L KARP

NON-CONSTANT DISCOUNTING (NCD)

OBJECTIVES

FOR GENERAL PROBLEM W/ NCD:

1) OBTAIN DPE

2) COMPARE TO A CONTROL PROBLEM W/ CONSTANT DISCOUNTING

~~3)~~

SPECIALIZE TO STATE-INDEPENDENT UTILITY, EQUATION OF MOTION LINEAR IN CONTROL (NON-LINEAR IN STATE)

1) OBSERVATIONAL NON-EQUIVALENCE

2) NON-UNIQUENES OF MPE STEADY STATE

3) RELATION TO "PRICE-TAKING" (LINEAR IN STATE)

4) PARETO RANKING OF STEADY STATE

DESCRIPTION OF PROBLEM

DISCOUNT FACTOR

$$\theta(\tau) = e^{\left(-\int_0^\tau r(s)\,ds\right)}$$

$$r(s) \equiv \bar{r} \quad \text{FOR} \quad s \geq T$$

$$r'(s) \leq 0 \quad \text{FOR} \quad s < T$$

STATE EQUATION

$$\dot{s} = f(s, x)$$

s : STATE VARIABLE

x : CONTROL VARIABLE

PAYOFF (at $\tau = 0$)

$$\int_0^\infty \theta(\tau)\, U(s_{t+\tau}, x_{t+\tau})\, d\tau$$

DISCRETE APPROXIMATION

$$\sum_{i=0}^{\infty} \theta(i\epsilon)\, U(s_{t+i\epsilon}, x_{t+i\epsilon})\,\epsilon$$

$$s_{t+\epsilon} = s_t + f(s_t, x_t)\,\epsilon$$

FIX $T < \infty$

DISCOUNT RATE FALLS FOR

$n = T/\epsilon$ PERIODS

FIND ~~MPE~~ DPE FOR MPE

OF DISCRETE STAGE PROBLEM,

LET $\epsilon \to 0$ TO OBTAIN

CONTINUOUS TIME LIMITING DPE

DEF'N.

$\chi(s)$ MP POLICY RULE

$H(s_t) \equiv U(s_t, \chi(s_t))$ EQUIL. FLOW PAYOFF

$$K(s_t) \equiv \int_0^T \theta(\tau) \underbrace{\left(r(\tau) - \bar{r}\right)}_{\geq 0} H(s_{t+\tau}) d\tau$$

LIMITING DPE:

$$\bar{r}\, w(s_t) = \max_{x} \left[U(s_t, x) + w'(s_t)\, f(s_t, x) \right] - K(s_t)$$

THIS IS ALSO DPE FOR

$$\text{MAX} \int_0^\infty e^{-\bar{r}\tau} \left(U(s_{t+\tau}, x_{t+\tau}) - K(s_{t+\tau}) \right) d\tau$$

$$\text{s.t.} \quad \dot{s} = f(s, x)$$

NECESSARY CONDITION (E.E.) MAY NOT BE SUFFICIENT

AT A STEADY STATE S_∞

$$f(S_\infty, x(S_\infty)) = 0$$

STABILITY $\Longleftrightarrow$

$$Z(S_\infty) \equiv f_s + f_x \, x'(s) < 0$$

SPECIAL CASE

$$U_s \equiv 0 \quad , \quad f(s, x) = F(s) - x$$

RAMSEY RULE

$$\eta(x_t) \frac{\dot{x}}{x} = F'(s_t) - \left(\bar{r} + \frac{K'(s_t)}{U'(x_t)} \right)$$

$$\eta \equiv \frac{-U''}{U'} x$$

$$K'(s_t) = \int_0^T \left(\theta(\tau) \left(r(\tau) - \bar{r} \right) U'(t+\tau)(H) \frac{\partial g(\tau, s_t)}{\partial s_t} \right) d\tau$$

$g(\tau, s_t)$ = EQUIL VALUE OF $S_{t+\tau}$

(INTERIOR)

STABLE STEADY STATE S_∞ REQUIRES

$$(1) \quad F'(S_\infty) = \bar{r} + \chi'(S_\infty) \int_0^T \theta(\tau)\left(r(\tau) - \bar{r}\right) e^{z\tau}\, d\tau$$

$$(2) \quad F(S_\infty) - \chi(S_\infty) = 0$$

WHERE $\quad z \equiv F'(S_\infty) - \chi'(S_\infty) < 0$

(1) & (2) COMPRISE TWO EQUATIONS IN THREE UNKNOWNS

0 SOLUTIONS OR A CONTINUUM —

AS $T \to \infty$, STEADY STATE

E.E. SIMPLIFIES TO

$$1 = \chi'(S_\infty) \int_0^\infty \theta(\tau)\, e^{Z\tau}\, d\tau$$

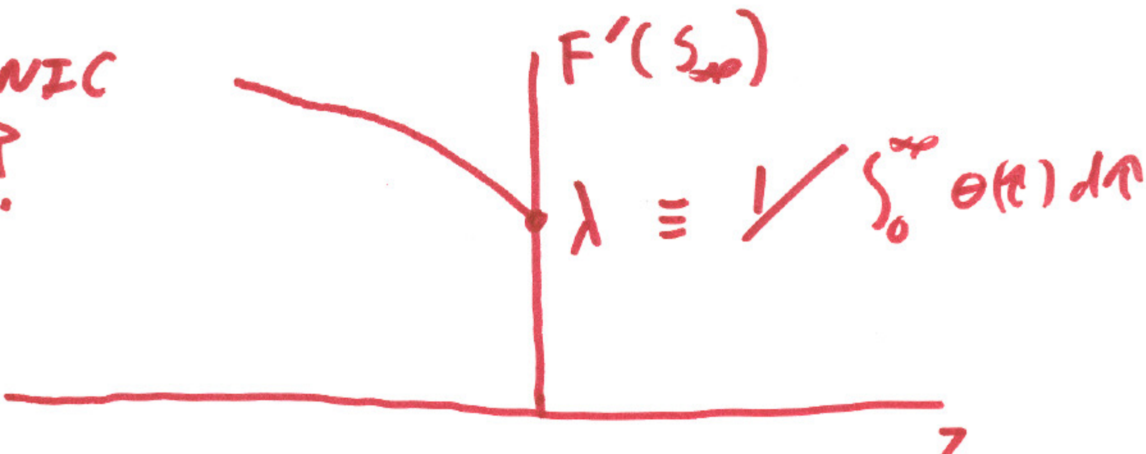
$$\Longrightarrow \quad \underbrace{\bar{r} < F'(S_\infty) < r_0}$$

NECESSARY, NOT SUFF.

RE-WRITE S.S. E.E. AS

$$0 = 1 - \left(F'(S_\infty) - Z\right) \int_0^\infty \theta(\tau)\, e^{Z\tau}\, d\tau$$

MONOTONIC ??



EXAMPLE

$$\theta(t) = \beta e^{-\gamma t} + (1-\beta) e^{-\delta t}$$

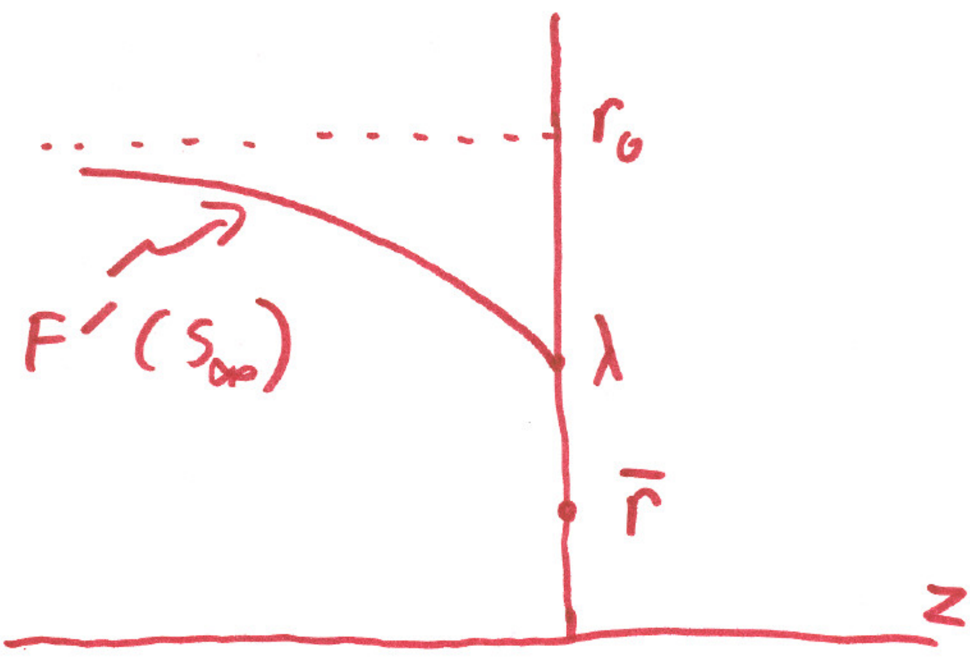
$$0 \le \gamma < \delta \qquad 0 \le \beta < 1$$

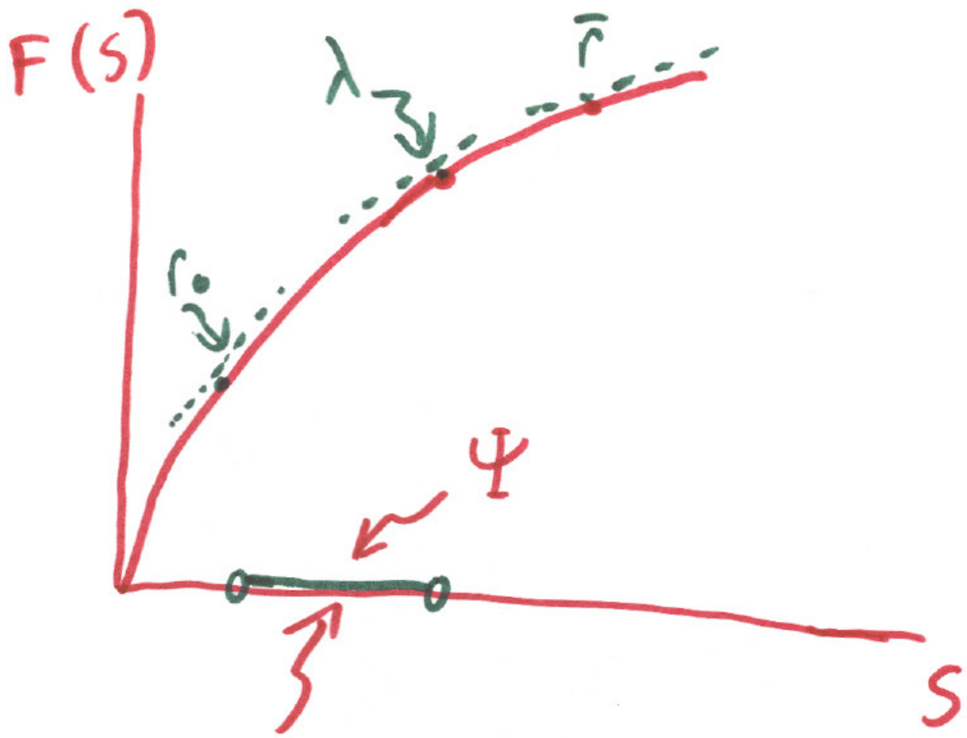
$$\bar{r} = \gamma \qquad \text{(LONG RUN)}$$

$$r_0 = \beta\gamma + (1-\beta)\delta \qquad \text{(SHORT RUN)}$$

$$\lambda = \frac{\delta\gamma}{(1-\beta)\gamma + \beta\delta}$$

$$r_0 > \lambda > \bar{r}$$





ASSUME
$F'(0) > r_0$

OPEN SET OF CANDIDATE STEADY STATE

UPPER BOUDARY SOLVES

$$F'(S) = \lambda$$

λ IS "OBSERVATIONALLY EQUIVALENT" DISCOUNT RATE IN BARRO'S LOGARITHMIC MODEL

HOW TO SELECT "THE RIGHT" STEADY STATE?

1) LOCAL LINEARITY ($\chi''(S_\infty)=0$)

2) TAKE LIMIT OF FINITE HORIZON PROBLEM.

3) REQUIRE $\chi(S; S_\infty)$ $\exists$ "GLOBALLY"

NOTATION:

$\chi(S; S_\infty)$ SATISFIES NECESSARY CONDITIONS FOR MPE & DRIVES STATE TO S_∞ FOR $S \approx S_\infty$

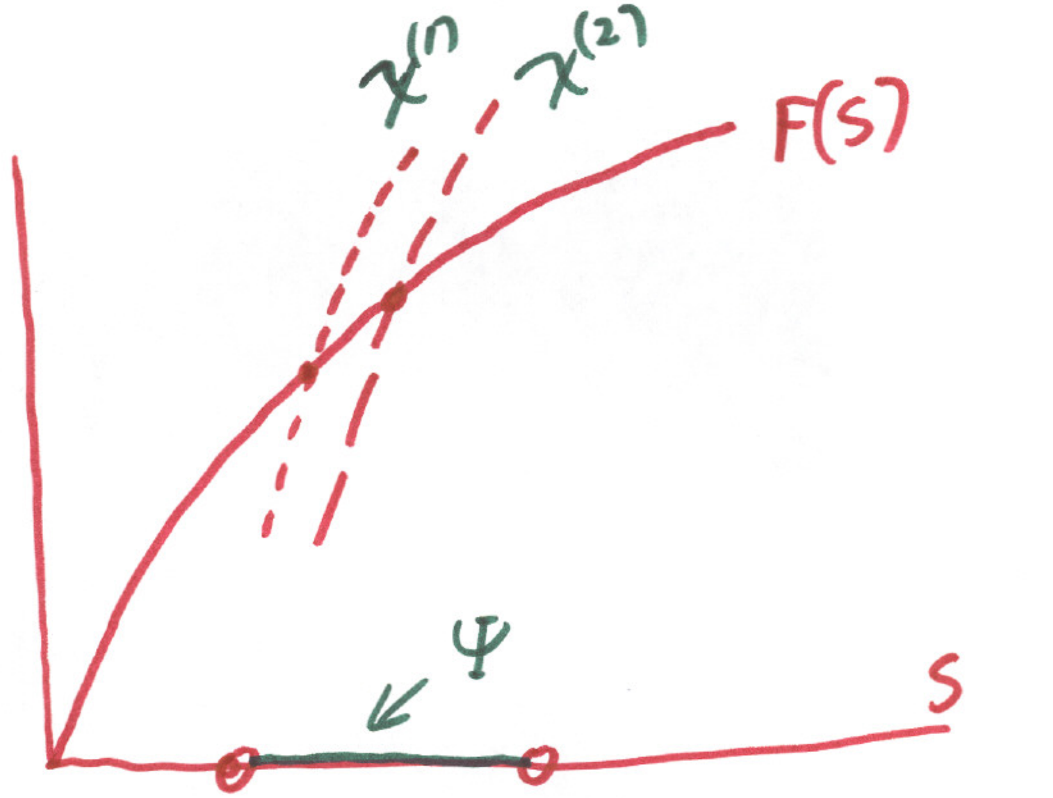
COMPARE NEIGHBORING POLICIES

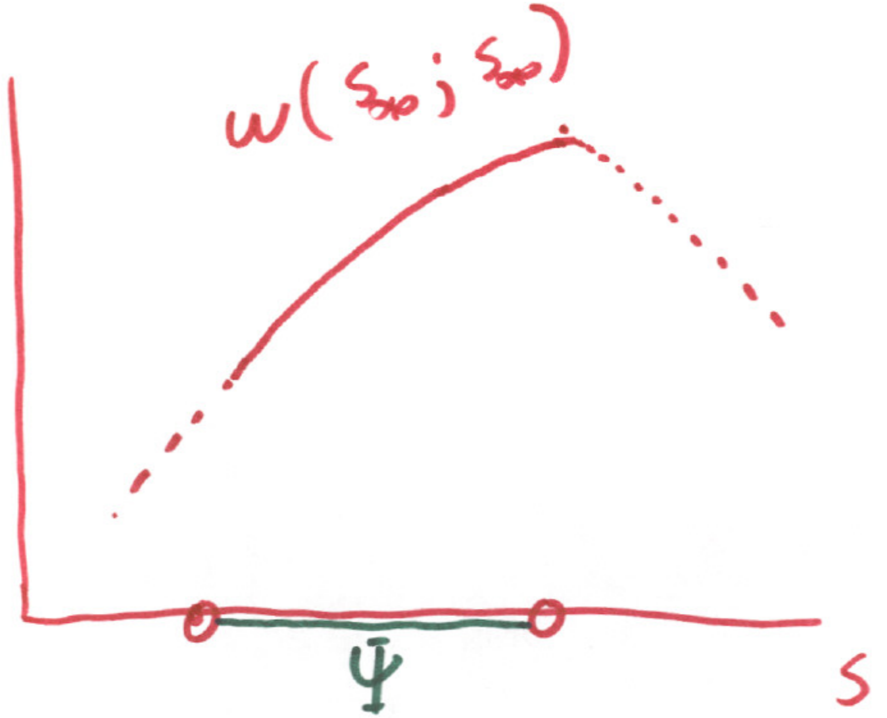
$x(s; s_\infty)$ + $x(s; s_\infty + \epsilon)$

FOR $s \approx s_\infty$

("LOCAL" PARETO DOMINANCE)

LESS CONSERVATIVE POLICY <u>NEVER</u> DOMINATES





MORE CONSERVATIVE POLICY

(I.E., HIGHER STEADY STATE)

"LOCALLY" PARETO DOMINATES

LESS CONSERVATIVE POLICY,

FOR s_{op} $\ni$: $F'(s_{op}) < \lambda$

THIS IS NOT A GLOBAL RESULT.

e.g. $S_\infty^1 > S_\infty^2$ AND

$S_\infty^1 \approx S_\infty^2$

THE GRAPHS OF $w(S; S_\infty^i)$

MIGHT CROSS