L. KARP, JUNE 15 2004 (1)

Gollier, Jullien & Treich

J Pub Econ 2000 vol 75 229–253

Unknown parameter, treat as r.v. $\tilde{z}$

w/ support $Z = (z_1, z_2, \ldots z_m)$

$\tilde{y}$ is an "experiment", a r.v. correlated w/ $\tilde{z}$

$$\pi_y(z_i) = pr(\tilde{z} = z_i \mid \tilde{y} = y)$$

Defn: Experiment $\tilde{y}$ is more informative than experiment $\tilde{y}'$ iff

for any f convex on Δ_Z

$$E_{\tilde{y}}\, f(\pi_{\tilde{y}}) \geq E_{\tilde{y}'}\, f(\pi_{\tilde{y}'}),$$

where $\Delta_Z = \left\{ \pi_y \in R^m_+ \mid \sum_{i=1}^m \pi_y(z_i) = 1 \right\}$

Note: Defn $\Rightarrow$

$$E_{\tilde{y}}\, \pi_{\tilde{y}} = E_{\tilde{y}'}\, \pi_{\tilde{y}'}$$

(Let $f = \gamma' \pi$, i.e. f is linear. So $-f$ is also linear. So if $\tilde{y}$ more informative than $\tilde{y}'$, Defn $\Rightarrow$

$$E_{\tilde{y}}\, \gamma' \pi_{\tilde{y}} \geq E_{\tilde{y}'}\, \gamma' \pi_{\tilde{y}'}$$

and

$$E_{\tilde{y}} - \gamma' \pi_{\tilde{y}} \geq E_{\tilde{y}'}\, \gamma' \pi_{\tilde{y}'}$$

$$\Rightarrow$$

$$E_{\tilde{y}}\, \pi_{\tilde{y}} = E_{\tilde{y}}\, \pi_{\tilde{y}'}$$

Motivation (explanation) for Defn.

Consider the problem

$$\max_{a} \underset{\tilde{z}}{E}\, R(a, \tilde{z}) = \max_{a} \sum_i \pi_{z_i} R(a, z_i)$$

optimal decision

$$a^* = a^*(\pi) \quad , \quad \pi = \begin{pmatrix} \pi_{z_1} \\ \pi_{z_2} \\ \vdots \\ \pi_{z_m} \end{pmatrix}$$

$$\rho(\pi) \equiv \sum_i \pi_{z_i} R\big(a^*(\pi), z_i\big)$$

Note that $\rho(\pi)$ is convex in π.

Demonstration :

Pick $\pi^1, \pi^2, \pi^\lambda = \lambda \pi^1 + (1-\lambda)\pi^2$,
$0 < \lambda < 1$

"$\rho(\pi)$ is convex in π" means:

$$\lambda\, \rho(\pi^1) + (1-\lambda)\,\rho(\pi^2) \geq \rho(\pi^\lambda)$$

$$LHS = \lambda \sum \pi^1_{z_i} R(a^*(\pi^1), z_i) + (1-\lambda) \sum \pi^2_{z_i} R(a^*(\pi^2), z_i)$$

$$\geq \lambda \sum \pi^1_{z_i} R(a^*(\pi^\lambda), z_i) + (1-\lambda) \sum \pi^2_{z_i} R(a^*(\pi^\lambda), z_i)$$

(because $a^*(\pi^\lambda)$ not optimal at π^1, π^2)

$$= \sum \pi^\lambda_{z_i} R(a^*(\pi^\lambda), z_i) = \rho(\pi^\lambda)$$

Definition implies that, whatever is my optimization problem, I obtain a (weakly) higher payoff if I have more information.

Example: $\tilde{z}$ has three possible realizations.

$\tilde{y}$ has three possible realizations, with

$$\pi_{y_1}(z) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \pi_{y_2}(z) = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \pi_{y_3}(z) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$\tilde{y}'$ is a degenerate r.v. that has the single realization resulting in $\pi_y(z) = \begin{pmatrix} p_1 \\ p_2 \\ 1-p_1-p_2 \end{pmatrix}$

Obviously $\tilde{y}$ is more informative than $\tilde{y}'$

The problem

$$\max_{c_1} \; U(c_1) + \underset{\tilde{y}}{E}\left\{ \max_{c_2 \geq 0} \underset{\tilde{x}|y}{E} \, v\left(c_2 - \tilde{x}(\delta c_1 + c_2)\right)\right\}$$

$\tilde{x}$ is damage parameter

c_i is consumption

δ persistence of "stock"

Define

$$C = \delta c_1 + c_2 \qquad \text{(cummulative stock)}$$

$$\tilde{Z} = 1 - \tilde{x}$$

Rewrite problem

$$\max_{c_1} \; U(c_1) + \underset{\tilde{y}}{E} \left\{ \max_{C \geq \delta c_1} \underset{\tilde{Z}|y}{E} \, v(-\delta c_1 + \tilde{Z} C) \right\}$$

The "value function"

$$j(c_1, \pi_y) = \max_{C \geq \delta c_1} \underset{\tilde{Z}|y}{E} \, v(-\delta c_1 + \tilde{Z} C)$$

$$= \underset{\tilde{Z}|y}{E} \, v\Big(-\delta c_1 + \tilde{Z} \underbrace{C^*(c_1, \pi_y)}_{\text{The optimal policy function.}}\Big)$$

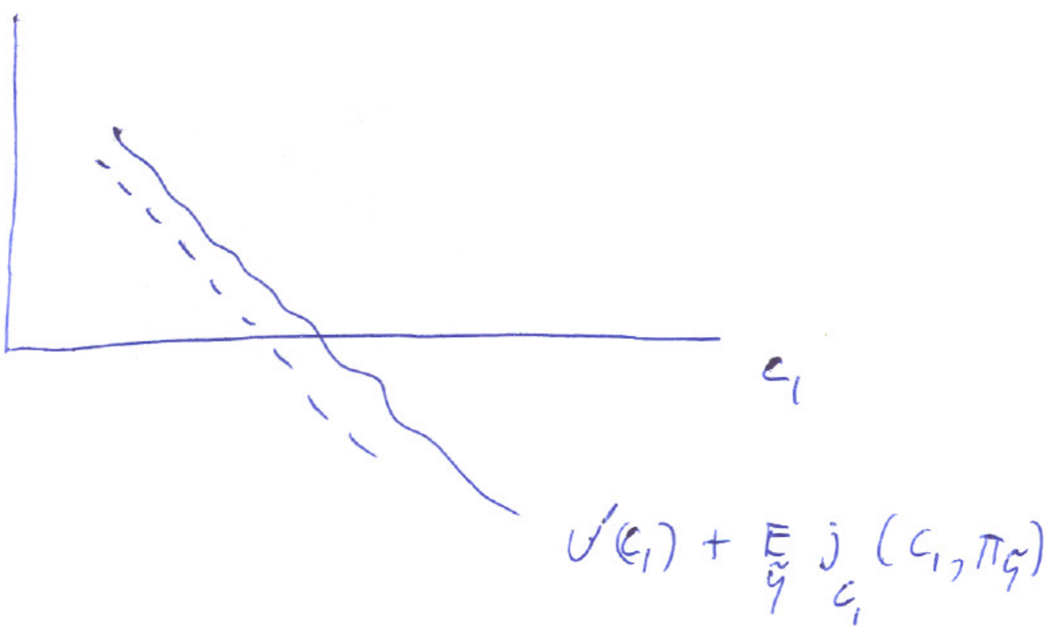
Assume c^* is interior. By envelope then

$$j_{c_1} = -s \underset{\tilde{z}|y}{E} v'\left(-s c_1 + \tilde{z}\, c^*(c_1, \pi_y)\right)$$

1st period problem.

$$\max_{c_1} \; U(c_1) + \underset{\tilde{y}}{E}\, j(c_1, \pi_{\tilde{y}})$$

F.O.C. $U'(c_1) + \underset{\tilde{y}}{E}\, j_{c_1}(c_1, \pi_{\tilde{y}}) = 0$



If $j_c(c_1, \pi)$ is concave in π, then from Definition, a more informative signal in the second period decreases $E(j_{c_1})$, decreasing optimal c_1. In this case, the anticipation of better information in future ~~of~~ decreases 1st period action.

The reverse holds if $j_c(c_1, \pi)$ is convex in π. (Epstein)

(HARA UTILITY)

Special case : $u(x) = \frac{\gamma}{1-\gamma}\left(\eta + \frac{x}{\gamma}\right)^{1-\gamma}$

$\eta = 0 \Rightarrow$ CRRA γ

$\gamma \to \infty \Rightarrow$ CARA η

If $v(\,)$ is HARA

Better information:

Reduces c_1 iff $0 \le \gamma \le 1$

(i.e. better info increases c_1 for $\gamma < 0$ or $\gamma > 1$)

If $v(\,)$ is not HARA, effect of better information is ambiguous.