L. KARP 7/16/04 ①

HYBERBOLIC DISCOUNTING PART 1

PDV PAYOFF

$$\int_0^\infty \phi(\tau)\, U(C_\tau)\, d\tau$$

$$r(t) \equiv -\frac{d}{dt} \mathrm{LN}\left(\phi(t)\, U'(C_t) \right)$$

$$= \rho(t) + v(C_t) \frac{\dot{C}_t}{C_t}$$

$r(t)$ = SOCIAL DISCOUNT RATE

$\rho(t)$ = PURE RATE of TIME PREFERENCE, $\frac{-\dot{\theta}}{\theta}$

$v(C_t) = -\frac{U''}{U'} C$

EXAMPLE of DECREASING $\rho(t)$

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TIME INCONSISTENCY

—

RUBINSTEIN'S CRITICISM IER '03

—

GOLLIER JET 02

AER 01

(WEITZMAN)

$$\Pi(t) = \underset{\tilde{r}}{E}\, e^{-\tilde{r} t}$$

$$r(t) \equiv - \frac{d \ln \Pi(t)}{dt}$$

$$r'(t) < 0$$

QUASI - HYPERBOLIC DISCOUNTING

TIME	0	1	2	3	4	....
DISCOUNT FACTOR	1	$\beta\delta$	$\beta\delta^2$	$\beta\delta^3$	$\beta\delta^4$	....

$$0 \le \delta < 1$$

$$0 \le \beta \begin{cases} = 1 & \Rightarrow \text{EXPONENTIAL DISCOUNTING} \\ < 1 & \Rightarrow \text{QUASI-HYPERBOLIC DISCOUNTING} \end{cases}$$

STATE EQN.

$$S_{t+1} = \eta S_t + Z_t$$

POLLUTION STOCK EMISSIONS

$0 \leq \eta \leq 1$ (DECAY FACTOR)

$$h(\overset{\ominus}{S_t}, \overset{\oplus}{Z_t})$$

SINGLE PERIOD PAYOFF

PDV of Payoff at t

$$h(S_t, Z_t) + \beta \sum_{\tau=1}^{\infty} \delta^{\tau} h(S_{t+\tau}, Z_{t+\tau})$$

FIND (DIFFERENTIABLE) MARKOV PERFECT EQUILIBRIUM, A FUNCTION $\chi(S_t)$

(HARRIS & LAIBSON)

DEFINE

$$H(s_t) \equiv h(s_t, \chi(s_t))$$

(THE EQUIL. SINGLE PERIOD PAYOFF)

$z_t = \chi(s_t)$ IS SOLUTION TO

$$W(s_t) = \max_z \Big\{ h(s_t, z) + \delta\left[W(s_{t+1}) - H(s_{t+1})(1-\beta)\right]\Big\}$$

$$\text{s.t. } s_{t+1} = \eta s_t + z_t$$

EULER EQN:

$$h_z(t) = -\delta\Big\{\beta h_s(t+1) - h_z(t+1)\Big(\eta + (1-\beta)\chi'(s_{t+1})\Big)\Big\}$$

PROP 1 (MONOTONICITY)

$$h_{sz} - \eta h_{zz} \geq 0 \quad \Rightarrow$$

(MONOTONIC STATE TRAJECTORY) $\quad \eta + \chi'(s) \geq 0$

$$h_{sz} - \eta h_{zz} \leq 0 \quad \Rightarrow$$

(OSCILLATING STATE TRAJECTORY) $\quad \eta + \chi'(s) \leq 0$

$\chi'(s) < 0$ CORRESPONDS TO "STRATEGIC SUBSTITUTES"

$\chi'(s) > 0$ STRATEGIC COMPLEMENTS

(EITHER ARE POSSIBLE)

STEADY STATE CONDITIONS

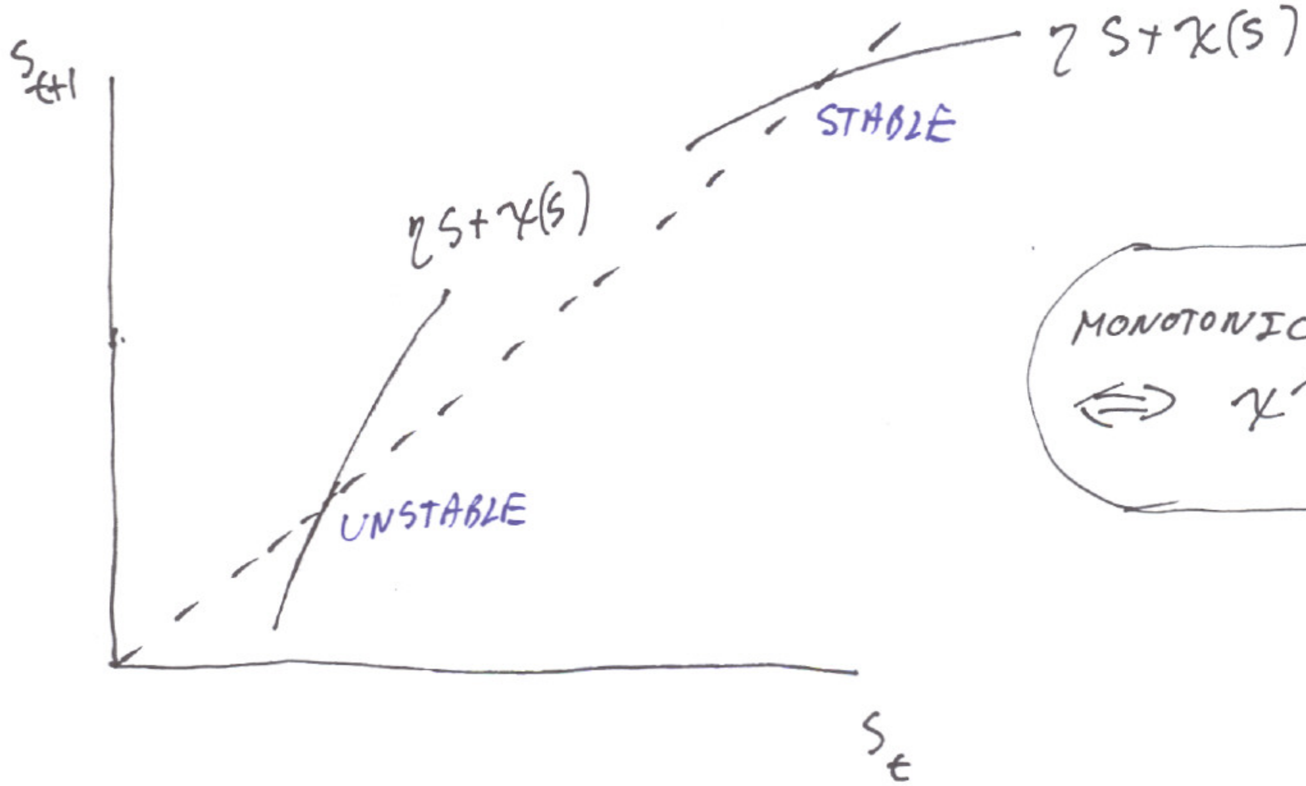
$$h_z(\infty) = -\delta\left\{\beta\, h_s(\infty) - h_z(\infty)\left(\eta + (1-\beta)\,\chi'(S_\infty)\right)\right\}$$

$$S_\infty = \eta\, S_\infty + \chi(S_\infty)$$

2 EQUATIONS IN
3 UNKNOWNS $(S_\infty, x_\infty, \chi'(S_\infty))$

STABILITY :

$$-(1+\eta) < \chi'(S_\infty) < 1-\eta$$



MONOTONICITY
$\Leftrightarrow \chi' > -\eta$

COMPARE EMISSIONS UNDER MPE

& "FULL COMMITMENT" (F.C.)

$$\boxed{\text{ASSUME} \quad h_{SZ} = 0}$$

F.C. BEGINS w/ HIGHER EMISSIONS

& HAS LOWER EMISSIONS IN S.S.

COMPARE WELFARE UNDER DIFFERENT MONOTONIC POLICIES,

$B(S; S^B_\infty)$ AND $C(S; S^C_\infty)$

RESPECTIVE EQUIL S.S.'s

$C \succsim B$ IF

$W(S; C) > W(S, B)$

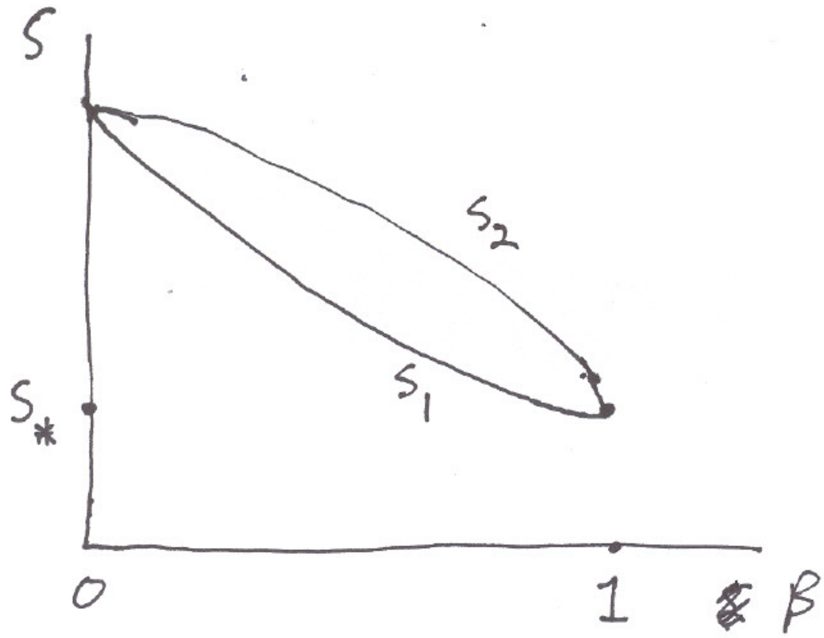
FOR S "BETWEEN" S^B_∞ & S^A_∞

NOTE : $S^C_\infty > S^B_\infty \Rightarrow C \not\succsim B$

(BECAUSE STOCK IS A 'BAD')

STABILITY + MONOTONICITY $\Longrightarrow$

MPE STEADY STATE $\in (s_1, s_2)$



FIX $\beta < 1$

CHOOSE s^a, s^b, s^c

$$s_* < s^a < s_1 < s^b < s^c < s_2$$

$$\chi(s; s^b) \succsim \chi(s; s^c)$$

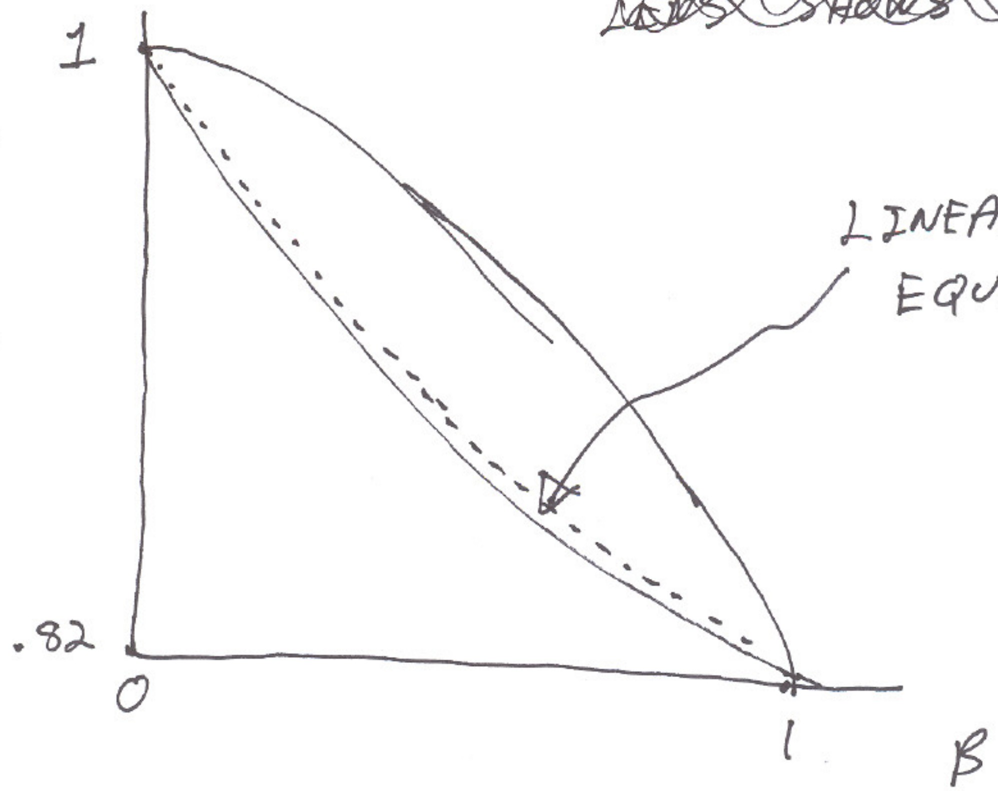
(MORE CONSERVATIVE MPE DOMINATES LESS CONSERVATIVE MPE)

$$A(s; s^a) \not\succsim \chi(s; s_1 - \epsilon), \quad \epsilon \text{ SMALL}$$

(COMMITMENT DOES NOT LEAD TO PARETO IMPROVEMENT)

GLOBAL WARMING (LINEAR QUADRATIC)

~~LENS SHOWS SET OF S.S.~~



% FIRST PERIOD ABATEMENT (RELATIVE TO B.A.U.)

r \ d	.01	.03	.05
0	25	10	5
.02	21	8	4
.04	17	7	3

$$\delta = e^{-10d}$$

$$\beta = e^{-10r}$$

GENERALIZATION :

$$\theta(\tau) \equiv \exp\left(- \int_0^{\tau} r(s)\, ds \right)$$

$$r'(s) \leq 0$$

$$r(s) = \bar{r}, \text{ A CONSTANT, FOR } s \geq T$$

$$T \leq \infty$$

$$\dot{s} = f(s, x)$$

PAYOFF : $\int_0^{\infty} \theta(\tau)\, U(s_{t+\tau}, x_{t+\tau})\, d\tau$

$\chi(s)$ a MPE POLICY

$$H(s) \equiv U(s, \chi(s))$$

$$K(s_t) \equiv \int_0^{T} \theta(\tau)\, (r(\tau) - \bar{r})\, H(s_{t+\tau})\, d\tau$$

$\chi(s)$ IS SOLUTION TO

$$\text{MAX} \int_0^{\infty} e^{-\bar{r}\tau} \left(U(s_{t+\tau}, x_{t+\tau}) - K(s_{t+\tau}) \right) d\tau$$

$$\text{s.t.} \quad \dot{s} = f(s, x)$$