Hotelling Revisited: Oil Prices and Endogenous Technological Progress

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Preliminary—Comments welcome

Abstract

This paper examines the Hotelling model of optimal exhaustible resource extraction in light of stock effects and technological progress. I hope to answer the question: why have petroleum and minerals prices been trendless despite resource scarcity? In particular, I examine how endogenous technology-induced shifts in the cost function must have evolved over time in order to maintain a constant market price for nonrenewable resources. According to my results, in the absence of stock effects, market prices remain constant when the cost shifts rise at the rate of interest. In the presence of a simple form of a stock effect, market prices remain constant when the *net* cost shifts rise at the *effective* rate of interest, where the effective rate of interest adjusts the interest rate to account for the stock effect.

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1 Introduction

The basic Hotelling model of nonrenewable resource extraction predicts that the shadow price of the resource stock, which is an economic measure of the scarcity of the resource, should grow at the rate of interest (Hotelling, 1931). This prediction is now known as the "Hotelling rule" (Krautkraemer, 1998). If the natural resource market is perfectly competitive, then the Hotelling rule implies that the market price minus marginal costs must grow at the rate of interest, and therefore that the natural resource price should be increasing over time if marginal costs are constant. In contrast to Hotelling's theoretical prediction, however, empirical studies have shown that mineral prices have been roughly trendless over time (see Krautkraemer, 1998, & references therein; Lin 2004a,b).[2] This paper attempts to reconcile Hotelling's theoretical model with empirical evidence on world mineral prices.

There are several possible reasons why the Hotelling rule may not be a good guide to the actual behavior of mineral prices over time. In this paper I focus on two such reasons. First, the simple Hotelling model assumes that the costs of extraction do not depend on the stock of reserve remaining in the ground. However, it is plausible that extraction costs increase as more of the resource is extracted and fewer reserves remain. For instance, extraction costs may increase inversely with the remaining stock of reserves if the resource needed to be extracted from greater depths or if well pressure declined as more of the reserve was depleted. Another possible explanation is that since different grades of oil may differ in their extraction costs, and since the cheaper grades are likely to be mined to exhaustion before the more expensive grades are mined, the cost of extraction may increase as the cheaper grades are exhausted. I use the term "stock effect" to refer to the dependence of extraction cost on the stock of reserve extracted. With a stock effect, the shadow price rises less slowly than the rate of interest, but the market price still increases over time (Tietenberg, 1996).

In addition to stock effects, a second reason why the Hotelling rule does not adequately describe the actual behavior of world mineral prices is that technological progress may occur that enhances the ability of firms to extract ore. Such technological progress would cause the extraction cost schedule to decrease over time, and may result in a U-shaped market price trajectory in which the resource price declines over some initial interval and then increases as the effect of a finite reserve outweighs the effect of a declining extraction cost (Krautkraemer, 1998).

This paper thus focuses on two diametrically opposed factors that may cause real world prices to diverge from the basic Hotelling rule: stock effects which increase extraction costs and are consistent with rising resource prices, and technological progress which lowers extraction costs and may cause prices to initially decline.[3]

[2] Some studies have shown evidence for a U-shaped price path (see e.g., Slade, 1982), but the coefficients on the quadratic trend are not robust to the period of estimation (Berck & Roberts, 1996).

[3] There are several other factors in addition to stock effects and technological progress that are ignored in the basic Hotelling model. A third reason why the Hotelling rule is not a good guide to the actual behavior of world mineral prices and extraction paths over time is that discoveries of new reserves may take place. Discoveries expand the known stock of oil, and therefore decrease the

I pursue the following research question: *Given extraction costs as a function of extraction and of the remaining resource stock, how must cost-reducing technological progress evolve in order to maintain a constant market price for the nonrenewable resource?* In other words, *if technological progress were an endogenous process that acted to stabilize resource prices, what would its time path look like?*

According to my results, in the absence of stock effects, market prices remain constant when shifts in the cost function rise at the rate of interest. In the presence of a simple form of a stock effect, market prices remain constant when the *net* cost shifts rise at the *effective* rate of interest, where the effective rate of interest adjusts the interest rate to account for the relative stock effect. For general cost and demand functions, a component of the cost shifter must still rise at the effective interest rate, where for this more general case the effective interest rate adjusts the market interest rate for not only the relative stock effect, but also for an elasticity effect, for the effect of stocks on marginal costs, and for the growth rate of demand.

The balance of this paper proceeds as follows. In Section 2, I present the basic Hotelling model. In Section 3, I derive and present my main results on endogenous technological progress. Section 4 concludes.

2 The Hotelling Model

In this section, I present my theoretical model of optimal nonrenewable extraction under perfect competition. The notation follows closely that used by Weitzman (2003).

Suppose there are T oil markets indexed by $t = 1, ..., T$. For each time t, the supply of oil is given by $E(t)$, the total extraction flow in units of oil per unit time at time t.[4] When the oil market is perfectly competitive, extraction $E(t)$ represents the total amount of oil extracted by all the firms in the market at a given point in time.[5] Let $S(t)$ denote the stock of oil remaining in the ground at time t:

$$S(t) = S(0) - \int_0^t E(\tau)d\tau \quad , \tag{1}$$

scarcity of the resource and, if there are stock effects, decrease costs as well. Moreover, discoveries can revise expectations about the future value of the resource stock, thereby generating new expected time paths for resource price and extraction. A fourth reason why the simple Hotelling rule may not apply to real-world mineral prices is that stock effects may exist not only on the cost side, but on the benefit side as well. Benefits may be a function of the remaining resource stock if, for example, the depletion of the resource causes a permanent loss of environmental amenities such as those from the recreational, scientific or aesthetic services generated by the reserve. A fifth complexity ignored by the Hotelling model is the capital-intensive nature of mineral industries. Adjustment costs to and capacity constraints on investments in extractive capital would alter some of the basic Hotelling implications. Market imperfections are a sixth potential source of the divergence between the Hotelling rule and the actual evidence on world mineral prices (Krautkraemer, 1998). A seventh complexity is that extraction costs may be uncertain; in contrast, the basic Hotelling model assumes that costs are deterministic.

[4] I assume that, at any given time t, all the oil extracted at time t is sold on the market at time t.

[5] I ignore any common access problems that may arise in perfect competition. In other words, I assume, as does Pindyck (1978), that there is a large number of identical firms that all ignore ignore each other, or, equivalently, that a social planner or a state-owned company has sole production rights and sets a competitive price.

where the initial stock $S(0)$ is taken as given.

The market price of oil at time t is $P(t)$. The demand for oil at time t when the market price is P is given by the demand function $D(P,t)$. Markets are assumed to clear, which means that, at each time t, the price $P(t)$ acts to equate supply and demand:

$$E(t) = D(P(t),t)\ \forall t. \tag{2}$$

The total benefits $U(\cdot,\cdot)$ from oil at time t is given by the area under the demand curve:

$$U(E(t),t) = \int_0^{E(t)} D^{-1}(x;t)dx, \tag{3}$$

where $D^{-1}(\cdot;t)$ is the inverse of the demand curve with respect to price. This area measures the gross consumer surplus, and is a measure of the consumers' total money-metricized willingness-to-pay. As shown in Weitzman (2003), using the area under the demand curve in place of revenue yields the same outcome as a perfectly competitive market.[6] For mathematical simplicity, I thus choose to model the perfectly competitive firm's maximization problem using the area under the demand curve.

The cost of extracting E units of oil at time t when there are S units of reserve remaining is given by $C(S,E,t)$. As explained in the introduction, I use the term *stock effect* to refer to the dependence $\frac{\partial C}{\partial S}(\cdot)$ of extraction cost on the stock S of reserve remaining, and this dependence is likely to be negative. I use the term *relative stock effect* to refer to the ratio $\frac{\left|\frac{\partial C}{\partial S}(\cdot)\right|}{\frac{\partial C}{\partial E}(\cdot)}$ of the absolute value $\left|\frac{\partial C}{\partial S}(\cdot)\right|$ of the stock effect over the marginal extraction cost $\frac{\partial C}{\partial E}(\cdot)$.

Let $p(t)$ denote the non-negative current-value shadow price measuring the value of a unit of reserve at time t. This shadow price is known by a variety of terms, including "marginal user cost", because it measures the opportunity cost of extracting the resource; "in situ value", because it measures the marginal value of leaving an additional unit of resource in the ground; "scarcity rent", because it is an economic measure of scarcity, and "dynamic rent", to reflect the difference between price and marginal extraction cost (Krautkraemer, 1998; Weitzman, 2003). As the reader will soon see, the shadow price plays a crucial role in the Hotelling model.

The competitive interest rate is ρ.

The social planner's optimal control problem, which yields the same solution as would arise in perfect competition, is to choose her extraction profile $\{E(t)\}$ to maximize the present discounted value of her entire

[6] This is because

$$P(t) = \frac{\partial U(E(t),t)}{\partial E},$$

so that the first-order conditions for the social planner's problem are the same as those that arise in perfect competition.

stream of net benefits, given her initial stock $S(0)$ and given the relationship between her extraction $E(t)$ and the stock remaining $S(t)$, and subject to the constraints that both extraction and stock are nonnegative. Her problem is thus given by:

$$\begin{aligned} &\max_{\{E(t)\}} \int_0^{\infty} \left(U(E(t),t) - C(S(t),E(t),t)\right) e^{-\rho t} dt \\ &\text{s.t.} \qquad \overset{\bullet}{S}(t) = -E(t) \qquad : p(t) \\ &\qquad\qquad E(t) \geq 0 \\ &\qquad\qquad S(t) \geq 0 \\ &\qquad\qquad S(0) = S_0 \ , \end{aligned} \tag{4}$$

where the multiplier $p(t)$ associated with the equation of motion for the stock $S(t)$ of oil remaining is precisely the shadow price $p(t)$ of the reserve.

From the Maximum Principle, the first-order necessary conditions for a feasible trajectory $\{S^*(t), E^*(t)\}$ to be optimal are:[7]

$$[\#1]: \qquad p(t) = P(t) - \frac{\partial C\left(S(t), E(t), t\right)}{\partial E} \tag{5}$$

$$[\#2]: \qquad \overset{\bullet}{p}(t) = \frac{\partial C\left(S(t), E(t), t\right)}{\partial S} + \rho p(t) \tag{6}$$

$$[\#3]: \qquad \lim_{t \to \infty} p(t) S(t) e^{-\rho t} = 0 \tag{7}$$

Condition [#1] states that, at each time t, the shadow price $p(t)$ must equal the competitive market price $P(t)$ minus the marginal cost of extraction $\frac{\partial C(S(t),E(t),t)}{\partial E}$; this condition is needed to ensure static optimality at each point in time. Condition [#2] governs how the shadow price $p\left(t\right)$ must evolve over time; conditions [#1] and [#2] combined are needed to ensure intertemporal optimality over all finite subperiods. Condition [#3], the transversality condition, is required for the solution to be dynamically optimal over the entire infinite horizon (Weitzman, 2003).

In order to solve the Hotelling resource extraction problem (4) for extraction and market price trajectories, one needs to make functional form assumptions on both the demand function $D(P,t)$ and the cost function $C(S,E,t)$. In particular, I make the following assumptions on the cost function $C(S,E,t)$:

[7] These same conditions can be derived using either the present-value or the current-value Hamiltonian, even though this is not a time-autonomous problem.

A1) (multiplicatively separable technological progress)

The cost function consists of two multiplicatively separable terms:

$$C(S, E, t) = \varphi(S, E) \cdot h(t) \tag{8}$$

where $\varphi(S, E) \geq 0$ is the time-independent component of the extraction cost, representing the costs in the absence of technological change; and $h(t) > 0$ is the time-varying component, capturing how the cost schedule shifts over time due to changes in technology.

A2) (negative stock effects)

Both total cost and marginal extraction costs decrease with the amount of reserve S remaining in the ground:

$$\frac{\partial C}{\partial S}(S, E, t) = \frac{\partial \varphi}{\partial S}(S, E) \leq 0 \tag{9}$$

and

$$\frac{\partial^2 C}{\partial S \partial E}(S, E, t) = \frac{\partial^2 \varphi}{\partial S \partial E}(S, E) \leq 0. \tag{10}$$

I call $h(t)$ the *cost shifter*. The lower the value of the cost shifter, the lower the costs of extraction. If the cost function shifts down over time, then $h'(t) \leq 0$.

Suppose endogenous technological progress acted to maintain prices at a constant price $\overline{P}$. I now ask: what must the trajectory of the technology-induced cost shifter $\{h(t)\}$ look like in order for the market price trajectory $\{P(t)\}$ to be constant at $\overline{P}$?

3 Endogenous Technological Progress

I now examine how endogenous technological progress must evolve so that resource prices remain constant first in the benchmark case of no stock effects, then in the simplest case of a stock effect, and lastly in the most general case where only the assumptions A1 of a multiplicatively separable cost shifter and A2 of negative stock effects are made on the cost function and no assumptions are made on the demand function.

3.1 The benchmark case: No stock effects

As a benchmark, I first derive the trajectory that the technology-induced cost shifter $h(t)$ must follow in order to maintain a constant market price for oil when costs do not depend on the remaining resource stock S.

Proposition 1 *Under A1, when marginal extraction costs are nonzero, but when extraction costs neither depend on the remaining resource stock (i.e., $\frac{\partial C}{\partial S}(\cdot) = 0$) nor are convex in the rate of extraction (i.e., $\frac{\partial^2 C}{\partial E^2}(\cdot) = 0$), then, in order to maintain a constant market price (i.e., $P(t) = \overline{P}\ \forall t$), the extraction cost function must shift upwards at the rate of interest of interest:*

$$\frac{\dot{h}(t)}{h(t)} = \rho. \tag{11}$$

Proof. When there are no stock effects (i.e., $\frac{\partial C}{\partial S}(\cdot,\cdot,\cdot) = 0$), then condition [#2] yields the Hotelling rule that the shadow price rises at the rate of interest:

$$\frac{\dot{p}(t)}{p(t)} = \rho. \tag{12}$$

When combined with condition [#1], this means that the market price minus marginal costs must increase at the rate of interest:

$$\frac{\frac{d}{dt}\left(P(t) - \frac{\partial C}{\partial E}(X(t), E(t), t)\right)}{P(t) - \frac{\partial C}{\partial E}(X(t), E(t), t)} = \rho, \tag{13}$$

which yields, after rearranging terms, the following equation for the growth rate of market price in the absence of stock effects:

$$\frac{\dot{P}(t)}{P(t)} = (1 - \theta(t))\rho + \theta(t)\frac{\frac{d}{dt}\left(\frac{\partial C}{\partial E}(X(t), E(t), t)\right)}{\frac{\partial C}{\partial E}(X(t), E(t), t)} \tag{14}$$

where the weight $\theta(t)$ is defined as:

$$\theta(t) \equiv \frac{-\frac{\partial C}{\partial E}(X(t), E(t), t)}{P(t)}, \tag{15}$$

which is non-zero when marginal extraction costs are nonzero.

Under A1, when there are no stock effects and costs are linear in extraction, equation (14) reduces to:

$$\frac{\dot{P}(t)}{P(t)} = (1 - \theta(t))\rho + \theta(t)\frac{\dot{h}(t)}{h(t)}. \tag{16}$$

In order for market price to be constant (i.e., $\dot{P}(t) = 0$), we need $h(t)$ to rise at rate ρ. ■

The intuition is as follows. As seen in Krautkraemer (1998), the growth rate of the resource price in the absence of stock effects is a weighted average of the discount rate and the growth rate in marginal extraction cost, where the weights $\theta(t)$ are given by the ratio of marginal extraction cost to price. The first term on the right-hand side of equation (14) measures the effect of scarcity while the second term measures the effect of changes in marginal extraction cost. If the marginal extraction cost $\frac{\partial C}{\partial E}(\cdot)$ is constant over time, then only the scarcity effect applies and market price still grows but at a rate less than the discount rate. If technological change acts to decrease marginal extraction costs, then it is possible for the declining cost effect to dominate the scarcity effect, especially at the beginning of an extraction horizon, and therefore for prices to decline.[8]

Thus, in the absence of extraction costs, the effect of scarcity would cause the market price to rise at rate ρ. In order to cancel out this effect so that market prices are constant, we would need the cost shifter $h(t)$ to rise at rate ρ as well. Thus, when there are no stock effects and when costs are linear in extraction, the cost shifter $h(t)$ must rise at the rate of interest ρ.

I now examine how the cost shifter $h(t)$ must evolve in order to maintain a constant market price when stock effects are present.

3.2 The basic case: Simple stock effect

To examine how endogenous technological progress must evolve to keep the market price for oil constant in the presence of stock effects, I first use a simple cost function that exhibits a stock effect. In particular, I assume that the cost function takes the following form:

B1) (simple stock effect)

Extraction costs are linear in extraction E and exponential in the remaining reserve stock S:

$$\varphi(S,E) = F(S)\cdot E \tag{17}$$

where the exponential stock-dependent component $F(S)$ of the cost function is given by:

$$F(S) = \Psi e^{-\sigma S}, \tag{18}$$

with $\Psi \geq 0$ and $\sigma \geq 0$.

[8] As the marginal extraction cost decreases over time, however, the scarcity effect is given greater weight and may eventually dominate, causing prices to increase. Thus, cost-decreasing technological progress may result in a U-shaped price path (Krautkraemer, 1998).

The *stock effect parameter* σ measures the extent of the dependence of cost on the remaining stock of reserve; the greater the σ, the more severe the dependence.

There is some empirical support for B1: using world data on proven and estimated reserves and on extraction costs compiled by the East-West Center Energy Program to test a variety of function forms for the oil extraction costs, Chakravorty, Roumasset and Tse (1997) found that the cost function that best fit the data was of the form:

$$C(S,E) = \Psi e^{\sigma(S_0 - S)} E$$

where, when S is in units of billion mmBtu and costs are in units of dollars per mmBtu, the parameter values are given by $\Psi = 0.1774$ and $\sigma = 0.000217$.[9]

Let the *stock effect growth rate* g to be defined as the growth rate of the stock-dependent component $F(S)$ of the cost function:

$$g(t) \equiv \frac{\frac{d}{dt}F(S(t))}{F(S(t))}. \tag{19}$$

It turns out that the stock effect growth rate g is a measure of the stock effect, as seen in the following Lemma:

Lemma 2 *(i) The stock effect growth rate g is equal to the relative stock effect:*

$$g(t) = \frac{\left|\frac{\partial C}{\partial S}(\cdot)\right|}{\frac{\partial C}{\partial E}(\cdot)}. \tag{20}$$

(ii) The stock effect growth rate g is also proportional to the stock effects parameter σ:

$$g(t) = \sigma E(t). \tag{21}$$

Proof. (i) $g(t) \equiv \frac{\frac{d}{dt}F(S(t))}{F(S(t))} = \frac{-F'(S(t))E(t)}{F(S(t))} = \frac{\left|\frac{\partial C}{\partial S}(\cdot)\right|}{\frac{\partial C}{\partial E}(\cdot)}$. (ii) $g(t) \equiv \frac{\frac{d}{dt}F(S(t))}{F(S(t))} = \sigma E(t)$. ■

As the reader will see shortly, the stock effect growth rate plays a crucial role in my main results.

In addition to assumption B1 on the cost function, for this basic case I also make the following assumption on demand:

[9] The cost function estimated by Chakravorty et al. (1997) differs from the one implied by A1 and B1, however, because it does not include the time-dependent term $h(t)$.

B2) (stationary demand)

Demand is stationary:

$$D(P,t) = D(P) \ \ \forall t.$$

If endogenous technological progress acted to maintain prices at a constant price $\overline{P}$, then under B2, extraction rates $E(t)$ would be constant at $\overline{E} \equiv D(\overline{P})$ and the reserve remaining at time t would be given by $S(t) = S_0 - \overline{E}t \equiv \overline{S}(t)$. By Lemma 2, when market prices are constant, the stock effect growth rate $g(t)$ is constant at $\overline{g} = \sigma\overline{E}$.

Before I proceed to the central result of my paper, let me define several concepts that will be critical to its interpretation.

First, I define the **effective interest rate** R as the following:

$$R \equiv \rho - g. \tag{22}$$

The effective interest rate R is composed of two terms: (1) the discount rate ρ, which is a measure of the *scarcity effect*; and (2) the stock effect growth rate g, which, as seen in Lemma 2, is a measure of the *relative stock effect*. It adjusts the market discount rate ρ downward to account for the stock effect. All else equal, a higher relative stock effect decreases the effective discount rate. Under B2, g is constant at $\sigma\overline{E}$ by Lemma 2. As a consequence, R is constant under B2 as well.

The second key concept I will introduce is that of the net cost shifter. I define the **net cost shifter** $\widetilde{h}(t)$ as:

$$\widetilde{h}(t) \equiv h(t) - \frac{\overline{P}}{F(S(t))}. \tag{23}$$

The net cost shifter is the cost shifter $h(t)$ net of the ratio of price $\overline{P}$ to the stock-dependent component $F(S(t))$ of the cost function.

I now present the central result of my paper.

Theorem 3 *Under A1, A2, B1 and B2, the technology-induced cost shifter $h(t)$ would act so as to maintain the market resource price $P(t)$ constant at $\overline{P}$ if the net cost shifter were to rise at the effective rate of interest:*

$$\widetilde{h}(t) = \widetilde{h}(0)e^{Rt}. \tag{24}$$

Proof. From [#1],

$$\overset{\bullet}{p}(t) = \overset{\bullet}{P}(t) - \frac{\partial^2 C(\cdot)}{\partial E^2}\overset{\bullet}{S}(t) - \frac{\partial^2 C(\cdot)}{\partial S \partial E}\overset{\bullet}{E}(t) - \frac{\partial^2 C(\cdot)}{\partial t \partial E}$$

When combined with [#2],

$$-\frac{\partial C(\cdot)}{\partial S} + \rho p(t) = \overset{\bullet}{P}(t) - \frac{\partial^2 C(\cdot)}{\partial E^2}\overset{\bullet}{S}(t) - \frac{\partial^2 C(\cdot)}{\partial S \partial E}\overset{\bullet}{E}(t) - \frac{\partial^2 C(\cdot)}{\partial t \partial E}$$

When combined with [#1],

$$-\frac{\partial C(\cdot)}{\partial S} + \rho\left(P(t) - \frac{\partial C(\cdot)}{\partial E}\right) = \overset{\bullet}{P}(t) - \frac{\partial^2 C(\cdot)}{\partial E^2}\overset{\bullet}{S}(t) - \frac{\partial^2 C(\cdot)}{\partial S \partial E}\overset{\bullet}{E}(t) - \frac{\partial^2 C(\cdot)}{\partial t \partial E}$$

$$\Rightarrow P(t) = \frac{1}{\rho}\overset{\bullet}{P}(t) + \frac{1}{\rho}\frac{\partial C(\cdot)}{\partial S} + \frac{\partial C(\cdot)}{\partial E} + \frac{1}{\rho}\frac{\partial^2 C(\cdot)}{\partial E^2}E(t) - \frac{1}{\rho}\frac{\partial^2 C(\cdot)}{\partial S \partial E}\overset{\bullet}{E}(t) - \frac{1}{\rho}\frac{\partial^2 C(\cdot)}{\partial t \partial E}$$

Suppose market price were constant at $\overline{P}$. Then, under B2,

$$\begin{aligned}
\overline{P} &= \frac{1}{\rho}\frac{\partial C(\cdot)}{\partial S} + \frac{\partial C(\cdot)}{\partial E} + \frac{1}{\rho}\frac{\partial^2 C(\cdot)}{\partial E^2}\overline{E} - \frac{1}{\rho}\frac{\partial^2 C(\cdot)}{\partial t \partial E} \\
&= h(t)\left(\frac{1}{\rho}\frac{\partial \varphi(\cdot)}{\partial S} + \frac{\partial \varphi(\cdot)}{\partial E} + \frac{1}{\rho}\frac{\partial^2 \varphi(\cdot)}{\partial E^2}\overline{E}\right) - \frac{1}{\rho}\frac{\partial \varphi(\cdot)}{\partial E}\overset{\bullet}{h}(t) \\
&= h(t)\left(\frac{1}{\rho}F'(S(t))\overline{E} + F(S(t))\right) - \frac{1}{\rho}F(S(t))\overset{\bullet}{h}(t).
\end{aligned}$$

where the second step comes from A1 and the third step comes from B1. Rearranging terms, one gets the following first-order differential equation for $h(t)$:

$$\overset{\bullet}{h}(t) - Rh(t) = -\frac{\overline{P}}{F(S(t))},$$

which has as its solution:

$$h(t) = \frac{\overline{P}}{F(S(t))} + \left(h(0) - \frac{\overline{P}}{F(S_0)}\right)e^{Rt}, \tag{25}$$

which yields the desired result under A2.

To confirm that [#3] is satisfied, note that:

$$
\begin{aligned}
\lim_{t\to\infty} p(t)S(t)e^{-\rho t} &= \lim_{t\to\infty}\left(P(t)-\frac{\partial C}{\partial E}\left(S(t),E(t),t\right)\right)S(t)e^{-\rho t}\\
&= \lim_{t\to\infty}\left(P(t)-\frac{\partial\varphi\left(\cdot\right)}{\partial E}h(t)\right)S(t)e^{-\rho t}\\
&= \lim_{t\to\infty}\left(\overline{P}-F(S(t))h(t)\right)\left(S_0-\overline{E}t\right)e^{-\rho t}\\
&= \lim_{t\to\infty} -F(S(t))h(t)\left(S_0-\overline{E}t\right)e^{-\rho t}\\
&= \lim_{t\to\infty} -F(S(t))\left(\frac{\overline{P}}{F(S(t))}+\left(h(0)-\frac{\overline{P}}{F(S_0)}\right)e^{Rt}\right)\left(S_0-\overline{E}t\right)e^{-\rho t}\\
&= \lim_{t\to\infty} -F(S(t))\left(h(0)-\frac{\overline{P}}{F(S_0)}\right)\left(S_0-\overline{E}t\right)e^{gt}\\
&= \lim_{t\to\infty} -\Psi e^{-\sigma S(t)}\left(h(0)-\frac{\overline{P}}{\Psi e^{-\sigma S_0}}\right)\left(S_0-\overline{E}t\right)e^{gt}\\
&= \lim_{t\to\infty} -\Psi e^{-\sigma(S_0-\overline{E}t)}\left(h(0)-\frac{\overline{P}}{\Psi e^{-\sigma S_0}}\right)\left(S_0-\overline{E}t\right)e^{gt}\\
&= \lim_{t\to\infty} -\Psi e^{-\sigma S_0}\left(h(0)-\frac{\overline{P}}{\Psi e^{-\sigma S_0}}\right)\left(S_0-\overline{E}t\right)\\
&= 0
\end{aligned}
$$

where the first equality comes from [#1] and the second comes from A1, and the third falls from B1-B2 and assuming that market price $P(t)$ is constant at $\overline{P}$. ■

We saw from Proposition 1 that in the absence of stock effects and when the costs are linear in extraction, then the cost shifter $h(t)$ must rise at the rate of interest ρ in order to offset the effects of scarcity on price. According to Proposition 3, when stock effects are present in the simple form posited by B1 (where the cost function is still linear in extraction), then the *net* cost shifter $\widetilde{h}(t)$ must rise at the *effective* rate of interest R.

The intuition is as follows. The effective interest rate R is the market interest rate ρ minus the stock effect growth rate g. All else equal, a higher the scarcity effect, as measured by ρ, would require the net cost shifter to rise faster, while a more severe stock effect, as measured by g, would require the net cost shifter to rise more slowly, or even decay. If the cost shifter is decaying over time, this means that technological progress is shifting the cost function downwards over time. Thus, in order to keep the market price constant, technological progress must cause the cost function to shift downwards faster over time if stock effects were present than it would if stock effects were absent.

What does the cost shifter $h(t)$ look like over the infinite horizon? The answer depends critically on the sign of the effective interest rate R, as can be seen in the following Corollary:

Corollary 4 *Under A1, A2, B1, and B2,* *(i) If the market interest rate is greater than the stock effect growth rate (i.e., $R > 0$), then* $\lim_{t\to\infty} h(t) = \infty$.

(ii) If the market interest rate is less than the stock effect growth rate (i.e., $R < 0$), then $\lim_{t\to\infty} h(t) = 0$.

(iii) If the market interest rate equals the stock effect growth rate (i.e., $R = 0$), then $\lim_{t\to\infty} h(t) = h(0) - \frac{\overline{P}}{\Psi e^{-\sigma S_0}}$.

Proof. $\lim_{t\to\infty} h(t) = \lim_{t\to\infty} \left(h(0) - \frac{\overline{P}}{F(S_0)}\right) e^{Rt}$. ■

Thus, if the scarcity effect were *greater* than the relative stock effect, then, in order to maintain a constant market price, costs would need to shift upwards until they eventually became infinite. If, on the other hand, the scarcity effect were *less* than the stock effect, then the endogenous technology would need to shift the cost function downwards until cost were eventually equal to zero. In the knife-edge case in which the scarcity effect exactly offset the stock effect, then, assuming that $h(0) \neq \frac{\overline{P}}{F(S_0)}$, the cost function would be finite and nonzero over the infinite horizon.

Having examined endogenous technological progress in a simple case of a stock effect and stationary demand, I now generalize my results to any cost function satisfying assumptions A1 of multiplicatively separable technological progress and A2 of negative stock effects, and to any demand function.

3.3 The general case of fully flexible cost and demand

In this section I examine endogenous technological progress with minimal restrictions on the cost function and no restrictions on demand. In particular, the only assumptions that I make on the cost function $C(S, E, t)$ are assumption A1 of multiplicatively separable technological progress and assumption A2 of negative stock effects. The demand function $D(P, t)$ can be of any form; in particular, unlike in the previous section, demand can be non-stationary.

When demand is non-stationary, then under a constant market price $\overline{P}$, the extraction rate at time t would be given by $E(t) = D(\overline{P}, t) \equiv \overline{E(t)}$ and the remaining resource stock at time t would be given by $S(t) = S_0 - \overline{E(t)}t \equiv \overline{S(t)}$.

Let $R(t; \overline{P})$ denote the effective interest rate at time t when market price is constant at $\overline{P}$. $R(\cdot)$ is given by:

$$R(t, \overline{P}) = \rho - \frac{\left|\frac{\partial \varphi\left(\overline{S(t)}, \overline{E(t)}\right)}{\partial S}\right|}{\frac{\partial \varphi\left(\overline{S(t)}, \overline{E(t)}\right)}{\partial E}} + \frac{\frac{\partial^2 \varphi\left(\overline{S(t)}, \overline{E(t)}\right)}{\partial E^2} D(\overline{P}, t)}{\frac{\partial \varphi\left(\overline{S(t)}, \overline{E(t)}\right)}{\partial E}} + \frac{\left|\frac{\partial^2 \varphi\left(\overline{S(t)}, \overline{E(t)}\right)}{\partial S \partial E}\right| \frac{\partial D(\overline{P}, t)}{\partial t}}{\frac{\partial \varphi\left(\overline{S(t)}, \overline{E(t)}\right)}{\partial E}} \tag{26}$$

As before, the first term in the expression for the effective interest rate is the interest rate ρ, which is a measure of the scarcity effect. All else equal, a higher actual interest rate increases the effective interest rate. Also as before, the second term is the non-negative stock effect growth rate $g(t)$, which is a measure of relative stock effect:

$$g(t) = \frac{\left|\frac{\partial\varphi\left(\overline{S(t)},\overline{E(t)}\right)}{\partial S}\right|}{\frac{\partial\varphi\left(\overline{S(t)},\overline{E(t)}\right)}{\partial E}}.$$

All else equal, a higher stock effect growth rate decreases the effective interest rate.

There are now two terms in addition to the interest rate and the stock effect growth rate that affects the effective interest rate $R(\cdot)$. The first additional term (which is the third term in (26)) is the non-negative elasticity of the marginal cost with respect to extraction, a term which measures the curvature of the cost function with respect to extraction and which I will call the *elasticity effect*. All else equal, a greater elasticity effect increases the effective interest rate. The second additional term (which is the last term in (26)) measures both the effects of the reserve stock on marginal costs and the growth rate of demand. All else equal, both a greater effect of the stock on marginal extraction costs and greater demand growth increases the effective interest rate.

To derive my most general result, I will need to make the following assumption on the parameters:

C1) (transversality condition)

$$\lim_{t\to\infty} -\frac{\partial\varphi\left(\overline{S(t)},\overline{E(t)}\right)}{\partial E}\left(h(0)-\rho\overline{P}\int_0^t\left(\frac{1}{\frac{\partial\varphi\left(\overline{S(t)},\overline{E(t)}\right)}{\partial E}}e^{-\int^{\tau}R(v,\overline{P})dv}d\tau\right)\right)\overline{S(t)}e^{\int^{t}R(\tau,\overline{P})d\tau-\rho t}=0.$$

My general result is as follows.

Proposition 5 *Using only A1, A2 and C1, the expression for the cost shifter needed to maintain a constant resource price is given by:*

$$h(t)=\left(h(0)-\rho\overline{P}\int_0^t\left(\frac{1}{\frac{\partial\varphi(\cdot)}{\partial E}}e^{-\int^{\tau}R(v,\overline{P})dv}d\tau\right)\right)e^{\int^{t}R(\tau,\overline{P})d\tau}.$$

Proof. From [#1] and [#2],

$$P(t)=\frac{1}{\rho}\dot{P}(t)+\frac{1}{\rho}\frac{\partial C(\cdot)}{\partial S}+\frac{\partial C(\cdot)}{\partial E}+\frac{1}{\rho}\frac{\partial^2C(\cdot)}{\partial E^2}E(t)-\frac{1}{\rho}\frac{\partial^2C(\cdot)}{\partial S\partial E}\dot{E}(t)-\frac{1}{\rho}\frac{\partial^2C(\cdot)}{\partial t\partial E}$$

Suppose market price were constant at $\overline{P}$. Then

$$\overline{P}=\frac{1}{\rho}\frac{\partial C(\cdot)}{\partial S}+\frac{\partial C(\cdot)}{\partial E}+\frac{1}{\rho}\frac{\partial^2C(\cdot)}{\partial E^2}\overline{E(t)}-\frac{1}{\rho}\frac{\partial^2C(\cdot)}{\partial S\partial E}\dot{\overline{E(t)}}-\frac{1}{\rho}\frac{\partial^2C(\cdot)}{\partial t\partial E},$$

which yields the desired result under A1 and A2.

To check if [#3] is satisfied, note that:

$$\begin{aligned}
\lim_{t\to\infty} p(t)S(t)e^{-\rho t} &= \lim_{t\to\infty}\left(P(t) - \frac{\partial C}{\partial E}(S(t),E(t),t)\right)S(t)e^{-\rho t} \\
&= \lim_{t\to\infty}\left(P(t) - \frac{\partial \varphi(\cdot)}{\partial E}h(t)\right)S(t)e^{-\rho t} \\
&= \lim_{t\to\infty}\left(\overline{P} - \frac{\partial \varphi(\cdot)}{\partial E}h(t)\right)\overline{S(t)}e^{-\rho t} \\
&= \lim_{t\to\infty} -\frac{\partial \varphi(\cdot)}{\partial E}h(t)\overline{S(t)}e^{-\rho t} \\
&= \lim_{t\to\infty} -\frac{\partial \varphi(\cdot)}{\partial E}\left(h(0) - \rho\overline{P}\int_0^t\left(\frac{1}{\frac{\partial \varphi(\cdot)}{\partial E}}e^{-\int^{\tau} R(v,\overline{P})dv}d\tau\right)\right)\overline{S(t)}e^{\int^t R(\tau,\overline{P})d\tau-\rho t} \\
&= 0.
\end{aligned}$$

where the first equality comes from [#1], the second comes from A1, and the third falls from assuming that market price $P(t)$ is constant at $\overline{P}$, and where for the last step it is sufficient to assume C1. ■

Although the expression for the cost shifter trajectory $\{h(t)\}$ that maintains a constant market price in the most general case of Proposition 5 is more complicated than that for the simple stock effect of Theorem 3, the former still shares some of the features of the latter. In particular, the cost shifter in the general case still consists of a term (i.e., $h(0)e^{\int^t R(\tau,\overline{P})d\tau}$) that rises at the effective interest rate $R(\cdot)$ and another term (i.e., $-\rho\overline{P}\int_0^t\left(\frac{1}{\frac{\partial\varphi(\cdot)}{\partial E}}e^{\int_\tau^t R(v,\overline{P})dv}d\tau\right)e^{\int^t R(\tau,\overline{P})d\tau}$) that evolves over time. Thus, as before, the effective interest rate still governs the evolution of the cost shifter, at least in part. Also as before, the effective interest rate adjusts the market interest rate ρ downward to account for the relative stock effect g, although now the effective interest rate is also adjusted upward to account for the elasticity effect, the effect of stocks on marginal extraction costs, and the growth rate of demand.

4 Conclusion

This paper examines the Hotelling model of optimal exhaustible resource extraction in light of stock effects and technological progress. I hope to answer the question: why have petroleum and minerals prices been trendless despite resource scarcity? In particular, I examine how endogenous technology-induced shifts in the cost function must have evolved over time in order to maintain a constant market price for nonrenewable resources.

According to my results, in the absence of stock effects, market prices remain constant when cost shifts rise at the rate of interest. In the presence of a simple form of a stock effect, market prices remain constant when the *net* cost shifts rise at the *effective* rate of interest, where the effective rate of interest adjusts the interest rate to account for the relative stock effect. For general cost and demand functions, a component of the cost shifter must still rise at the effective interest rate, where for this more general case the effective interest rate adjusts the market interest rate for not only the relative stock effect, but also for an elasticity effect, for the effect of stocks on marginal costs, and for the growth rate of demand.

The results of this paper reconcile Hotelling's theoretical model with empirical evidence on the trendless nature of world mineral prices and may have important implications for future examinations of endogenous technological progress.

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