotonically to their steady-state values.[13]

Asset dynamics

The equilibrium sectoral outputs in terms of the aggregate stocks of capital, K and H, are similiar to those of the small open economy:

$$Y_1 = \frac{-r_K(1-\beta)K + r_H\beta H}{(1-\tau\sigma)(\beta-\alpha)}; \qquad Y_2 = \frac{r_K(1-\alpha)K - r_H\alpha H}{p(\beta-\alpha)} \tag{71}$$

Aggregate wealth, in terms of physical capital as numéraire, is given by:

$$W = K + pH \tag{72}$$

Differentiating this expression with respect to time and noting: (i) the accumulation equation (57); (ii) the equilibrium sectoral outputs defined in (71); (iii) the definition of aggregate consumption C; (iv) the definition of $p = \acute{u}/\acute{q}$; and (v) the optimality conditions (63) and (64), the following relationship describes the rate of aggregate wealth accumulation:

$$\dot{W}(t) = r_K W(t) + T(t) - C(t) \tag{73}$$

where $T = \sigma\tau Y_1$. The wealth accumulation depends on C and T. C evolves according to (66), which, as before, depends upon the rate of inflation of the relative price, and, consequently, depends upon the relative sectoral capital intensities, as previously discussed. Thus, as in the open economy, the profile of asset accumulation depends upon wether $\beta \gtreqless \alpha$. Again, we focus on $\alpha > \beta$.

In the steady-state, p is constant at $\tilde{p}$ and, consequently, so does $r_K(\tilde{p})$. Consumption grows at the steady-state growth rate when the pollution growth

[13] If $\beta > \alpha$ the unstable behavior of p would be transmitted accordingly to all these variables.

rate is constant, which is the same as the capital growth rate in the steady-state. Moreover, the capital growth rate is constant only when C and K grow at the same rate. Thus, consumption grows at the steady-state growth rate:

$$\frac{\dot{C}}{C} = \frac{\tilde{r}_K - \rho}{1 - \gamma\,(1-\mu)} = \tilde{\Phi} \tag{74}$$

Furthermore, in the steady-state, the shares of physical and human capital in the two sectors (K_1/K, H_1/H) are constant and C, K, H grow at the common rate $\tilde{\Phi}$ ($\frac{\dot{C}}{C} = \frac{\dot{K}}{K} = \frac{\dot{H}}{H} = \frac{\dot{P}}{P} = \tilde{\Phi} = g$). Since $\tilde{r}_K$ is negatively affected by the pollution tax, $\frac{\partial \tilde{\Phi}}{\partial \tau} < 0$, as expected. Thus, the larger the tax, the lower g will be, that is, the lower is the growth rate of the economy, as well as pollution growth.

Also, aggregate wealth grows at the same rate as consumption, in the steady-state. T grows at $\tilde{\Phi}$ since $\dot{Y}_1/Y_1 = \tilde{\Phi}$. Substituting $C_t = C_0 e^{\tilde{\Phi} t}$ and $T_t = T_0 e^{\tilde{\Phi} t}$ in (73) and solving, we get[14]

$$W_t = (C_0 - T_0)\,\frac{e^{\tilde{\Phi} t}}{\tilde{r}_K - \tilde{\Phi}} \tag{75}$$

Thus, the consumption-wealth ratio is constant in the steady-state:

$$\frac{C}{W} = \frac{\tilde{r}_K - \tilde{\Phi}}{\left(1 - \frac{T_0}{C_0}\right)} \tag{76}$$

We need to check the conditions under which the transversality conditions for K and H hold.

The time path of $p_t H_t$ around the steady-state is given by:

$$\begin{aligned} p_t H_t \quad = \quad & \left[p_0 H_0 + \frac{\left[(1-\alpha)\,\tilde{r}_K K_0 + (1-\theta)\,(\alpha-\beta)\,C_0\right]\tilde{p}}{\tilde{p}\tilde{\Phi}\,(\alpha-\beta) - \tilde{r}_H \alpha}\right] e^{\frac{\tilde{r}_H \alpha}{\tilde{p}(\alpha-\beta)}} \\ & - \frac{\left[(1-\alpha)\,\tilde{r}_K K_0 + (1-\theta)\,(\alpha-\beta)\,C_0\right]\tilde{p}}{\tilde{p}\tilde{\Phi}\,(\alpha-\beta) - \tilde{r}_H \alpha} e^{\tilde{\Phi} t} \end{aligned} \tag{77}$$

From the transversality condition on H, (65), $\lim_{t\to\infty} \acute{u} H e^{-\rho t} = \lim_{t\to\infty} \acute{q} p H e^{-\rho t} = 0$. Since $\acute{q}_t = \acute{q}_0 e^{(\rho - \tilde{r}_K)t}$ in the steady-state, we obtain $\lim_{t\to\infty} \acute{q}_0 p H e^{-\tilde{r}_k t} = 0$, which reduces to $\lim_{t\to\infty} p H e^{-\tilde{r}_K t} = 0$. Applying this to the two exponential terms in (77), the following conditions must hold: (i) $(r_H \alpha)\,/\,(p\,(\alpha-\beta)) - r_K < 0$ and

[14] From the tranversality condition on H: $r_K > \tilde{\Phi}$, as it will be shown later.

(ii) $r_K > \tilde{\tilde{\Phi}}$. However, condition (i) is never met for $\alpha > \beta$ since $r_K = r_H/p$ in the steady-state. In this case the term in the first parentheses in (77) must be zero:

$$p_0 H_0 + \frac{\left[(1-\alpha)\,\tilde{r}_K K_0 + (1-\theta)\,(\alpha-\beta)\,C_0\right]\tilde{p}}{\tilde{p}\tilde{\tilde{\Phi}}\,(\alpha-\beta) - \tilde{r}_H\alpha} = 0 \tag{78}$$

The conditions imposed also guarantee that $\lim_{t\to\infty} qKe^{-\rho t} = 0$. So, the transversality condition on aggregate wealth is also met: $\lim_{t\to\infty} qWe^{-\rho t} = 0$.

As previously discussed, the growth rate of H is the same as the growth rate of K, in the steady-state. The ratio of physical to human capital will converge to a balanced growth path along which:

$$\frac{K}{H} = \frac{-\tilde{p}\tilde{\tilde{\Phi}}\,(\alpha-\beta) + \alpha r_H\,(\tilde{p})}{(1-\alpha)\,r_K\,(\tilde{p}) + (1-\theta)\,(\alpha-\beta)\,\frac{C_0}{K_0}} \tag{79}$$

with $-\tilde{p}\tilde{\tilde{\Phi}}\,(\alpha-\beta) + \alpha r_H\,(\tilde{p}) > 0$ for the nonnegativity of this ratio. Also, $\frac{r_H}{r_K}\left(\frac{\beta}{1-\beta}\right) < \frac{-\tilde{p}\tilde{\tilde{\Phi}}(\alpha-\beta)+\alpha r_H(\tilde{p})}{(1-\alpha)r_K(\tilde{p})+(1-\theta)(\alpha-\beta)\frac{C_0}{K_0}} < \frac{r_H}{r_K}\left(\frac{\alpha}{1-\alpha}\right)$ must hold, so that the steady-state ratio K/H lies within the limits defined in (33).

In summary, with a tax on pollution affecting symetrically K and H in sector 1, the steady-state is characterized by a lower relative price of good 2, $\tilde{p}$, which increases consumption of good 2 relative to consumption of good 1, and by a lower growth rate of the economy $\tilde{\tilde{\Phi}}$. Since $r_K = r_H/p$ evaluated at the steady-state and r_K are negatively affected by the pollution tax, the real rate of return on human capital measured in terms of the numéraire, $\frac{\tilde{r}_H}{\tilde{p}}$, decreases. This means that $\tilde{r}_H$ decreases more than proportionally to $\tilde{p}$. Therefore, the relative capital intensity ratio in sector 2, K_2/H_2, decreases, as well as K_1/H_1. This implies that $\frac{\tilde{r}_K}{\tilde{p}}$ must increase, that is, $\tilde{r}_K$ decreases, but less than proportionally to $\tilde{p}$.

3 Optimal Policy

In this section, we will determine how pollution affects optimal growth.

3.1 Small Open Economy

The social planner's problem consists of choosing C_T, C_N, K_T, K_N, H_T, H_N, I, $\dot{H}$ to maximize the present value of the stream of utility, as follows:

$$U = \int_0^{\infty} \frac{1}{\gamma} \left(C_T^{\theta} C_N^{1-\theta} Q^{\mu} \right)^{\gamma} e^{-\rho t} dt \tag{80}$$

subject to the constraints (1), (2), (3), (6), (7), (10),

$$\dot{B} = Y_T + pY_N + rB - C_T - pC_N - \Gamma(I, K) - p\dot{H} \tag{81}$$

and the initial stocks of assets K_0, H_0 and B_0. In this case, pollution is not exogenous, and the optimal pollution tax rate can be obtained by comparing the results obtained in this case with those in the competitive equilibrium examined in Section 2.

The current value Hamiltonian for the social planner´s problem is:[15]

$$\begin{aligned} H \quad = \quad & \frac{1}{\gamma} \left(C_T^{\theta} C_N^{1-\theta} (\sigma a K_T^{\alpha} H_T^{1-\alpha})^{-\mu} \right)^{\gamma} + \acute{q_p} I + \lambda_p [a K_T^{\alpha} H_T^{1-\alpha} + \\ & + pb K_N^{\beta} H_N^{1-\beta} + rB - C_T - pC_N - I \left(p^* + h \frac{I}{2K} \right) - p\dot{H}] + \acute{u_p} \dot{H} \end{aligned} \tag{82}$$

The first-order conditions with respect to the decision variables, after some algebra, are given by:

$$\theta C_T^{\theta\gamma - 1} C_N^{\gamma(1-\theta)} (\sigma a K_T^{\alpha} H_T^{1-\alpha})^{-\mu\gamma} = \lambda_p \tag{83}$$

$$(1-\theta) C_T^{\theta\gamma} C_N^{\gamma(1-\theta)-1} (\sigma a K_T^{\alpha} H_T^{1-\alpha})^{-\mu\gamma} = \lambda_p p \tag{84}$$

$$-\frac{\mu\alpha}{\theta} \frac{C_T}{K_T} + a\alpha K_T^{\alpha-1} H_T^{1-\alpha} = pb\beta K_N^{\beta-1} H_N^{1-\beta} \tag{85}$$

$$-\frac{\mu(1-\alpha)}{\theta} \frac{C_T}{H_T} + a(1-\alpha) K_T^{\alpha} H_T^{-\alpha} = pb(1-\beta) K_N^{\beta} H_N^{-\beta} \tag{86}$$

$$p^* + h\frac{I}{K} = q_p \tag{87}$$

$$\frac{\acute{u_p}}{\lambda_p} = p \tag{88}$$

The first term on the left hand side of equations (85) and (86) reflects the marginal social damage of pollution.

[15] Subscript p refers to the social planner's problem.

The optimality conditions with respect to B, K and H lead to similar conditions to those obtained in the competitive equilibrium:

$$\rho - \frac{\dot{\lambda}_p}{\lambda_p} = r \tag{89}$$

$$\frac{\dot{q}_p}{q_p} + \frac{pb\beta K_N^{\beta-1} H_N^{1-\beta}}{q_p} + \frac{(q_p - p^*)^2}{2hq_p} = r \tag{90}$$

$$\frac{\dot{p}}{p} + \frac{pb\,(1-\beta)\,K_N^{\beta} H_N^{-\beta}}{p} = r \tag{91}$$

Given the similarity of the first-order conditions with respect to C_T and C_N both in the social planner´s problem and in the competitive equilibrium, it is easily shown that the relationships between the two types of consumption and aggregate consumption still hold (see (23) and (24)). Therefore, aggregate consumption grows according to:

$$\frac{\dot{C}}{C} = \frac{r - \rho - \gamma\,(1-\theta)\,\frac{\dot{p}}{p} - \mu\gamma\left[\alpha\frac{\dot{K}_T}{K_T} + (1-\alpha)\,\frac{\dot{H}_T}{H_T}\right]}{1-\gamma} \tag{92}$$

Thus, from (90) and (85) evaluated at the steady-state, the growth rate of consumption in the steady-state is given by:

$$\frac{\dot{C}}{C} = \frac{\frac{a\alpha\tilde{\omega}^{\alpha-1} - \frac{\mu\alpha}{\theta}\frac{C_T}{K_T}}{\tilde{q}_p} + \frac{(\tilde{q}_p - p^*)^2}{2h\tilde{q}_p} - \rho - \mu\gamma\tilde{\phi}}{1-\gamma} \tag{93}$$

The term $-\frac{\mu\alpha}{\theta}\frac{C_T}{K_T}$ summarizes the adverse effect of capital accumulation on the environment and, consequently, on the efficient growth path. That is, an increase in the level of capital is associated with larger emissions, which decreases environmental quality and, therefore, decreases utility. The optimal environmental policy is summarized in the following proposition.

Proposition 3 *The optimal pollution tax vanishes asymptotically.*

Proof. *The planner chooses τ to maximize social utility (80) subject to the representative agent behavior, as given by (12)-(21). Substitution of (12)-(21) into (83)-(91) shows that the optimal tax is a function of the marginal social damage of pollution and in the neighborhood of the steady-state can be written as:*

$$\tilde{\tau} = \mu\frac{C}{P} = \mu\frac{C_0 e^{\tilde{\Phi}t}}{P_0 e^{\tilde{\phi}t}} \tag{94}$$

Since $\tilde{\phi} > \tilde{\Phi}$, asymptotically, the result follows. ■

Therefore, with no environmental policy in the rest of the world and perfect capital mobility, the externality is eliminated and the long-run optimal tax for the small open economy is zero.

3.2 Closed economy

Similarly, the social planner's problem can be stated for the closed economy case. The planner chooses C_1, C_2, K_1, K_2, H_1, H_2 to maximize:

$$U = \int_0^\infty \frac{1}{\gamma}(C_1^\theta C_2^{1-\theta} Q^\mu)^\gamma e^{-\rho t} dt \tag{95}$$

subject to the constraints (1), (2), (3), (10),

$$\dot{K} = Y_1 + pY_2 - C_1 - pC_2 - p\dot{H} \tag{96}$$

and the initial stocks of assets K_0, H_0.

The current value Hamiltonian for this optimization problem is:

$$\begin{aligned} H &= \frac{1}{\gamma}\left[C_1^\theta C_2^{1-\theta}(\sigma a K_1^\alpha H_1^{1-\alpha})^{-\mu}\right]^\gamma \\ &\quad + \acute{q_p}\left[aK_1^\alpha H_1^{1-\alpha} + pbK_2^\beta H_2^{1-\beta} - C_1 - pC_2 - p\dot{H}\right] + \acute{u_p}\dot{H} \end{aligned} \tag{97}$$

The first-order necessary conditions for an optimal solution, after some algebra, are given by:

$$\theta C_1^{\theta\gamma-1} C_2^{\gamma(1-\theta)} \left(\sigma a K_1^\alpha H_1^{1-\alpha}\right)^{-\mu\gamma} = \acute{q_p} \tag{98}$$

$$(1-\theta)\, C_1^{\theta\gamma} C_2^{\gamma(1-\theta)-1} \left(\sigma a K_1^\alpha H_1^{1-\alpha}\right)^{-\mu\gamma} = \acute{q_p} p \tag{99}$$

$$-\frac{\mu\alpha}{\theta}\frac{C_1}{K_1} + a\alpha K_1^{\alpha-1} H_1^{1-\alpha} = pb\beta K_2^{\beta-1} H_2^{1-\beta} \tag{100}$$

$$-\frac{\mu(1-\alpha)}{\theta}\frac{C_1}{H_1} + a(1-\alpha) K_1^\alpha H_1^{-\alpha} = pb(1-\beta) K_2^\beta H_2^{-\beta} \tag{101}$$

The optimality conditions with respect to K and H are similar to those of the competitive equilibrium:

$$\frac{\dot{\acute{q_p}}}{\acute{q_p}} = \rho - pb\beta K_2^{\beta-1} H_2^{1-\beta} \tag{102}$$

$$\frac{\dot{\acute{u_p}}}{\acute{u_p}} = \rho - \frac{pb(1-\beta) K_2^\beta H_2^{-\beta}}{p} \tag{103}$$

Accordingly, consumption along the efficient path grows at the rate:

$$\frac{\dot{C}}{C}=\frac{pb\beta K_2^{\beta-1}H_2^{1-\beta}-\rho-\gamma\left(1-\theta\right)\frac{\dot{p}}{p}-\mu\gamma\left[\alpha\frac{\dot{K}_1}{K_1}+\left(1-\alpha\right)\frac{\dot{H}_1}{H_1}\right]}{1-\gamma} \tag{104}$$

Thus, from (100) evaluated at the steady-state, the stationary growth rate of consumption is:

$$\frac{\dot{C}}{C}=\frac{a\alpha\tilde{\omega}^{\alpha-1}-\frac{\mu\alpha}{\theta}\frac{C_1}{K_1}-\rho}{1-\gamma\left(1-\mu\right)} \tag{105}$$

where the term $-\frac{\mu\alpha}{\theta}\frac{C_1}{K_1}$ summarizes the negative externality caused by pollution. The optimal environmental policy is summarized in the following proposition.

Proposition 4 *The first best-policy consists of a constant and positive tax on production of the polluting good given by $\tilde{\tau}=\mu C_0/P_0$, where C_0 and P_0 represent the initial conditions on consumption and pollution, in the neighborhood of the steady-state, respectively.*

Proof. *The competitive equilibrium steady-state growth rate of consumption is given by (74), which according to (61) evaluated at the steady-state, can be rewritten as:*

$$\frac{\dot{C}}{C}=\frac{\tilde{r}_K-\rho}{1-\gamma\left(1-\mu\right)}=\frac{\left(1-\tilde{\tau}\sigma\right)a\alpha\tilde{\omega}^{\alpha-1}-\rho}{1-\gamma\left(1-\mu\right)} \tag{106}$$

The efficient steady-state growth rate of consumption is given by (105).

Equating the competitive equilibrium path to the optimal social path and solving for τ we obtain the optimal pollution tax $\tilde{\tau}=\mu C_0/P_0$. ■

The tax is constant and positive in the steady-state, since all variables grow at the same growth rate. The optimal tax is positive since the pre-tax competitive path lies above the efficient path. This is in contrast to the small open economy, where the optimal tax rate is zero asymptotically.

4 Separable preferences

4.1 Small open economy

In the previuos sections, we have assumed non separable preferences between consumption and environmental quality. However, if the individual attributes a very high value to a good environmental quality he may not be willing to trade-off pollution for consumption in the margin. If this is the case, preferences are better represented by a separable utility function. Ultimately, the choice of the utility function is an empirical question.

Let us define the separable utility function:[16]

$$U = \frac{1}{\gamma}\left(C_T^{\theta} C_N^{1-\theta}\right)^{\gamma} - \frac{P^{\mu}}{\mu}, \qquad \mu > 0, \quad 0 \leq \theta \leq 1 \tag{107}$$

For the utility function to be increasing in C_T, C_N and decreasing in P, and strictly concave in C_T, C_N and P it has to be the case that $\mu > 1$, and $\gamma < 1$.

Solving for the competitive equilibrium, the current value Hamiltonian is:

$$\begin{aligned} H \quad = \quad & \left(\frac{1}{\gamma}\left(C_T^{\theta} C_N^{1-\theta}\right)^{\gamma} - \frac{P^{\mu}}{\mu}\right) + \acute{q} I + \lambda[(1-\tau\sigma)\, a K_T^{\alpha} H_T^{1-\alpha} + \qquad (108) \\ & pbK_N^{\beta} H_N^{1-\beta} + rB + T - C_T - pC_N - I\left(p^{*} + h\frac{I}{2K}\right) - p\dot{H}] + \acute{u}\dot{H} \end{aligned}$$

The first-order conditions with respect to C_T and C_N become:

$$\theta C_T^{\theta\gamma - 1} C_N^{\gamma(1-\theta)} = \lambda \tag{109}$$

$$(1-\theta)\, C_T^{\theta\gamma} C_N^{\gamma(1-\theta)-1} = \lambda p \tag{110}$$

The remaining optimality conditions are similar to those previously obtained with a non separable utility function (see (14)-(21)).

The relationships between C_T, C_N and C still hold (see (23)). However, given the separability of the utility function, the aggregate consumption growth rate is not affected by pollution growth:

$$\frac{\dot{C}}{C} = \frac{r - \rho - \gamma(1-\theta)\frac{\dot{p}}{p}}{1-\gamma} = \psi(t) \tag{111}$$

In the steady-state, the growth rate of consumption $\tilde{\psi}$ is determined by parameters that are constant: the rate of return on foreign bonds, r, the domestic rate of time preference, ρ, and the intertemporal elasticity of substitution, $\frac{1}{1-\gamma}$.

The discussion surronding the competitive equilibrium for a small open economy with non separable preferences still applies. The main difference relies on the growth rate of consumption in the steady-state and its implications.

[16] See Stokey [21].

Solving for the social planner´s problem, the current value Hamiltonian is:

$$\begin{aligned} H &= \left(\frac{1}{\gamma}\left(C_T^{\theta}C_N^{1-\theta}\right)^{\gamma} - \frac{\left(\sigma a K_T^{\alpha}H_T^{1-\alpha}\right)^{\mu}}{\mu}\right) + q_p' I + \lambda_p[aK_T^{\alpha}H_T^{1-\alpha} + \\ & pbK_N^{\beta}H_N^{1-\beta} + rB - C_T - pC_N - I\left(p^* + h\frac{I}{2K}\right) - p\dot{H}] + u_p'\dot{H} \end{aligned} \tag{112}$$

The first-order conditions with respect to the decision variables are given by:

$$\theta C_T^{\theta\gamma-1}C_N^{\gamma(1-\theta)} = \lambda_p \tag{113}$$

$$(1-\theta)\, C_T^{\theta\gamma}C_N^{\gamma(1-\theta)-1} = \lambda_p p \tag{114}$$

$$-\left(\sigma a K_T^{\alpha}H_T^{1-\alpha}\right)^{\mu}\alpha K_T^{-1} + \lambda_p\left[a\alpha K_T^{\alpha-1}H_T^{1-\alpha} - pb\beta K_N^{\beta-1}H_N^{1-\beta}\right] = 0 \tag{115}$$

$$-\left(\sigma a K_T^{\alpha}H_T^{1-\alpha}\right)^{\mu}(1-\alpha)\, H_T^{-1} + \lambda_p\left[a(1-\alpha)\, K_T^{\alpha}H_T^{-\alpha} - pb(1-\beta)\, K_N^{\beta}H_N^{-\beta}\right] = 0 \tag{116}$$

$$p^* + h\frac{I}{K} = q_p \tag{117}$$

$$\frac{u_p'}{\lambda_p} = p \tag{118}$$

The usual arbitrage conditions hold (see (89)-(91)). Similarly, the first term on the left hand of (115) and (116) reflects the marginal social damage of pollution. As in the competitive equilibrium, the growth rate of aggregate consumption is not affected by the growth rate of pollution.

Similarly, we can show that the optimal tax is a function of the marginal social damage of pollution and it is given as follows:

$$\tau = \frac{1}{\theta^{\theta\gamma}(1-\theta)^{(1-\theta)\gamma}}p^{(1-\theta)\gamma}C^{1-\gamma}P^{\mu-1} \tag{119}$$

The existence of a balanced growth path for capital with a constant pollution tax is only possible in a stationary economy ($\dot{C}/C = \dot{K}/K = 0$). So, $\tilde{\phi} = \tilde{\Phi} = 0$ must hold, [17] corresponding to the highest possible pollution tax rate. This is a well known result in the literature for a closed economy (see Stokey [21], or Reis [19]). Separability of the utility function is associated with a null growth rate of the economy, in the presence of a production externality. However, in a small

[17] τ comes from the solution for $\tilde{q} = p^*$, evaluated at $r = \rho$, or, alternatively, τ is the tax level for which the growth rate of consumption and the growth rate of capital are the same in the steady-state, $\tilde{\psi} = \tilde{\phi} = \frac{r-\rho}{1-\gamma} = 0$.

open economy, the interest rate, r, is exogenous, and, thus, cannot be adjusted, in contrast to the closed economy case. Therefore, the optimal pollution tax rate may not exist in a small open economy when the utility function is separable, except when $r = \rho$. This is exactly equivalent to the result for a closed economy, as we will show below. This has important consequences from a policy point of view.

4.2 Closed Economy

If we assume a separable utility function, the current value Hamiltonian for the competitive equilibrium is given by:

$$\begin{aligned} H &= \left(\frac{1}{\gamma} \left(C_1^{\theta} C_2^{1-\theta} \right)^{\gamma} - \frac{P^{\mu}}{\mu} \right) + q\acute{}[(1 - \tau\sigma)\, aK_1^{\alpha} H_1^{1-\alpha} + \\ & \quad pbK_2^{\beta} H_2^{1-\beta} + T - C_1 - pC_2 - p\dot{H}] + u\acute{}\dot{H} \end{aligned} \tag{120}$$

The first-order conditions with respect to C_1 and C_2 are:

$$\theta C_1^{\theta\gamma-1} C_2^{\gamma(1-\theta)} = q\acute{} \tag{121}$$

$$(1-\theta)\, C_1^{\theta\gamma} C_2^{\gamma(1-\theta)-1} = q\acute{}p \tag{122}$$

The remaining optimality conditions are similar to those previously obtained with a non separable utility function (see (61)-(64)). Given the optimality conditions with respect to the two types of consumption, the aggregate consumption growth rate is:

$$\frac{\dot{C}}{C} = \frac{r_K - \rho - \gamma\,(1-\theta)\,\frac{\dot{p}}{p}}{1-\gamma} \tag{123}$$

Thus, the steady-state growth rate of the economy is $g = \frac{\tilde{r}_K - \rho}{1-\gamma}$. The remaining features of the model are similar to those of the competitive equilibrium with a non separable utility function.

Solving for the social planner´s problem, the current value Hamiltonian is:

$$\begin{aligned} H &= \left(\frac{1}{\gamma} \left(C_1^{\theta} C_2^{1-\theta} \right)^{\gamma} - \frac{\left(\sigma a K_1^{\alpha} H_1^{1-\alpha} \right)^{\mu}}{\mu} \right) + \\ & \quad + q\acute{}_p \left[aK_1^{\alpha} H_1^{1-\alpha} + pbK_2^{\beta} H_2^{1-\beta} - C_1 - pC_2 - p\dot{H} \right] + u\acute{}_p \dot{H} \end{aligned} \tag{124}$$

The first-order necessary conditions for an optimal solution are:

$$\theta C_1^{\theta\gamma-1} C_2^{\gamma(1-\theta)} = q\acute{}_p \tag{125}$$

$$\left(1-\theta\right)C_1^{\theta\gamma}C_2^{\gamma(1-\theta)-1}=q_p^{'}p \tag{126}$$

$$-\left(\sigma aK_1^{\alpha}H_1^{1-\alpha}\right)^{\mu}\alpha K_1^{-1}+q_p^{'}\left[a\alpha K_1^{\alpha-1}H_1^{1-\alpha}-pb\beta K_2^{\beta-1}H_2^{1-\beta}\right]=0 \tag{127}$$

$$-\left(\sigma aK_1^{\alpha}H_1^{1-\alpha}\right)^{\mu}\left(1-\alpha\right)H_1^{-1}+q_p^{'}\left[a\left(1-\alpha\right)K_1^{\alpha}H_1^{-\alpha}-pb\left(1-\beta\right)K_2^{\beta}H_2^{-\beta}\right]=0 \tag{128}$$

The optimality conditions with respect to the stock variables are the same as (102) and (103). Similarly to the competitive equilibrium, the growth rate of aggregate consumption is not affected by the growth rate of pollution.

The optimal tax is given by:

$$\tau=\frac{1}{\theta^{\theta\gamma}\left(1-\theta\right)^{(1-\theta)\gamma}}p^{(1-\theta)\gamma}C^{1-\gamma}P^{\mu-1} \tag{129}$$

The existence of a balanced growth path with a constant tax in this economy is only possible in a stationary economy ($\dot{C}/C=\dot{K}/K=0$). Thus, $\tilde{r}_K=\rho$ must hold, meaning that the first-best solution is implemented through:

$$0<\tau=\frac{1}{\sigma}-\frac{\rho^{\frac{\beta-\alpha+1}{\beta}}}{\sigma a^{\frac{\beta-\alpha+1}{\beta}}\alpha\delta^{\frac{(\alpha-1)(\beta-\alpha)}{\beta}}\left(1-\alpha\right)^{\frac{1-\alpha}{\beta}}}\leq 1 \tag{130}$$

Given the environmental externality, the first-best policy consists of imposing the highest possible constant tax on the production of the polluting good, that is, the one for which growth is eliminated. This is a feasible outcome since the growth rate of the economy is endogenously determined. This result contrasts to the one obtained for the closed economy with non separable preferences, where the first-best policy was compatible with a positive growth rate of the economy. These findings support our previous comments according to which separability reflects more conservative preferences of society towards environmental quality.

5 Trade Policy: the case of a tariff

We now consider trade policy. We go back to the initial set up of a small open economy with non separable preferences, and with no pollution tax. Since we are in the case of a small open economy, we know that the optimal policy is a zero tariff. However, in the real world, economies often use trade policy in a second-best context, as an instrument of environmental policy.

Because countries differ in their location, proximity to suppliers and existing trade barriers, domestic prices will not be identical to world prices:

$$p_T = \epsilon p_T^* \tag{131}$$

where ϵ measures the importance of trade frictions and p_T^* is the common world price of Y_T. We assume that the country imports Y_T ($C_T > Y_T$), and is subject to a commercial tariff, so that $\epsilon > 1$. A movement of ϵ toward 1 captures a reduction in trade frictions. Because ϵ is greater than 1 for a dirty good importer this implies $\partial\epsilon/\epsilon < 0$.

In the previous sections, we assumed free trade ($p_T = p_T^*$). Now, with trade frictions, and, in particular, with a tariff on the imported good, $\frac{p_N}{p_{T^*}}$ increases in equilibrium, since the tariff discriminates against the traded good in the domestic market.

Following similar arguments to those used in previous sections, based on the expressions for the marginal products of the two types of capital, we can show that both $r_K(\tilde{p})$ and $r_H(\tilde{p})$ increase. In the steady-state, $r_H(\tilde{p})$ increases in the same proportion as $\tilde{p}$, so that $\frac{r_H(\tilde{p})}{\tilde{p}} = r$. Also, the capital intensities do not change in both sectors. The higher return to traded capital increases the price of installed capital $\tilde{q}$. Hence, the growth rate of capital increases and, consequently, the growth rate of consumption decreases. The higher ϵ, the higher the difference between capital and consumption growth rates in the steady-state.

We can also show that the optimal tariff is zero, since there is a balanced growth path only when $\tilde{\phi} > \tilde{\Phi}$. Thus, free trade is the first-best solution in a small open economy, asymptotically.

However, if the commercial tariff already exists and cannot be removed for political reasons, should the planner use a pollution tax to decrease production of the polluting good and compensate the effect of the tariff? From our previous analysis, we know that in this second-best world, the tax should exactly match the tariff rate, asymptotically. Therefore, the second-best pollution tax is larger than the first-best one, which we have shown to be zero in Proposition 3, but lower than in a static world. In a similar (second-best) static context, Fredriksson [12] has shown that the second-best pollution tax should be larger than the tariff.

6 Conclusion

This paper presents a two-sector endogenous growth model with physical and human capital where pollution is generated as a by-product of the production of the good relatively intensive in physical capital. The competitive equilibrium of this economy with a pollution tax levied on the output of the good that pollutes is examined in two cases: (i) small open economy with physical capital as the traded capital and the human capital as the nontraded one, facing a perfect international bond market, and (ii) closed economy.

In the case of the open economy, and in contrast to the case without pollution, the rate of growth of consumption is also determined by the growth rate of capital. The results obtained depend on the relative intensities of the two sectors in the two types of capital. In particular, assuming that the traded sector is relatively intensive in traded capital, both asset prices will follow transitional paths before eventually converging to their respective steady-state equilibria. This also holds in the closed economy.

In the presence of a tax levied on the output that pollutes, the growth rate of capital decreases, while the consumption growth rate increases, since it also depends on the growth rate of capital. As the growth rate on capital has to be at least the one of consumption, there might exist a threshold tax level for which the two tax rates are equated (on capital and consumption). However, we show that the optimal pollution tax vanishes asymptotically when preferences are non separable. In contrast, with separability, no optimal pollution tax may exist.

In the case of the closed economy, a positive pollution tax also decreases the steady-state growth rate of the economy (and pollution growth), yet keeping positive growth, except in the case of separable preferences where the optimal pollution tax eliminates growth. This reflects the more conservative preferences towards environment embedded in separable preferences.

Finally, we show how the optimal pollution tax rate interacts with trade policy, in particular, in the presence of a tariff on imports. In this second-best world, the optimal pollution tax is positive in the long-run and should be equated to the tariff rate. This is in contrast to the results found in a first-best world, as well as those obtained in a static second-best context, where Fredriksson [12] has shown that the second-best pollution tax should be larger than the tariff.

7 Appendix A - Competitive equilibrium for the closed economy with adjustment costs

We study the competitive equilibrium for the closed economy, keeping pollution exogenous. In order to make the results comparable with those obtained for the small open economy, we also consider adjustment costs for capital.[18]

The representative agent´s problem now becomes:

$$Max \int_0^\infty \frac{1}{\gamma}(C_1^\theta C_2^{1-\theta} Q^\mu)^\gamma e^{-\rho t} dt \tag{a.1}$$

$$\begin{aligned}
s.t.\ K &= K_1 + K_2 && \text{(a.2)}\\
H &= H_1 + H_2 && \text{(a.3)}\\
\dot{K} &= I_K && \text{(a.4)}\\
\dot{H} &= I_H && \text{(a.5)}\\
(1-\tau\sigma)Y_1 + T &= C_1 + I_K\left(1 + h\frac{I_K}{2K}\right) && \text{(a.6)}\\
Y_2 &= C_2 + I_H && \text{(a.7)}\\
Y_1 &= aK_1^\alpha H_1^{1-\alpha} && \text{(a.8)}\\
Y_2 &= bK_2^\beta H_2^{1-\beta} && \text{(a.9)}\\
Q &= \frac{1}{P} && \text{(a.10)}\\
& K_0,\ H_0 \text{ given} && \text{(a.11)}
\end{aligned}$$

The current value Hamiltonian for this optimization problem is:

$$\begin{aligned}
H &= \frac{1}{\gamma}\left[\left((1-\tau\sigma)aK_1^\alpha H_1^{1-\alpha} + T - I_K\left(1 + h\frac{I_K}{2K}\right)\right)^\theta \left(bK_2^\beta H_2^{1-\beta} - I_H\right)^{1-\theta} P^{-\mu}\right]^\gamma \\
&\quad + q I_K + u I_H
\end{aligned} \tag{a.12}$$

[18] In the context of a Ramsey model with adjustment costs, see Abel and Blanchard ([1]).

After making some substitutions, the first-order conditions with respect to the decision variables K_1, H_1, I_K, I_H are given by:

$$(1-\tau\sigma)a\alpha K_1^{\alpha-1}H_1^{1-\alpha} = \frac{(1-\theta)C_1}{\theta C_2}(b\beta K_2^{\beta-1}H_2^{1-\beta}) \tag{a.13}$$

$$(1-\tau\sigma)aK_1^{\alpha}(1-\alpha)H_1^{-\alpha} = \frac{(1-\theta)C_1}{\theta C_2}(bK_2^{\beta}(1-\beta)H_2^{-\beta}) \tag{a.14}$$

$$(C_1^{\theta}C_2^{1-\theta}P^{-\mu})^{\gamma}\theta C_1^{-1}\left(1+h\frac{I_K}{K}\right) = q\acute{} \tag{a.15}$$

$$(C_1^{\theta}C_2^{1-\theta}P^{-\mu})^{\gamma}(1-\theta)C_2^{-1} = u\acute{} \tag{a.16}$$

Defining aggregate consumption as before, the same demand functions for C_1 and C_2 are obtained (see (23) and (24)).[19]

We can rewrite the first-order conditions with respect to K_1 and H_1, as follows:

$$(1-\tau\sigma)a\alpha K_1^{\alpha-1}H_1^{1-\alpha} = pb\beta K_2^{\beta-1}H_2^{1-\beta} = r_K \tag{a.17}$$

$$(1-\tau\sigma)aK_1^{\alpha}(1-\alpha)H_1^{-\alpha} = pbK_2^{\beta}(1-\beta)H_2^{-\beta} = r_H, \tag{a.18}$$

which are exactly the same as in the small open economy.

The first-order conditions with respect to I_K and I_H yield the following relationship between p (the relative price of good 2 in terms of good 1) and $u\acute{}/q\acute{}$ (the opportunity cost of H in terms of the opportunity cost of K):

$$p = \frac{u\acute{}}{q\acute{}}\left(1+h\frac{I_K}{K}\right) \tag{a.19}$$

The optimality conditions with respect to K and H are:

$$\frac{\dot{q\acute{}}}{q\acute{}} = \rho - \frac{1}{\left(1+h\frac{I_K}{K}\right)}\frac{h}{2}\left(\frac{I_K}{K}\right)^2 - \frac{r_K}{\left(1+h\frac{I_K}{K}\right)} \tag{a.20}$$

$$\frac{\dot{u\acute{}}}{u\acute{}} = \rho - \frac{r_H}{p} \tag{a.21}$$

[19] Solving the utility maximizing problem: $Max\ \frac{1}{\gamma}\left(C_1^{\theta}C_2^{1-\theta}Q^{\mu}\right)^{\gamma}$ *s.t.* $C_1 + pC_2 = W$, we get the following relationship between the two types of consumption $\frac{\theta C_2}{(1-\theta)C_1} = \frac{1}{p}$, which together with the definition of aggregate consumption leads to the consumption of the two goods in (23).

Combining the time derivative of (a.15) with (24) implies that aggregate consumption grows at the following rate:

$$\frac{\dot{C}}{C} = \frac{\frac{1}{\left(1+h\frac{I_K}{K}\right)}\frac{h}{2}\left(\frac{I_K}{K}\right)^2 + \frac{r_K}{\left(1+h\frac{I_K}{K}\right)} - \rho - (1-\theta)\gamma\frac{\dot{p}}{p} + \frac{h\frac{d\left(\frac{I_K}{K}\right)}{dt}}{\left(1+h\frac{I_K}{K}\right)} - \mu\gamma\frac{\dot{P}}{P}}{1-\gamma} \qquad \text{(a.22)}$$

Dividing equation (a.17) by (a.18) we get the same relationship between the capital intensities in the two sectors as before (26). Substituting this equation into (a.17) yields:

$$\omega = \delta\left(\frac{1}{1-\tau\sigma}\right)^{\frac{1}{\alpha-\beta}} p^{\frac{1}{\alpha-\beta}} \qquad \text{(a.23)}$$

or,

$$p = \Delta(1-\tau\sigma)\omega^{\alpha-\beta} \qquad \text{(a.24)}$$

where $\Delta = \left[\left(\frac{a}{b}\right)\left(\frac{\alpha}{\beta}\right)^{\beta}\left(\frac{1-\alpha}{1-\beta}\right)^{1-\beta}\right]$. Therefore, the expressions for r_K and r_H in terms of p are the same as (28) and (29), respectively. Likewise, similar expressions are obtained for given absolute levels of the capital stocks instantaneously employed in the two sectors (see (30), (31)).

A.1. Price Dynamics

In this case, in order to have a positive consumption growth rate in the steady-state, $\dot{\acute{q}}/\acute{q}$ must be negative, since from (a.22),

$$\frac{\dot{C}}{C} = \frac{-\frac{\dot{\acute{q}}}{\acute{q}} - \mu\gamma\frac{\dot{P}}{P}}{1-\gamma} \qquad \text{(a.25)}$$

where $\frac{\dot{P}}{P} > 0$.

Assuming that $\dot{\acute{q}}/\acute{q} = -\Lambda$ in the steady-state, and since $p = \frac{\acute{u}}{\acute{q}}\left(1+h\frac{I_K}{K}\right)$, we get:

$$\frac{\dot{p}}{p} = \frac{\dot{\acute{u}}}{\acute{u}} - \frac{\dot{\acute{q}}}{\acute{q}} + \frac{h\frac{d\left(\frac{I_K}{K}\right)}{dt}}{\left(1+h\frac{I_K}{K}\right)} \qquad \text{(a.26)}$$

or,

$$\begin{aligned}\frac{\dot{p}}{p} \quad = \quad & -(1-\tau\sigma)^{\frac{\beta}{\beta-\alpha}}a(1-\alpha)\delta^{\alpha}p^{\frac{\beta}{\alpha-\beta}} + \frac{1}{\left(1+h\frac{I_K}{K}\right)}\frac{h}{2}\left(\frac{I_K}{K}\right)^2 + \qquad \text{(a.27)} \\ & + \frac{(1-\tau\sigma)^{\frac{1-\beta}{\alpha-\beta}}a\alpha\delta^{\alpha-1}p^{\frac{\alpha-1}{\alpha-\beta}}}{\left(1+h\frac{I_K}{K}\right)} + \frac{h\frac{d\left(\frac{I_K}{K}\right)}{dt}}{\left(1+h\frac{I_K}{K}\right)}\end{aligned}$$

The growth rate of p depends not only on p, but also on the growth rate of capital. Therefore, $\dot{p}/p$ is constant if and only if (i) p is constant and (ii) $I_K/K = \dot{K}/K$ is constant. So, in the steady-state $\dot{p}/p = 0$. Using (a.26) and assuming that $\dot{q}/q = -\Lambda$ in the steady-state, we have

$$(1-\tau\sigma)^{\frac{\beta}{\beta-\alpha}} a(1-\alpha)\delta^{\alpha} p^{\frac{\beta}{\alpha-\beta}} = \rho + \Lambda. \tag{a.28}$$

Thus, the relative price of the nontraded good in the steady-state is:

$$\tilde{p} = (1-\tau\sigma)\left(\frac{\rho+\Lambda}{a(1-\alpha)\delta^{\alpha}}\right)^{\frac{\alpha-\beta}{\beta}} \tag{a.29}$$

A.2. Characterization of Steady-State

Given that (i) $\frac{\dot{C}_1}{C_1} = \frac{\dot{C}_2}{C_2} = \frac{\dot{C}}{C}$, (ii) $\frac{\dot{Y}_1}{Y_1} = \frac{\dot{K}_1}{K_1} = \frac{\dot{H}_1}{H_1} = \frac{\dot{K}_2}{K_2} = \frac{\dot{H}_2}{H_2} = \frac{\dot{K}}{K} = \frac{\dot{H}}{H} = \frac{\dot{Y}_2}{Y_2} = \frac{\dot{P}}{P}$, (iii) $\frac{\dot{K}}{K}$ is constant, then all variables grow at the same constant rate $g = \tilde{\Phi} = \frac{\dot{C}}{C} = \frac{\dot{K}}{K} = \frac{\dot{H}}{H} = \frac{\dot{P}}{P}$. From (a.22) evaluated at the steady-state, we obtain the following expression

$$\mu\gamma\tilde{\Phi} + \rho + \tilde{\Phi}(1-\gamma) = \frac{1}{\left(1+h\tilde{\Phi}\right)}\frac{h}{2}\tilde{\Phi}^2 + \frac{(1-\tau\sigma)a\alpha\delta^{\alpha-1}\left(\frac{\rho+\Lambda}{a(1-\alpha)\delta^{\alpha}}\right)^{\frac{\alpha-1}{\beta}}}{1+h\tilde{\Phi}} \tag{a.30}$$

that implicitly defines the growth rate along the balanced growth path in terms of the parameters of the model, where $r_K(\tilde{p}) = (1-\tau\sigma)a\alpha\delta^{\alpha-1}\left(\frac{\rho+\Lambda}{a(1-\alpha)\delta^{\alpha}}\right)^{\frac{\alpha-1}{\beta}}$, and $\Lambda = [1-\gamma(1-\mu)]\,\tilde{\Phi}$ from (a.25).

From (a.30), we can determine how the different parameters in the model affect g. In this case, and assuming that $\alpha > \beta$, by totally differentiating equation (a.30), we can show that $\frac{\partial\tilde{\Phi}}{\partial\tau} < 0$, as long as $\mu > \frac{1}{2}$. Thus, the larger the tax the lower g will be, that is, the lower is the growth rate of the economy as well as pollution growth. In contrast to the small open economy case, the growth rate of consumption also decreases, as consumption and capital now grow at the same rate along the balanced growth path.

In summary, with a tax on pollution affecting symetrically K and H in sector 1, the steady-state is characterized by a lower growth rate of the economy $\tilde{\Phi}$, associated with a lower Λ, and a lower relative price of good 2 in terms of good

1, $\tilde{p}$. The decrease in $\tilde{p}$ increases consumption of good 2 relative to consumption of good 1. Using (a.26) evaluated at the steady-state and combining it with (a.21) implies that the real rate of return on human capital measured in terms of the numéraire, $\frac{\tilde{r}_H}{\tilde{p}}$, decreases. This means that $\tilde{r}_H$ decreases more than proportionally to p̃. Therefore, the relative capital intensity ratio in sector 2, K_2/H_2, decreases, as well as K_1/H_1. This implies that $\frac{\tilde{r}_K}{\tilde{p}}$ must increase ($\tilde{r}_K$ decreases, but less than proportionally to $\tilde{p}$).

Using the constraints of the representative agent's problem, and the definitions of Y_1 and Y_2 (71), we obtain

$$\frac{C}{K} = \frac{\Phi - \frac{r_K(1-\beta)}{(1-\tau\sigma)(\alpha-\beta)} + \frac{h}{2}\Phi^2 - \frac{r_H r_K(1-\alpha)\beta}{(1-\tau\sigma)p(\beta-\alpha)^2\left[\Phi - \frac{r_H\alpha}{p(\alpha-\beta)}\right]}}{\frac{(1-\theta)\beta r_H}{(1-\tau\sigma)(\alpha-\beta)p\left[\Phi-\frac{r_H\alpha}{p(\alpha-\beta)}\right]} - \theta} \tag{a.31}$$

and

$$\begin{aligned}\frac{K}{H} &= \frac{\Phi - \frac{r_H\alpha}{p(\alpha-\beta)}}{-\frac{r_K(1-\alpha)}{p(\alpha-\beta)} - \frac{1-\theta}{p}\frac{C}{K}} = \\ &= \frac{\Phi - \frac{r_H\alpha}{p(\alpha-\beta)}}{-\frac{r_K(1-\alpha)}{p(\alpha-\beta)} - \frac{1-\theta}{p}\left[\frac{\Phi - \frac{r_K(1-\beta)}{(1-\tau\sigma)(\alpha-\beta)} + \frac{h}{2}\Phi^2 - \frac{r_H r_K(1-\alpha)\beta}{(1-\tau\sigma)p(\beta-\alpha)^2\left[\Phi-\frac{r_H\alpha}{p(\alpha-\beta)}\right]}}{\frac{(1-\theta)\beta r_H}{(1-\tau\sigma)(\alpha-\beta)p\left[\Phi-\frac{r_H\alpha}{p(\alpha-\beta)}\right]} - \theta}\right]}\end{aligned} \tag{a.32}$$

evaluated at the steady-state. Therefore, these ratios are both constant.

The next step will be to study the convergence of the system to the balanced growth path.

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