A Simple Dynamic Model of the Environmental Kuznets Curve*

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We employ a simple dynamic macroeconomic model in the spirit of Andreoni and Levinson (2001) to investigate a number of important issues related to the Environmental Kuznets Curve. By focusing on the social solution, we are able to derive analytical solutions for the critical thresholds of income and point in time at which pollution starts to decline. The consequences of external effects and public policies on these critical thresholds are investigated numerically by simulating the transition process. It turns out that the impact of even small market failures is tremendous and hence there is a strong role for public policy. Moreover, we show that an observed N-shaped pollution-income relation (PIR) can be plausibly explained from the interaction of public policy measures and the intrinsic properties of the model. The model implies that this N-shaped PIR is indeed an M-shaped PIR.

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1 Introduction

The Environmental Kuznets Curve (EKC) hypothesis states that there is an inverted U-shaped relationship between environmental degradation and the level of income. Since the seminal contribution of Grossman and Krueger (1993), this pattern has been intensively debated in empirical terms (recent reviews are provided by Dasgupta et al. 2002 or Cavlovic et al. 2000).[1] The EKC has also captured large attention from policymakers and theorists. To some extent, this is due to the fact that the EKC hypothesis implies that pollution diminishes once a critical threshold level of income is reached. As a consequence, there is the hope that - loosely speaking - the environmental problem sooner or later peters out as the economy grows.

A number of models have been developed to provide theoretical explanations of the EKC pattern. Two major strands can be distinguished.[2] The first class of models stresses shifts in the use of production technologies. In Stokey (1998), the dirtiest but most productive technology is used at low levels of income. The economic reason simply is that marginal utility of consumption is higher than marginal disutility of pollution. Economic growth is accompanied with increasing environmental degradation. Behind a critical threshold, cleaner but less productive technologies are implemented and a decoupling of economic growth and environmental degradation occurs. In Smulders and Bretschger (2000), technology shifts need not be environmental friendly. In their model, technology shifts lead inter alia to more pollution during initial phases of economic development. As the process of economic development proceeds, technology shifts become more environmental friendly. The causes of these shifts are the availability of new general purpose technologies.

The second class of EKC models focuses on the abatement technology, which captures the fact that pollution can be alleviated by devoting resources to improve environmental quality. In Selden and Song (1995), abatement is zero initially and starts to increase once economic development has created enough consumption and environmental damage (through capital accumulation) to merit expenditures on abatement. Similar results are presented by Chimeli and Braden (2002). Formulating a simple growth model with environmental quality (a stock variable), the authors show that capital accumulation dominates at early stages of economic development and environmental effort is of secondary importance. Subsequently, abatement becomes more relevant, attracts more resources and economic growth declines. John and Pecchenino (1994) draw comparable conclusions using an OLG model. Again, the economy eventually switches from a corner solution with no environmental effort and increasing environmental degradation to a solution where abatement is positive

[1] The empirical evidence of the EKC hypothesis is mixed. Most estimations with cross-country data support the hypothesis. Whereas estimations with times series data, which are to be preferred for econometrical reasons, are less optimistic; no clear curve pattern can be found (see e.g. Egli 2003).

[2] A third strand of models stresses structural changes within an economy, see de Groot (1999). However, the underlying mechanism is restricted to developing countries and does not apply to mature economies. As a result, this mechanism has not attracted great attention in the EKC literature.

and economic growth comes along with increasing environmental quality. Brock and Taylor (2004) amend the Solow growth model to include emissions, abatement and a stock of pollution. Assuming an appropriate rate of (external) technological progress in the abatement, they show that an EKC may result along the transition to the balanced growth path.[3]

The models summarised in the second class share two features: First, pollution is a function of the capital stock, which evolves sluggishly over time. This is not plausible to the extent that a decrease in pollution would require to reduce the stock of capital in these modes. Instead, it appears more plausible to assume that pollution is related to some control variables, which are allowed to change instantaneously. Second, the explanation of the EKC relies on some form of discontinuities.[4] This implies that there must be abrupt changes in either the technological opportunities or the economic incentives at some specific point in time. Apart from the empirical plausibility of such discontinuities, the question arises whether an EKC can be explained without relying on discontinuities.

Another prominent approach which focuses on the importance of the abatement technology is the static Andreoni and Levinson (2001) (thereafter AL) model. Assuming that the abatement technology exhibits increasing returns to scale (IRS), AL show that an inverted U-shaped pattern between pollution and income results. This approach has several important advantages: First, by focusing on the degree of returns to scale in abatement, AL are able to summarise a large part of the literature dealing with very different mechanisms (e.g. a shift in technology or a shift in institutions). All these mechanisms require a form of IRS (e.g. due to fixed costs). By modelling the abatement technology directly, the authors show that the degree of IRS is indeed crucial for the explanation of an EKC. Second, the model is fairly simple so that analytical results can be derived.

The present paper contributes to the literature on dynamic EKC models along several dimensions: First, we generalise the static AL model and set up a simple dynamic model of the EKC in the spirit of AL. Thereby, we can show that basic results derived by AL are also valid within a dynamic set-up. Second, focusing on the social planner's problem we derive closed-form solutions for the resulting dynamic system. This enables us to determine analytically the critical level of income and point in time at which pollution starts to decline. As a result, the economic determinants behind these critical thresholds can be identified. Third, we introduce external effects into the model and investigate the consequences of public policies on the shape of the pollution-income relation (PIR). Unfortunately, the decentral solution does not allow a closed-form solution of the resulting dynamic system. Nonetheless, we investigate the effects of public policy measures on the critical threshold values numerically by simulating the transition process. On this occasion, we distinguish between isolated policy measures and a comprehensive policy programme diminishing all market

[3]Moreover, it should be noted that most approaches stress the importance of a sufficiently high income elasticity of demand for environmental quality. It can be shown, however, that a high income elasticity for environmental quality is indeed helpful for an EKC conformable pattern, but it is neither sufficient nor necessary (McConnell 1997).

[4]This does not apply to the Green Solow model of Brock and Taylor (2004).

distortions simultaneously.

The remainder of this paper is organised as follows: In Section 2, the basic AL model is sketched. In Section 3, a general dynamic EKC model in the spirit of AL is set up. We first solve the problem of the social planner, then determine the market solution and finally derive optimal taxes within a general framework. In Section 4, a specific dynamic EKC model is employed to investigate a number of important issues. The critical level of income and point in time at which pollution peaks are determined analytically. Next, the transition process of the market solution is simulated and the consequences of public policy measures for the critical thresholds are investigated. Finally, Section 5 summarises the main results and concludes.

2 The Andreoni and Levinson EKC model

In their seminal paper, Andreoni and Levinson (2001) set up a simple static model to derive sufficient conditions for an EKC analytically. We sketch the AL model below to provide a reference point for the following discussion.

Utility of the representative agent depends on consumption C and pollution P. The general utility function may be expressed as:

$$U = U(C, P), \tag{1}$$

where $U(C, P)$ is quasiconcave in C and $-P$ and both arguments $(C, -P)$ are normal goods. Pollution is a function of consumption and environmental effort E according to:

$$P = C - B(C, E). \tag{2}$$

Pollution increases one by one with consumption (gross pollution) as represented by the first term on the RHS. On the other hand, pollution decreases due to abatement as represented by the second term of the RHS. $B(C, E)$ is the abatement technology, which is increasing in both arguments consumption C and environmental effort E. Both "inputs" are essential for abatement, i.e. $B(0, E) = B(C, 0) = 0$. One the one hand, it is clear that abatement requires a positive amount of environmental effort, i.e. $E > 0$. On the other hand, effective abatement necessarily requires pollution, i.e. $C > 0$. Otherwise, cleaning up would simply be ineffective.

The final basic equation is a standard budget constraint given by:

$$M = C + E, \tag{3}$$

where M denotes the resources available (income) and is spent either on consumption or environmental effort.

AL show that there are two conditions which guarantee the existence of an EKC (AL, 2001, p. 277). The first condition (related to the preference side of the model) states that the marginal willingness to pay to clean up the last speck of pollution does not go to zero as income approaches infinity. As AL notice, this is a rather weak condition; it is easily satisfied since pollution abatement can be regarded as a normal good. The second condition (related to the abatement technology) states that there must be IRS in abatement. Both conditions together are sufficient for the existence of an EKC.

Using the following parameterisation $U(C,P) = C - zP$ with $z = 1$ and $B(C,E) = C^\alpha E^\beta$, AL show that an EKC results provided that $\alpha+\beta > 1$. This result follows immediately from the pollution function in terms of M, which has the following shape:

$$P(M) = \frac{\alpha}{\alpha+\beta}M - \left(\frac{\alpha}{\alpha+\beta}\right)^\alpha \left(\frac{\beta}{\alpha+\beta}\right)^\beta M^{\alpha+\beta} \quad (4)$$

The preceding equation results from the fact that $P = C - C^\alpha E^\beta$ and $C^* = \frac{\alpha}{\alpha+\beta}M$ and $E^* = \frac{\beta}{\alpha+\beta}M$, where C^* and E^* are the optimal level of consumption and environmental effort. Equation 4 implies that $P(M)$ is concave in M provided that $\alpha+\beta > 1$. Hence, IRS in abatement defined by $\alpha+\beta > 1$ represent a necessary condition for the existence of an EKC.

3 A general dynamic EKC model

In this section, we set up and concisely discuss the general dynamic EKC model, which is employed in the course of the paper. At first, the socially controlled economy is considered. Subsequently, the decentral equilibrium is derived taking external effects associated with polluting consumption and environmental effort into account. Finally, optimal taxes are determined.

3.1 The social planner's problem

The social planner maximizes welfare of the representative household, who derives utility from consumption C and disutility from pollution P. The instantaneous utility function is given by $U(C,P)$ with $U_C > 0$, $U_{CC} < 0$, $U_P < 0$ and $U_{PP} < 0$.[5] Pollution is modelled as flow pollution and results from the difference between gross pollution $G(C,\bar{C})$ and abatement $B(C,E,\bar{E})$, i.e. $P(C,\bar{C},E,\bar{E}) = G(C,\bar{C}) - B(C,E,\bar{E})$, where E is environmental effort and a "bar" above a variable denotes its economywide average level. Although it is not necessary to distinguish between individual and average levels at this stage, we use this formulation to enable a direct comparison with the market economy.

As the above pollution function shows, we model pollution to result from consumption. It is more common to assume that pollution results from production (e.g. Xepapadeas, 2004). At a microeconomic level, the appropriate kind of modelling would clearly depend on the specific activity under study. Within the current macroeconomic framework both assumptions appear plausible in principle. Moreover, there are other theoretical studies which assume that consumption generates pollution (Andreoni and Levinson, 2001 and John and Pecchenino, 1994). Most importantly, however, polluting consumption represents a simplifying assumption, which does not affect the qualitative results

[5] We do not restrict the cross derivative $U_{CP} = U_{PC}$ at this stage.

on the PIR.[6]

Final output is produced with a constant returns technology $F(K)$ employing capital K as the sole input factor. The social planner's problem may be expressed as follows (time index omitted):

$$\max_{\{C,\bar{C},E,\bar{E}\}} \int_0^\infty U(C,P)e^{-\rho t}\,dt \tag{5}$$

$$\begin{aligned} s.t.\;\; P(C,\bar{C},E,\bar{E}) &= G(C,\bar{C}) - B(C,E,\bar{E}) && (6)\\ \dot{K} &= F(K) - C - E - \delta K && (7)\\ K(0) &= K_0. && (8)\end{aligned}$$

The dynamic problem possesses two (independent) choice variables ($C = \bar{C}$ and $E = \bar{E}$) and one state variable (K).

Pollution should be considered to be restricted by $P \geq 0$; there is no pollution stock so that flow pollution cannot become negative. Since we are interested in an inverted U-shaped PIR, we restrict our attention to interior solutions. The dynamic problem above can be easily extended to allow for border solutions with $P = 0$.[7]

The current-value Hamiltonian reads as follows:

$$H = U[C, P(C,\bar{C},E,\bar{E})] + \lambda[F(K) - C - E - \delta K] \tag{9}$$

The necessary first-order conditions are given by:[8]

$$H_C = U_C + U_P(P_C + P_{\bar{C}}) - \lambda = 0 \quad\Longleftrightarrow\quad U_C + U_P(P_C + P_{\bar{C}}) = \lambda \tag{10}$$

$$H_E = U_P(P_E + P_{\bar{E}}) - \lambda = 0 \quad\Longleftrightarrow\quad U_P(P_E + P_{\bar{E}}) = \lambda \tag{11}$$

$$\dot{\lambda} = -H_K + \rho\lambda = -\lambda(F_K - \delta) + \rho\lambda \quad\Longleftrightarrow\quad \dot{\lambda} = -\lambda(F_K - \delta - \rho) \tag{12}$$

$$\dot{K} = H_\lambda = F(K) - \delta K - C - E. \tag{13}$$

Equation (10) shows that along the optimal growth path marginal utility of consumption must equal the shadow price of capital. The marginal utility of consumption comprises two components: (i) direct utility from consumption U_C and (ii) disutility from pollution $U_P(P_C + P_{\bar{C}})$. Notice that this disutility term is composed of an "internal" and an "external" effect. Moreover, it should be remembered that P_C (as well as $P_{\bar{C}}$) captures a gross pollution effect G_C and an abatement effect B_C. Similarly, equation (11) indicates that marginal utility from environmental effort $U_P(P_E + P_{\bar{E}})$ must equal the shadow price of capital. Equation (12) shows that for $F_K - \delta - \rho > 0$ the shadow price of capital vanishes at the rate $F_K - \delta - \rho$. Finally, equation (13) reproduces the flow budget constraint.

[6]Within the current framework, polluting production has two unfavourable consequences: (i) the model then shows transitional dynamics and (ii) a balanced growth path does not exist. See the appendix for details.

[7]A less technical and economically more plausible possibility to avoid $P < 0$ is to restrict the degrees of IRS in abatement such that pollution would be constant in the long run.

[8]In addition, the transversality condition $\lim_{t\to\infty} e^{-\rho t}\lambda K = 0$ must hold. Moreover, we assume that the necessary conditions are also sufficient for a maximum of the utility functional.

In order to give a rigourous interpretation of the model dynamics, we derive the Keynes-Ramsey rule (KRR) for optimal consumption and environmental effort. The problem at this stage lies in the fact that the resulting expressions would be fairly complex for the original model stated above. Therefore, we use a slightly more compact formulation. Substituting the net pollution function $P = P(C, \bar{C}, E, \bar{E})$ into the utility function $U = U(C, P)$ gives $U = U[C, P(C, \bar{C}, E, \bar{E})]$. Moreover, noting that $C = \bar{C}$ and $E = \bar{E}$ we express the (transformed) utility function as $V(C, E)$. This is admissible provided that we keep in mind that, for instance, $V_C = U_C + U_P(P_C + P_{\bar{C}})$ when interpreting the results. With this formulation equations (10) and (11) become:

$$V_C = \lambda \tag{14}$$

$$V_E = \lambda. \tag{15}$$

The KRR of optimal C and E results from logarithmic differentiation of equations (14) and (15), eliminating $\hat{\lambda} := \frac{\dot{\lambda}}{\lambda}$ using (12) and solving for $\hat{C} := \frac{\dot{C}}{C}$ and $\hat{E} := \frac{\dot{E}}{E}$. This procedure finally yields:

$$\frac{\dot{C}}{C} = \frac{\sigma_{CE} - \sigma_{EE}}{\sigma_{CE}\sigma_{EC} - \sigma_{CC}\sigma_{EE}}(F_K - \delta - \rho) \tag{16}$$

$$\frac{\dot{E}}{E} = \frac{\sigma_{CC} - \sigma_{EC}}{\sigma_{CC}\sigma_{EE} - \sigma_{CE}\sigma_{EC}}(F_K - \delta - \rho) \tag{17}$$

where $\sigma_{CC} := -\frac{V_{CC}C}{V_C} > 0$, $\sigma_{CE} := -\frac{V_{CE}E}{V_C} < 0$, $\sigma_{EC} := -\frac{V_{EC}C}{V_E} < 0$ and $\sigma_{EE} := -\frac{V_{EE}E}{V_E} > 0$.[9]

The above differential equations prove that the steady state level of capital (or the long-run growth rate) is independent of the social planner's concern about pollution. This can be recognised by considering the RHS of equations (16) and (17). For the neoclassical model ($F_K > 0$, $F_{KK} < 0$) steady state requires $\frac{\dot{C}}{C} = \frac{\dot{E}}{E} = 0$ implying that $F_K = \delta + \rho$. Hence, the level of capital which satisfies this condition is the same as the one resulting from the underlying growth model without pollution ($U_P = 0$). Next consider an endogenous growth framework leading to sustained growth ($F_K =$ const. and $F_K - \delta - \rho > 0$). Provided that $\lim_{t\to\infty} \sigma_{EC} = 0$ ($\lim_{t\to\infty} \sigma_{CE} = 0$), the asymptotic KRR simplify to read:

$$\frac{\dot{C}}{C} = \frac{1}{\sigma_{CC}}(F_K - \delta - \rho) \tag{18}$$

$$\frac{\dot{E}}{E} = \frac{1}{\sigma_{EE}}(F_K - \delta - \rho). \tag{19}$$

Once more, the long-run outcome is independent of the social planner's concern about pollution. Notice that the above equations would hold true for each point

[9]Remember that $V(C, E) = U[C, P(C, E)]$ and hence $V_{CE} = U_P P_{CE} > 0$ since $U_P < 0$ and $P_{CE} < 0$.

in time if C and E enter the (transformed) utility function additively separable, i.e. $\sigma_{CE} = \sigma_{EC} = 0$.

To interpret the KRR displayed above, we consider the KRR in the Dorfman (1969, p. 825) form [resulting from equations (14), (15) and (12)]:

$$F_K - \delta - \sigma_{CE}\frac{\dot{E}}{E} = \rho + \sigma_{CC}\frac{\dot{C}}{C} \tag{20}$$

$$F_K - \delta - \sigma_{EC}\frac{\dot{C}}{C} = \rho + \sigma_{EE}\frac{\dot{E}}{E} \tag{21}$$

Holding an additional unit of capital during a short period of time causes a rising consumption and environmental effort profile. Along the optimal consumption and environmental effort path, the rate of consumption and the rate of environmental effort must be chosen such that the marginal benefits (displayed on the LHS) equals the marginal costs (on the RHS). Considering equation (20), the marginal benefits comprise the net marginal product of capital ($F_K - \delta$) as well as the increase in the marginal utility of consumption due to an increase in E ($\sigma_{CE}\frac{\dot{E}}{E} < 0$ for $\dot{E} > 0$). Marginal costs cover the time preference rate and the reduction in the marginal utility due to an increase in C ($\sigma_{CC}\frac{\dot{C}}{C} > 0$ for $\dot{C} > 0$); "the psychic cost of saving". An analogous interpretation holds for equation (21).

3.2 The decentral economy

We introduce two kinds of externalities such that the decentral allocation departs from the social planner's solution. On the one hand, polluting consumption is partly not taken into account by the representative individual, i.e. there is a (negative) pollution externality. On the other hand, the benefits from environmental effort do also affect the society as a whole and consequently there is a (positive) externality associated with environmental effort.

The pollution function $P = G(C, \bar{C}) - B(C, E, \bar{E})$ captures these effects. External effects are associated with the economywide averages of consumption $\bar{C}$ and environmental effort $\bar{E}$. These average levels are considered as exogenous from the perspective of the representative household.

Since we assume that consumption is polluting, the external effect results from household activities. Regarding environmental effort, we can interpret the model in the sense that either households or firms conduct abatement. For ease of modelling, we assume that households conduct abatement.

The dynamic problem of the representative household may then be expressed as follows:

$$\max_{\{C,E\}} \int_0^\infty U(C, P)e^{-\rho t}\, dt \tag{22}$$

$$\begin{aligned} s.t. \quad P(C, \bar{C}, E, \bar{E}) &= G(C, \bar{C}) - B(C, E, \bar{E}) && (23)\\ \dot{K} &= rK - (1 + \tau_C)C - (1 + \tau_E)E - \delta K + T && (24)\\ K(0) &= K_0, && (25) \end{aligned}$$

where r denotes the net interest rate and τ_C and τ_E represent taxes (subsidies). Overall tax revenues are redistributed in a lump-sum manner according to a balanced-budget rule, i.e. $T = \tau_C C + \tau_E E$.[10]

The current-value Hamiltonian for the problem of the representative household reads as follows:

$$H = U[C, P(C, \bar{C}, E, \bar{E})] + \lambda[rK - (1 + \tau_C)C - (1 + \tau_E)E - \delta K + T] \quad (26)$$

The necessary first-order conditions are given by:[11]

$$H_C = U_C + U_P P_C - \lambda(1 + \tau_C) = 0 \quad \Longleftrightarrow \quad \frac{U_C + U_P P_C}{1 + \tau_C} = \lambda \quad (27)$$

$$H_E = U_P P_E - \lambda(1 + \tau_E) = 0 \quad \Longleftrightarrow \quad \frac{U_P P_E}{1 + \tau_E} = \lambda \quad (28)$$

$$\dot{\lambda} = -H_K + \rho\lambda = \lambda r + \rho\lambda \quad \Longleftrightarrow \quad \dot{\lambda} = \lambda(r + \rho) \quad (29)$$

$$\dot{K} = H_\lambda = rK - (1 + \tau_C)C - (1 + \tau_E)E + T. \quad (30)$$

Considering the first-order conditions for the control variables [equations (27) and (28)], it becomes evident that there are two differences between the market allocation and the social solution. First, the representative household takes only internal effects into account. Second, taxes (subsidies) on consumption τ_C and environmental effort τ_E do play a role when deciding on the optimal level of C and E. Consider a tax on consumption, i.e. $\tau_C > 0$. In this case, the LHS of equation (27) diminishes due to the introduction of the tax considered. Holding the shadow price of capital constant, equation (27) requires that the marginal utility of consumption must increase. This can be accomplished only if the level of consumption is reduced. An analogous interpretation (with $\tau_E < 0$) applies to equation (28).

The social solution (described above) can be decentralised by an appropriate set of optimal taxes (shown below). In this case, the KRR for C and E are given by equations (16) and (17).

Finally, the representative firm employs the single input factor physical capital using a constant returns to scale technology to produce a homogenous good, which is sold in competitive markets. From the solution to the firm's static optimisation problem, we obtain a standard expression for the (net) interest rate:

$$r = F_K - \delta$$

where $F_K > 0$ is the marginal product of capital and $\delta > 0$ the constant rate of depreciation.

[10] Optimal taxes are determined below.

[11] Once more, the transversality condition $\lim_{t\to\infty} e^{-\rho t}\lambda K = 0$ must hold and we assume that the necessary conditions are also sufficient.

3.3 Optimal taxes

Optimal taxes τ_C^* and τ_E^* result from the comparison of the social first-order conditions (10) and (11) to the decentral first-order conditions (27) and (28). It is readily shown that an optimal tax scheme reads as follows:

$$\tau_C^* = -\frac{U_P P_{\bar{C}}}{U_C + U_P(P_C + P_{\bar{C}})} > 0 \tag{31}$$

$$\tau_E^* = -\frac{P_{\bar{E}}}{P_E + P_{\bar{E}}} < 0 \tag{32}$$

Let us start with the interpretation of τ_E^*, which is straightforward. Equation (32) shows that the optimal subsidy on environmental effort is given by the share of the external marginal effect of environmental effort on pollution $P_{\bar{E}} < 0$ to the overall (i.e. internal and external) marginal effect of environmental effort on pollution $P_E + P_{\bar{E}} < 0$. Similarly, the optimal consumption tax τ_C^* is the share of the external marginal consumption effect on utility $U_P P_{\bar{C}} < 0$ to the overall marginal effect of consumption on utility given by $U_C + U_P(P_C + P_{\bar{C}}) > 0$.[12]

4 A specific dynamic EKC model

4.1 Parameterisation

To further analyse the characteristics of the dynamic EKC model, we have to parameterise instantaneous utility $U(C, P)$, gross pollution $G(C, \bar{C})$, abatement $B(C, E, \bar{E})$ and the production function $F(K)$. We assume the following functional forms:

$$U(C, P) = \log(C - zP) \quad \text{with} \quad z > 0,\, C \geq zP \tag{33}$$

$$G(C, \bar{C}) = C^{\phi}\bar{C}^{\omega} \quad \text{with} \quad 0 < \phi, \omega < 1 \tag{34}$$

$$B(C, E, \bar{E}) = C^{\alpha}E^{\beta}\bar{E}^{\eta} \quad \text{with} \quad 0 < \alpha, \beta, \eta < 1 \tag{35}$$

$$F(K) = AK \quad \text{with} \quad A > 0 \tag{36}$$

where z is a preference parameter indicating the importance of pollution in the instantaneous utility function, C^{ϕ} represents the internal effect of consumption on gross pollution, whereas $\bar{C}^{\omega}$ is the corresponding external effect.[13] Similarly, E^{β} is the internal and $\bar{E}^{\eta}$ the external effect of environmental effort on abatement. Finally, A is a constant productivity parameter.

Let us concisely motivate the instantaneous utility function shown in equation (33). Since pollution is defined by $P = C - C^{\alpha}E^{\beta}$ we get $U(C, P) = \log(C^{\alpha}E^{\beta})$ provided that $z = 1$. This formulation has the advantage that C and E enter utility additive separable, which enables an analytical solution in the case of the social economy (without external effects). Two issues

[12] Notice that $U_C + U_P(P_C + P_{\bar{C}}) = \lambda > 0$.

[13] We assume that $\omega + \phi = 1$. This restriction enables to solve the differential equation system resulting from the socially optimal solution analytically. Moreover, we keep this restriction to compare the market allocation to the social solution.

should be noticed in this respect: First, the preceding utility function requires $C - zP \geq 0$, otherwise utility would be a complex number. For $z \leq 1$ this restriction is automatically satisfied since C is gross pollution and P is net pollution (gross pollution minus abatement). Second, the utility function implies $U_{CP} = \frac{1}{(C-zP)^2} > 0$. This property appears counterintuitive at first glance. However, this is due to the fact that a rise in P acts as if C is reduced and hence marginal utility of consumption increases with P.

4.2 Analytical results

In this section, we derive the PIR analytically and discuss the determinants of the critical levels of income and points in time at which pollution starts to decline. We focus on the social solution and assume that $z = 1$. This allows us to derive analytical results. The consequences of external effects as well as $z \neq 1$ are investigated in a second step by simulating the transition process (see Section 4.3).

4.2.1 The time path of pollution $P(t)$ and the PIR $P(Y)$

For the linear growth model one can readily derive analytical solutions for the time path of the endogenous variables. Applying the general first-order conditions from Section 3.1 [equations (10) to (13)] to the parameterised model [equations (33) to (36)] and taking into account that in the social solution $\tau_C = \tau_E = 0$ and therefore $T = 0$, we get the following solutions for K and λ:

$$K = K_0 e^{(A-\delta-\rho)t} \tag{37}$$

$$\lambda = \frac{\alpha+\beta+\eta}{K_0\rho} e^{-(A-\delta-\rho)t} \tag{38}$$

Using equations (10), (11) and (38) and noting equations (33) to (35), one can formulate an analytical expression for the time path of pollution:

$$P(t) = \frac{K_0 e^{-(-A+\delta+\rho)t}\alpha\rho}{\alpha+\beta+\eta} - \left[\left(\frac{K_0 e^{-(-A+\delta+\rho)t}\alpha\rho}{\alpha+\beta+\eta}\right)^{\alpha} \cdot \left(\frac{K_0 e^{-(-A+\delta+\rho)t}(\beta+\eta)\rho}{\alpha+\beta+\eta}\right)^{\beta+\eta}\right] \tag{39}$$

Next, we determine the PIR, which may be expressed as follows:

$$P(Y) = cY - (cY)^{\alpha}(eY)^{\beta+\eta} \tag{40}$$

Now we need to determine the consumption rate $c := \frac{C}{Y}$ and the "environmental effort rate" $e := \frac{E}{Y}$ along the BGP. To accomplish this task, we consider the growth rate of capital $\hat{K} := \frac{\dot{K}}{K}$ using equations (7), (36) and (37):

$$\hat{K} = A - \delta - \rho = A - \delta - \frac{C}{K} - \frac{E}{K} \tag{41}$$

Together with equations (10) and (11) this immediately yields the balanced growth values of c and e to read as follows:

$$c = \frac{\alpha\rho}{A(\alpha+\beta+\eta)} \qquad e = \frac{(\beta+\eta)\rho}{A(\alpha+\beta+\eta)} \tag{42}$$

4.2.2 An illustration

We illustrate the EKC $P(Y)$ in Figure 1 plot (a) and the time path of pollution in Figure 1 plot (b). The underlying baseline set of parameters is in line with usual growth model calibrations (e.g. Ortigueira and Santos, 1997). In addition, we assume that there are IRS in the abatement technology, i.e. $\alpha+\beta+\eta > 1$.[14]

$$\alpha = 0.6,\ \beta = 0.45,\ \eta = 0.05,\ \delta = 0.06,\ \rho = 0.04,\ A = 0.12, \phi = 0.9,\ \omega = 0.1$$

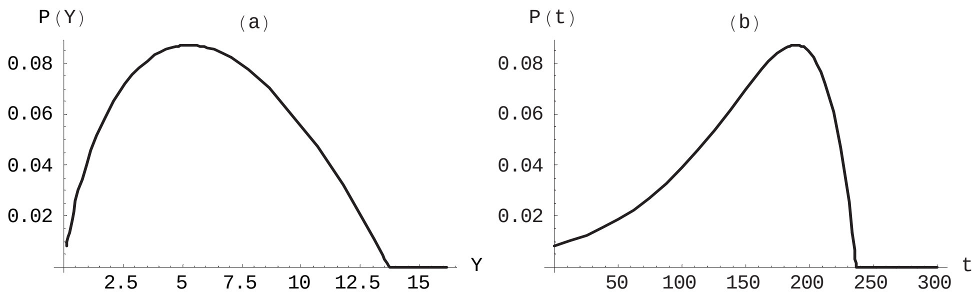

Figure 1: $P(Y)$ and $P(t)$ with IRS in abatement ($\alpha+\beta+\eta > 1$)

As can be seen in Figure 1 plot (a), pollution first rises with income, then declines and eventually becomes zero. This EKC represents a balanced growth phenomenon.[15] Although pollution does not grow with constant rate (as is required by a BGP definition), the illustrated pollution path is nonetheless a BGP since pollution results from two endogenous variables (consumption and environmental effort), which do grow at constant rates. The required time span until pollution reaches its peak and becomes zero is quite long. The whole "EKC story" takes nearly 250 years as is displayed in plot (b) of Figure 1. In the next sections, we will explicitly focus on the time span and on the corresponding level of income required until pollution reaches its maximum or becomes zero.

The EKC pattern displayed in Figure 1 plot (a) is in line with empirical evidence as reported by Grossman and Krueger (1995) according to which the pollution-income relation is asymmetric with an upper tail that declines relatively gradually.

[14] AL (2001, Section 4) give convincing evidence for increasing returns to scale in abatement.

[15] Employing a neoclassical growth model it can be shown that the EKC can also result from transitional dynamics (see the appendix).

4.2.3 Explicit answers to several important questions

There are several important questions related to economic growth and the environment. Some of the most important questions in this context are the following: (i) If an EKC can be shown to exist theoretically, how long does it take until pollution starts to decline? Similarly, provided that pollution vanishes, at which point in time does this occur? (ii) At what levels of income does pollution reach its peak and finally vanishes? (iii) Provided that the optimal long-run stock of overall pollution is finite, how large is this optimal long-run stock of overall pollution?

The answers to these questions are obviously of outstanding importance. Hence, it would be highly desirable to give explicit answers, based on a dynamic macroeconomic model. In addition, it would be clearly instructive to see the economic determinants behind these results and to show how these results change with the parameters of the underlying model.

Let us at first turn to the critical point in time at which pollution reaches its maximum. From the analytical expression for the time path of pollution (equation (39)), we are able to derive this critical point in time denoted as t^*:

$$t^* = -\frac{\log[K_0^{\alpha+\beta+\eta-1}\alpha^{\alpha-1}(\beta+\eta)^{\beta+\eta}(\alpha+\beta+\eta)^{2-\alpha-\beta-\eta}\rho^{\alpha+\beta+\eta-1}]}{(\alpha+\beta+\eta-1)(A-\delta-\rho)}. \quad (43)$$

Provided that we impose the restriction $\alpha = \beta + \eta$, the preceding equation can be simplified to read:

$$t^* = -\frac{\log(K_0^{2\alpha-1}2^{2(1-\alpha)}\alpha\rho^{2\alpha-1})}{(2\alpha-1)(A-\delta-\rho)}. \quad (44)$$

From the preceding solutions for t^*, we obtain the following comparative static results as shown in Table 1. These are largely based on the general case, which does not assume $\alpha = \beta + \eta$. The only exception is $\frac{\partial t^*}{\partial \alpha}$, which is based on the solution for t^* assuming $\alpha = \beta + \eta$.[16]

Table 1: Comparative Static Results for t^*

	$\frac{\partial t^*}{\partial x}$ for $x = K_0, A, \delta, \rho, \alpha$	
K_0	$-\frac{1}{K_0(A-\delta-\rho)}$	< 0
A	$-\frac{t^*}{A-\delta-\rho}$	< 0
δ	$\frac{t^*}{A-\delta-\rho}$	> 0
ρ	$-\frac{1-t^*\rho}{\rho(A-\delta-\rho)}$	?
α	$\frac{1+\alpha[\log(4)-2]+2\alpha\log(\alpha)}{(1-2\alpha)^2\alpha(A-\delta-\rho)}$	> 0

[16]Moreover, the comparative static results are based on the assumption $t^* > 0$. This implies that $K_0 < K^*$.

The first row shows that t^* is smaller, the higher the initial level of capital K_0. First notice that there is a critical level of capital K^* at which pollution reaches its peak.[17] This critical level is, of course, independent of $K(0)$. Hence, the larger $K(0)$, the closer the economy starts in relation to the critical level K^* and the smaller is the required period of time until K^* is reached.

The second row indicates that t^* is smaller, the higher the productivity of capital in final output production A. The reason simply lies in the fact that the growth rate of the economy increases with A. Therefore, the time span required to reach the critical level of capital K^* falls as A increases. This effect can be directly recognised by inspecting the denominator of equation (43). Furthermore, it should be noticed that K^* is independent of A.

According to the third row t^* increases with the capital depreciation rate δ. Similarly to the previous case, an increase in δ reduces the growth rate and thereby increases the time span required to reach the critical level of capital K^*.

The impact of the time preference rate ρ on t^* is generally unclear as indicated by the fourth row. This is due to the fact that there are two opposing mechanisms. First, an increase in ρ reduces (other things equal) the growth rate and hence increases t^*. Second, as ρ increases K^* falls as can be recognised by inspecting equation (46) below. The economic reason is due to the fact that pollution is solely determined by C and E, which in turn are determined by K. The relation between C and E on the one hand and K on the other is determined by the consumption rate and environmental effort rate as given by equation (42). Since both the consumption rate and environmental effort rate increase with ρ, the implied level of K at which pollution peaks decreases with ρ. This effect in turn reduces t^*. Whether the first or the second effect dominates depends on the specific set of parameters.

The last row shows that, for $0 < \alpha < 1$, t^* increases with α.[18] For ease of interpretation, let us assume that $\alpha = \beta + \eta$ such that $C = E$.[19] This result appears counter-intuitive at first glance. To understand this pattern, first notice that the relevant range of consumption is $0 < C < 1$; only within this range an EKC can be explained based on IRS. This does not mean, however, that the relevant range within which an EKC occurs is marginally small. We can choose the dimension of measurement for the numeraire good (which is Y; the price of C in terms of Y is unity) such that 1 corresponds to a fairly large number in empirical terms. Within this range an increase in α (i.e. an increase in the degree of IRS) lowers the abatement output (holding the inputs C and E constant). As a result, the maximum level of pollution occurs at a higher C-level. Moreover, since a higher C-level unambiguously implies a higher level of critical capital K^*, the time span required to reach this critical level of capital

[17] Pollution is a function of C and E only. Moreover, the policy functions for C and E indicate that both control variables are solely determined by K. Hence, we may write $P = P(K)$. Notice, however, that this is not an assumption but rather a result of the model.

[18] When α approaches 0.5 both the numerator and the denominator converge to zero but the limiting value does exist and is positive.

[19] An analogous, though slightly more complicated, reasoning would apply to the case $\alpha \neq \beta + \eta$.

increases.

We can also determine the point in time at which pollution becomes zero. This point in time is denoted as t^{**} and reads as follows:

$$t^{**} = \frac{\log[K_0^{1-\alpha-\beta-\eta}\alpha^{1-\alpha}(\beta+\eta)^{-\beta-\eta}(\alpha+\beta+\eta)^{\alpha+\beta+\eta-1}\rho^{1-\alpha-\beta-\eta}]}{(\alpha+\beta+\eta-1)(A-\delta-\rho)}. \quad (45)$$

The striking feature here is the similarity between t^* in equation (43) and t^{**} as shown above. This is not surprising since, for example, a higher initial level of capital $K(0)$ reduces the time span required until pollution vanishes. Similarly, an increase in A or a decrease in δ fosters economic growth and therefore reduces the time span required until pollution vanishes. It should also be noticed that t^{**} is independent of α provided that $\alpha = \beta + \eta$.

Having determined the critical points in time t^* (maximum pollution) and t^{**} (pollution vanishes), we now are in the position to determine and discuss the critical levels of income at which pollution peaks and the critical level of income at which pollution vanishes. In the empirical EKC literature, the income associated with the maximum pollution is intensively debated. The range of estimated incomes, however, is very large; not only across different measures of environmental degradation (that is not surprising), but also across different estimations equations and/or estimations techniques. Therefore, a theoretical determination of this critical income level should be clearly instructive.

Inserting the expression for t^* and t^{**} into the time path of income ($Y = AK$) and using equation (37), yields expressions for the associated levels of income, which are denoted as Y^* (maximum pollution) and Y^{**} (pollution vanishes):[20]

$$Y^* = \frac{A\alpha^{\frac{1-\alpha}{\alpha+\beta+\eta-1}}(\beta+\eta)^{-\frac{\beta+\eta}{\alpha+\beta+\eta-1}}(\alpha+\beta+\eta)^{1-\frac{1}{\alpha+\beta+\eta-1}}}{\rho} \quad (46)$$

$$Y^{**} = \frac{A\alpha^{\frac{1-\alpha}{\alpha+\beta+\eta-1}}(\beta+\eta)^{-\frac{\beta+\eta}{\alpha+\beta+\eta-1}}(\alpha+\beta+\eta)}{\rho} \quad (47)$$

These critical income levels are determined solely by the marginal product of capital A, the rate of time preference ρ and the production elasticities of consumption α and environmental effort in abatement β and η, respectively. They are independent of the capital depreciation rate δ and the initial capital stock K_0.

From the preceding solutions for Y^* in equation (46), we obtain the following comparative static results, which are shown in Table 2. The first and the second derivative are valid for the general case (which does not impose any restrictions on α, β and η); notice that in this case Y^* is taken from equation (46). The third derivative is based on $\alpha = \beta + \eta$ with Y^* valid for $\alpha = \beta + \eta$.

[20] It should be noticed that both the weight of pollution in the instantaneous utility function z and the intertemporal elasticity of substitution affect Y^*. These preference parameters, however, do not explicitly appear in the following result since they have been set equal to unity to simplify the analyses.

Table 2: Comparative Static Results for Y^*

	$\frac{\partial Y^*}{\partial x}$ for $x = A, \rho, \alpha$	
A	$Y^* \frac{1}{A}$	> 0
ρ	$Y^* \frac{-1}{\rho}$	< 0
α	$Y^* \frac{1+\alpha(\log[4]-2)+2\alpha\log[\alpha]}{(1-2\alpha)^2\alpha}$	> 0

The first row shows that Y^* increases with A. For ease of interpretation, let us assume that $\alpha = \beta + \eta$ such that $C = E$.[21] In order to understand this result, remember that (other things equal) the level of pollution depends only on consumption. Since an increase in A reduces the consumption rate [equation (42)], the required level of income for pollution to reach its maximum increases. The second row indicates that Y^* falls as ρ rises. An analogous reasoning is applicable here. The rate of consumption rises with ρ [equation (42)] and hence the required level of income for pollution to reach its maximum falls. The third row shows that Y^* rises as α increases. As before, first notice that the relevant range of consumption is $0 < C < 1$. Within this range an increase in α (i.e. an increase in the degree of IRS) lowers the abatement output (holding the inputs C and E constant). As a result, the maximum level of pollution occurs at a higher C -level. Moreover, since the rate of consumption is independent of α a higher C-level unambiguously implies a higher level of income, i.e. Y^* must be higher.

At this stage, it is worth considering the empirical evidence of the EKC. The evidence is strongest for local air quality indicators, such as suspended particular matters, sulphur dioxide, carbon monoxide or nitrogen oxides. However, the estimated income levels associated with the maximum pollution are very diverse. For sulphur dioxide the average level of income is about USD 5500, for suspended particular matters about USD 8400 and for nitrogen dioxides and carbon monoxide about USD 13000.[22] In our model, this diversity could be attributed to parameter heterogeneity across different pollutants, i.e. heterogeneity in z, α and β.

Using data on sulphur dioxide and nitrogen oxide emissions between 1929 and 1994 for the US states, List and Gallet (1999) estimated very different income turning points across the forty-eight considered states. In other words, the US states do not follow a uniform pollution path. In our model, heterogeneity across economies could be accounted for by country specific parameters, e.g. A, δ, ρ and z.

Turning to the fourth question, we determine the overall stock of pollution, which accumulates during the process of economic development. At a theoretical level it is of outstanding importance to know the economic determinants of

[21] An analogous, though slightly more complicated, reasoning would apply to the case $\alpha \neq \beta + \eta$.

[22] Theses income levels are calculated on the basis of the survey of Ekins (1997).

this overall level of pollution. The reason lies in the fact that there might be critical thresholds in the ecological system for the level of overall pollution. The overall stock of pollution is given by $R^* = R_0 + \int_{t=0}^{t^{**}} P(t)dt$, where R_0 denotes the initial overall stock of pollution inherited from the past and t^{**} is the point in time at which pollution vanishes. Evaluating the preceding definite integral we obtain:

$$\int_{t=0}^{t^{**}} P(t)dt = \frac{\alpha^{\frac{\beta+\eta}{\alpha+\beta+\eta-1}}(\beta+\eta)^{\frac{-\beta-\eta}{\alpha+\beta+\eta-1}}(\alpha+\beta+\eta-1) - K_0\alpha\rho}{(\alpha+\beta+\eta)(A-\delta-\rho)}. + \frac{K_0^{\alpha+\beta+\eta}\alpha^\alpha(\beta+\eta)^{\beta+\eta}(\alpha+\beta+\eta)^{-\alpha-\beta-\eta}\rho^{\alpha+\beta+\eta}}{(\alpha+\beta+\eta)(A-\delta-\rho)} \tag{48}$$

Once more, imposing the restriction $\alpha = \beta + \eta$ leads to a much clearer result:

$$\int_{t=0}^{t^{**}} P(t)dt = \frac{2\alpha - 1 - K_0\alpha\rho + 2^{-2\alpha}K_0^{2\alpha}\rho^{2\alpha}}{2\alpha(A-\delta-\rho)}. \tag{49}$$

From the preceding expression, we obtain the following comparative static results as shown in Table 3 (where it has been assumed that $\alpha = \beta + \eta$):

Table 3: Comparative Static Results for $\int_{t=0}^{t^{**}} P(t)dt$

	$\frac{\partial \int_{t=0}^{t^{**}} P(t)dt}{\partial x}$ for $x = K_0, A, \delta, \rho, \alpha$	
K_0	$-\frac{\rho(0.5-4^{-\alpha}K_0^{2\alpha-1}\rho^{2\alpha-1})}{A-\delta-\rho}$	< 0
A	$-\frac{\int_{t=0}^{t^{**}} P(t)dt}{A-\delta-\rho}$	< 0
δ	$\frac{\int_{t=0}^{t^{**}} P(t)dt}{A-\delta-\rho}$	> 0
ρ	$\frac{\int_{t=0}^{t^{**}} P(t)dt}{A-\delta-\rho} - \frac{K_0(1-2^{1-2\alpha}K_0^{2\alpha-1}\rho^{2\alpha-1})}{2(A-\delta-\rho)}$	?
α	$\frac{\int_{t=0}^{t^{**}} P(t)dt}{-\alpha} + \frac{2^{-1-2\alpha}\{2^{1+2\alpha}-4^\alpha K_0\rho+2K_0^{2\alpha}\rho^{2\alpha}[\log(\frac{K_0}{2})+\log(\rho)]\}}{\alpha(A-\delta-\rho)}$	?

The first row indicates that the overall stock of pollution decreases as K_0 increases (provided that $K_0 < K^{**}$). The reason lies simply in the fact that overall pollution is smaller, the closer the economy starts at the critical level of capital at which pollution vanishes.

The second row shows that R^* falls with A. This is due to the fact that the growth rate increases with A. The crucial aspect here lies in the fact that the economy passes through the pollution range more rapidly, the higher the growth rate. As a result, less pollution is accumulated during the course of economic development.

The third row shows that R^* increases as δ rises. The same interpretation as before applies since δ reduces the growth rate.

The sign of $\frac{\partial \int_{t=0}^{t^{**}} P(t)dt}{\partial \rho}$ (fourth row) is ambiguous. This is due the fact that there are two opposing effects at work. First, as ρ rises the growth rate falls and therefore R^* increases. Second, since both the consumption rate and environmental effort rate increase with ρ, the implied level of K at which pollution peaks decreases with ρ. This effect in turn reduces t^{**} and hence R^*. Which effect dominated is unclear.

The last row gives the impact of α on $\int_{t=0}^{t^{**}} P(t)dt$. The sign of this derivative cannot be determined in general. However, numerical exercises indicate that this relationship is positive. This is fairly plausible for two reasons: First, for any level of consumption pollution increases with α (remember that the relevant range is $0 < C < 1$). Second, the critical point in time at which pollution vanishes t^{**} is independent of α (compare to equation (45)).

4.3 Numerical analysis

So far we have focused on the social solution and assumed that $z = 1$ (i.e. consumption and pollution have the same weight in the utility function). However, it is clear that external effects (pollution and environmental effort externalities) may be important for the critical level of income and point in time at which pollution peaks.

With external effects, the market allocation and the social solution diverge. We will investigate the importance of external effects in turn. In addition, we also investigate the impact of the weight of pollution in utility. Since an analytical solution cannot be found in theses cases, we simulate the transition process of the market economy.

From the first-order conditions of the decentral solution [equations (27) to (30)] together with equations (33) to (36) we get a system of differential equations in K, C, E and λ. Applying the backward integration procedure (e.g. Brunner and Strulik, 2002), one can determine time paths of the endogenous variables.

4.3.1 The importance of external effects

At this stage, we analyse the quantitative importance of the external effects on the PIR and specifically on Y^* and t^*. It should be noticed that our baseline set of parameters (displayed above) implies fairly moderate external effects. More precisely, the share of the external pollution effect of consumption to the overall pollution effect of consumption amounts to $\frac{\omega}{\phi+\omega} = 0.1$ and the corresponding share for environmental effort is $\frac{\eta}{\beta+\eta} = 0.1$. Nevertheless, the impact on the resulting PIR are substantial, as is illustrated in Figure 2. The PIR labelled "social" shows the PIR resulting from the social solution, while the PIR labelled "market" shows the PIR resulting from the market allocation (ignore the curves marked by $\theta_C = 1$ and $\theta_E = 1$ for the moment). It is obvious that both Y^* and the maximum amount of pollution $P^* = P(Y^*)$ are highly sensitive with respect to the external effects. The market economy shows considerably larger values for Y^* and P^* compared to the social solution.

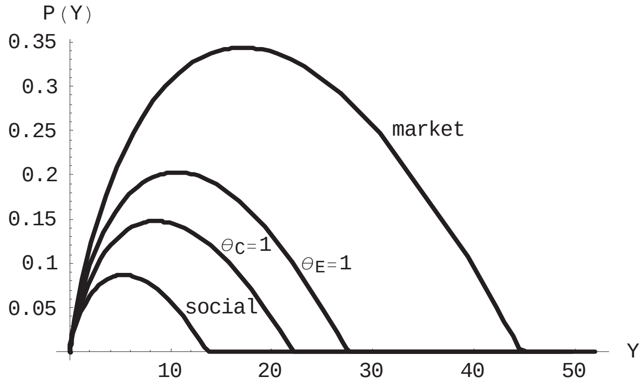

Figure 2: EKC; market versus social solutions

Moreover, Figure 2 illustrates the importance of the respective market failures, thereby demonstrating the impact of the respective policy instruments. The curve labelled as $\theta_C = 1$ shows a situation in which only the external effect of environmental effort is present (i.e. the external effect of polluting consumption is completely internalised) and the curve labelled as $\theta_E = 1$ shows a situation in which only the external effect of consumption on gross pollution is present (i.e. the external effect of environmental effort is completely internalised). The curves demonstrate that the consumption externality has a stronger impact on the shape of the resulting PIR. This can be recognised by fact that the curve $\theta_E = 1$ lies strictly above the curve $\theta_C = 1$ implying both a higher Y^* and P^*.

By imposing appropriate taxes on consumption and subsidies on environmental effort, the government can correct the disturbing external effects. The taxes imposed are specified as $\tau_C = \theta_C \tau_C^*$ and $\tau_E = \theta_E \tau_E^*$, where $\tau_C^* > 0$ and $\tau_E^* < 0$ are optimal taxes (defined in Section 3.3) and $\theta_C \geq 0$ and $\theta_E \geq 0$ indicate the extent of tax implementation. Moreover, a policy programme which diminishes all market distortions simultaneously is described by $\theta = \theta_C = \theta_E$. Setting $\theta = 0$ corresponds to the market solution, while $\theta = 1$ leads to the social planner's solution.

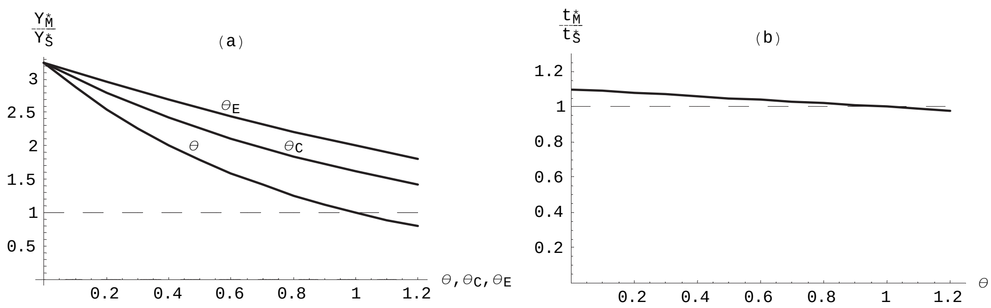

Figure 3: Y_M^*/Y_S^* and t_M^*/t_S^* in response to policy parameters

Figure 3 shows the sensitivity of Y^* and t^* in response to policy parameters.

Specifically, plot (a) displays the ratio of Y^* resulting from the market allocation to Y^* resulting from the social solution, denoted as Y_M^*/Y_S^*. Several points are worth being discussed: First, for $\theta = 0$ we get $Y_M^*/Y_S^* \cong 3.3$. The critical level of income resulting from the market allocation is accordingly more than three times higher than in the social solution. On the other hand, for $\theta = 1$ we get $Y_M^*/Y_S^* = 1$ since in this case the market allocation coincides with the social solution. Second, the shape of the θ-curve implies that the impact of policy actions on Y_M^*/Y_S^* are higher for low values of θ. Third, considering the θ_E-curve (along which $\theta_C = 0$) and the θ_C-curve (along which $\theta_E = 0$) shows that isolated policy measures are less effective in lowering Y_M^* relative to Y_S^*, which is not surprising. Fourth, the environmental effort subsidy (the θ_E-curve) is less effective than the consumption tax (the θ_C-curve).

Moreover, the θ-curve shows that public policy is basically able to reduce Y_M^* below Y_S^*. This would require to set $\theta > 1$. Of course, within the underlying model such a policy cannot be optimal. However, a more comprehensive framework which includes the ecological system could very well lead to a socially optimal Y^* which lies below Y_S^* resulting from the model under study.

Figure 3 pot (b) displays the corresponding relation for the critical point in time t_M^*/t_S^* in response to θ. For $\theta = 0$ we observe $t_M^*/t_S^* \cong 1.1$ and for $\theta = 1$ we get $t_M^*/t_S^* = 1$ (i.e. both solutions coincide). The striking feature here is the fact that t_M^*/t_S^* is much smaller than Y_M^*/Y_S^* for low values of θ. The reason behind this pattern is due to the fact that the economy exhibits exponential growth. Therefore, any $t_M^*/t_S^* > 1$ leads to a much larger Y_M^*/Y_S^*.

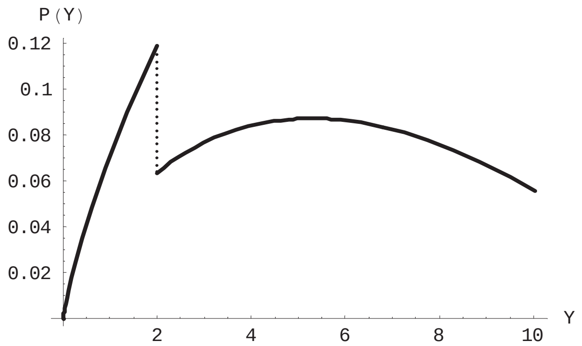

Figure 4: M-shaped EKC

There are a number of studies which argue that the PIR is not inverted U-shaped but instead is N-shaped at least for some pollutants (e.g. Grossman and Krueger, 1995, Section IV). This hypothesis would bear the important implication that pollution finally increases. With respect to this issue, the model under study provides two important insights. First, the model allows us to easily explain the observed N-shaped pattern. Imagine the economy develops at first along the upward sloping range of the EKC resulting from the market economy as shown in Figure 4. At some point in time, policy instruments are implemented to internalise external effects and pollution diminishes accordingly.[23]

[23] In the real world, the period of stark policy measure were the 1970s.

In the model, the economy jumps to the social EKC; of course, in reality this process is distributed over time. Provided that the economy is still below the critical threshold Y^*, pollution starts to increase again. As a result, we would observe an N-shaped PIR resulting from the interaction of policy actions and the intrinsic properties of the model. Second, the model under study does not imply that an observed N-shaped pattern must finally lead to a permanent increase in pollution. Instead, an M-shaped pattern results. As soon as the peak of pollution (on the "social EKC") is reached, pollution starts to decline.

4.3.2 The importance of the weight of pollution

We now investigate the importance of z by relaxing the assumption $z = 1$. This preference parameter reflects the love for a clean environment. A lower value of z means that a given amount of pollution causes less disutility and individuals will accordingly spend less on environmental effort and more on consumption. As a result, the PIR can be expected to shift outwards.

In general, it is important to understand the quantitative implications of this effect. Provided that the consequences of alternative z values are substantial, cross-country differences in the PIR (especially Y^* and t^*) can potentially be explained by differences in preferences. For instance, it is plausible to argue that the US have different preferences with respect to a clean environment compared Western Europe.

The quantitative consequences of alternative z values are described by Figure 5. Plot (a) shows the PIR as resulting from the social solution for $z = 1.1$, $z = 1$ and $z = 0.9$. It can immediately be recognised that Y^* is strongly affected by variations in z. This observation is summarised in plot (b), which displays Y^* relative to Y^* as resulting from $z = 1$ in response to z. For instance, Y^* is about five times larger for $z = 0.8$ compared $z = 1$. On the other hand, the ratio $Y^*/Y^*_{z=1}$ is about 0.24 for $z = 1.2$.[24]

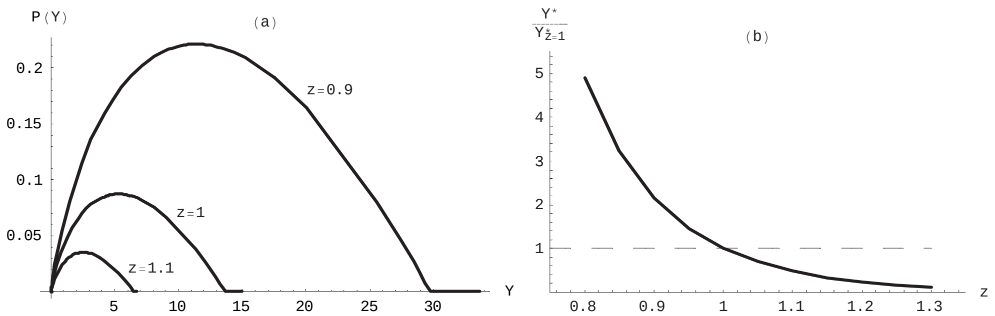

Figure 5: $P(Y)$ and $Y^*/Y^*_{z=1}$ in response to z

The results indicate that the priority attached to a clean environment within the political process may an important explanation for observable international differences in the PIR pattern.

[24] As for the external effects, the impact on the required time span to reach the critical level of income is much smaller.

5 Summary and conclusions

The paper at hand sets up a dynamic macroeconomic model, which combines several features of the static AL model with standard growth models in the vein of the Ramsey-Cass-Koopmans model. We show that an EKC arises naturally in the course of economic development. The resulting EKC represents a smooth development path and does not rely on abrupt changes (giving rise to discontinuities) as in most previous dynamic approaches. The analysis demonstrates that an EKC can be represented both as a transitional dynamics phenomenon as well as a balanced growth phenomenon. The main results can be summarised as follows:

(1) We confirm the basic finding of AL according to which IRS in abatement can explain an inverted U-shaped PIR. This is important to the extent that the analysis of AL ignores completely the intertemporal dimension of the problem. At a very general level, the AL model can hence be considered as a shortcut to specify basic conditions for the occurrence of an EKC.

(2) By focusing on the social solution, we derive closed-form solutions for the resulting dynamic system. This enables us to determine analytically the critical level of income and point in time at which pollution starts to decline. As a result, the economic determinants behind these critical thresholds can be identified. The determination of these critical thresholds may be used as an independent check of similar results, which have been derived empirically.

(3) We investigate the consequences of market failures and public policy numerically. By simulating the transition process of the market economy, we show that the critical thresholds are highly sensitive with respect to external effects. As a consequence, public policy is highly effective with respect to policy objectives such as lowering the level of income at which pollution starts to decline or reducing overall pollution.

(4) We show that an empirical pattern, which is observed for some specific pollutants (i.e. an N-shaped PIR) can be plausibly explained from the interaction of public policy measures and the intrinsic properties of the model. The resulting PIR comprises branches of the PIR resulting from the market economy and the PIR resulting from the social solution. This way of reasoning bears the stark implication that any observed N-shaped PIR may turn out to be indeed an M-shaped PIR implying that pollution eventually starts to diminish.

Finally, the present paper points to interesting questions for future research. For instance, it is well known that there are PIR with very different shapes in the real world depending on the specific pollutant under consideration. Some of these individual pollution paths fit the EKC pattern, while others do not. To shed light on the importance of pollution heterogeneity for the overall level of pollution and welfare, it would be clearly interesting to extend the model set up above to allow for different consumption activities (giving rise to the emission of different pollutants) as well as pollutant-specific abatement activities. Then the question whether the pollution structure affects the overall level of pollution can be investigated.

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Appendix I: Polluting production

We consider the consequences of modelling pollution to result from production (instead of pollution to result from consumption) for the pollution-income relation. In this case, pollution becomes a function of the capital stock (via the output technology).[25]

The net pollution function may be expressed as $P(K,E) = G(K) - A(K,E)$; we ignore external effects in this context. The instantaneous utility function then is $U = U[C, P(K,E)]$ and the transformed utility function may be expressed as $V = V(C,K,E)$. The Hamiltonian in this case can be written as:

$$H = V(C,K,E) + \lambda[F(K) - C - E - \delta K] \tag{50}$$

and the first-order conditions are given by

$$H_C = V_C - \lambda = 0 \quad \Longleftrightarrow \quad V_C = \lambda \tag{51}$$

$$H_E = V_E - \lambda = 0 \quad \Longleftrightarrow \quad V_E = \lambda \tag{52}$$

$$\dot{\lambda} = -H_K + \rho\lambda = -(V_K + \lambda F_K - \lambda\delta) + \rho\lambda \quad \Longleftrightarrow \quad \frac{\dot{\lambda}}{\lambda} = -\frac{V_K}{\lambda} - F_K + \delta + \rho \tag{53}$$

Obviously, only the condition $\dot{\lambda} = -H_K + \rho\lambda$ is affected by this second modelling procedure. From equations (51), (52) and (53) one obtains:

$$\frac{V_{CC}\dot{C}}{V_C} + \frac{V_{CE}\dot{E}}{V_C} = -\frac{V_K}{\lambda} - F_K + \delta + \rho \tag{54}$$

$$\frac{V_{EC}\dot{C}}{V_C} + \frac{V_{EE}\dot{E}}{V_E} = -\frac{V_K}{\lambda} - F_K + \delta + \rho \tag{55}$$

Assuming additive separability of C and E in $V(.)$ yields the KRR for optimal consumption to read:

$$\frac{\dot{C}}{C} = \frac{1}{\sigma_{CC}}(F_K - \delta - \rho + \frac{V_K}{\lambda}) \tag{56}$$

Considering a neoclassical growth model ($F_K > 0$, $F_{KK} < 0$), this condition suggests that the steady state level of capital $\widetilde{K}$ (associated with $\frac{\dot{C}}{C} = \frac{\dot{E}}{E} = 0$) is now affected by pollution (the basic model without pollution results from setting $V_K = U_P P_K = 0$ implying that $E = 0$). $\widetilde{K}$ must satisfy $F_K - \delta - \rho + \frac{V_K}{\lambda} = 0$ as well as $F(K) - C - E - \delta K = 0$ (C and E can be substituted by equations (51) and (52) as functions of λ).

The PIR is formally given by $P(Y) = G(Y) - A[Y, E(Y)]$. An EKC, i.e. an inverted U-shaped PIR, requires that two conditions hold: First, IRS in abatement and second growth must continue until pollution at least starts to decline. The first condition is not affected by the way how pollution is modelled. The second condition may be affected since the growth rate (in the neoclassical case the steady state) is influenced by the fact that the social planner cares about pollution. Without loss of generality we can assume that the economy under study is sufficiently productive and sustained growth is possible.

[25] The formulation $P(K)$ is often employed, whereas $P(C)$ is less frequent. Both formulations appear plausible. Moreover, at first glance one could expect that there should be no difference since $C = C[Y(K)]$. It will be shown that this conjecture is wrong.

Appendix II: Economic intuition

Consider once more the instantaneous utility function $u(C, P) = \log(C - zP)$ with $P = C - B(C, E)$. If we use the following parameterisation for the abatement technology $B(C, E) = C^\alpha E^\beta$ (both C and E are essential for abatement), we get $u(C, P) = \log[C - z(C - C^\alpha E^\beta)]$. Moreover, provided that $z = 1$ (pollution has the same weight as consumption), then we get a "reduced utility function" $u(C, P) = \log(C^\alpha E^\beta)$. Since $z = 1$ direct utility from consumption and indirect disutility from gross pollution exactly cancel out. Therefore, the abatement term $C^\alpha E^\beta$ remains as the sole argument in this "reduced utility function". In this case, consumption creates utility because it increases abatement (provided that $E > 0$), i.e. reduces pollution, and similarly environmental effort contributes to utility since it increases abatement (provided that $C > 0$).

The social planner now maximises the present value of discounted utility by choosing time paths for C and E. Notice that both C and E enter largely symmetric. This applies to the reduced utility function (for $z = 1$) as well as to the capital accumulation equation, where both C and E reduce capital accumulation one by one. In this respect, it is not surprising that the solutions for C and E are largely identical, i.e. both grow at the same rate.

The crucial aspect now lies in the definition of pollution: $P = C - C^\alpha E^\beta$. For instance, if both C and E are proportional to income Y (as for the AK model), then we immediately get a polynomial equation for $P(Y) = cY - (cY)^\alpha (eY)^\beta$, where $c = C/Y$ and $e = E/Y$ are constant. To simplify matters, assume further $\alpha = \beta$. In this case, $C = E$ and hence the pollution function becomes:

$$P(Y) = cY - c^{2\alpha} Y^{2\alpha} \tag{57}$$

Consider the case $\alpha > 0.5$ (i.e. IRS in abatement activities) and assume that $Y(0)$ is sufficiently small. Then $\alpha > 0.5$ implies that gross pollution cY is larger than abatement $c^{2\alpha} Y^{2\alpha}$. As the economy grows, gross pollution increases proportionally with income but abatement increases more than proportionally with income. The crucial aspect now lies in the fact that in the range $0 < C < 1$ and assuming IRS in abatement the absolute increase in gross pollution is initially larger than the absolute increase in abatement. This difference reaches a maximum and eventually approaches the lower boundary with $P = 0$ (at $C = 1$).

In summary, the hump-shaped pattern for pollution is due to the fact that it has been assumed that gross pollution increases proportionally (one by one) with consumption (which is fairly plausible) and abatement initially increases less than proportionally and subsequently rises more than proportionally with consumption (due to IRS). Notice that the symmetry between C and E implies that both grow at the same rate and hence we have an equiproportional variation in input factor in $B(C, E)$ such that the scale of abatement increases as C rises.