Notes on Kaye & Zhang.

ABATEMENT COST:

$$\frac{b}{2}\left(x_t^{b} - x_t\right)^2$$

$b > 0$

$x_t^{BAU} = \bar{x} + \theta_t$ = BAU EMISSION

$\theta_t \sim iid$

x_t = ACTUAL EMISSIONS.

UNDER QUOTA, REG. CHOOSES

X_t

UNDER TAX P_t,

$$X_t = \text{ARGMIN } \frac{b}{2}\left(X_t^{BAU} - X_t\right)^2 + P_t X_t$$

$$\Rightarrow X_t = \bar{X} - \frac{P_t}{b} + \tilde{\theta}_t$$

$$\equiv Z_t + \tilde{\theta}_t$$

EVOLUTION OF STOCK

$$S_{t+1} = \Delta S_t + X_t$$

$$0 < \Delta < 1$$

ENV. DAMAGES

$$\frac{G^*}{2}\left(S_t - \bar{S}\right)^2 w_t$$

$G^* > 0$, UNKNOWN

$w_t \sim iid$

$G_t = E_t(G^*)$

$\sigma^2_{G,t}$ = (VECTOR) of

higher moments

$x_t = (G_t, \sigma^2_{G,t})$

VECTOR OF SUBJECTIVE MOMENTS

ASSUMPTION ABOUT LEARNING

(a) $E_t \, G_{t+\tau} = G_t \qquad \tau \geq 0$

(b) $\sigma^2_{G,t}$ INDEPENDENT of

$$(S_{t-\tau}, X_{t-\tau}), \quad \tau \geq 0$$

n_t = number of times REGULATOR will learn about G^*

PAYOFF :

$$J(S_t, G_t, \sigma^2_{G,t}, n_t) = \text{MAX } E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \Big\{ f$$

$$+ a Z_{t+\tau} - \frac{b Z^2_{t+\tau}}{2} + \frac{\sigma^2_{\theta}}{2} - c S_{t+\tau} -$$

$$\frac{G^*_{\cancel{t+\tau}}}{2} S^2_{t+\tau} \Big\}$$

$$J(S_t, x_t, n_t) = \max_{z} \Big\{ \underline{f + az - \frac{bz^2}{2}}$$

$$\underline{+ \frac{\sigma_\theta^2}{2b} - cS_t - \frac{G_t S_t^2}{2}} +$$

$$\beta \, E_{x_{t+1}} \Big(E_{\theta_t} J(S_{t+1}, x_{t+1}, n_{t+1}) \Big) \Big\}$$

s.t.

$$S_{t+1} = \Delta S_t + z + \theta$$

$$n_{t+1} = \max(n_t - 1, 0)$$

$$x_{t+1} = \;.\;.\;.\;.\;.\; ?$$

COMPARE $n=0$, $n=1$

FOR $n=0$

$$J(S, G, \sigma_G^2, 0) \equiv \tilde{J}(s; G)$$

FOC :

$$n=0: \quad a - bz = -\beta \underline{E_\theta \tilde{J}_s(s'; G)}$$

$$n=1: \quad a - bz = -\beta E_{G'|G} \underline{\left(E_\theta \tilde{J}_s(s'; G)\right)}$$

$$n \geq 2: \quad a - bz = -\beta E_{x'|x} E_\theta J_s(s', x', n-1)$$

If $E_\theta \tilde{J}_s(s'; G)$ is convex in $G \Rightarrow$

learning increases z

$$a - bz + \beta E_{G'}\left(E_\theta \tilde{J}_s(s'; G')\right)$$

VALUE FUNCTION IS QUADRATIC IN S

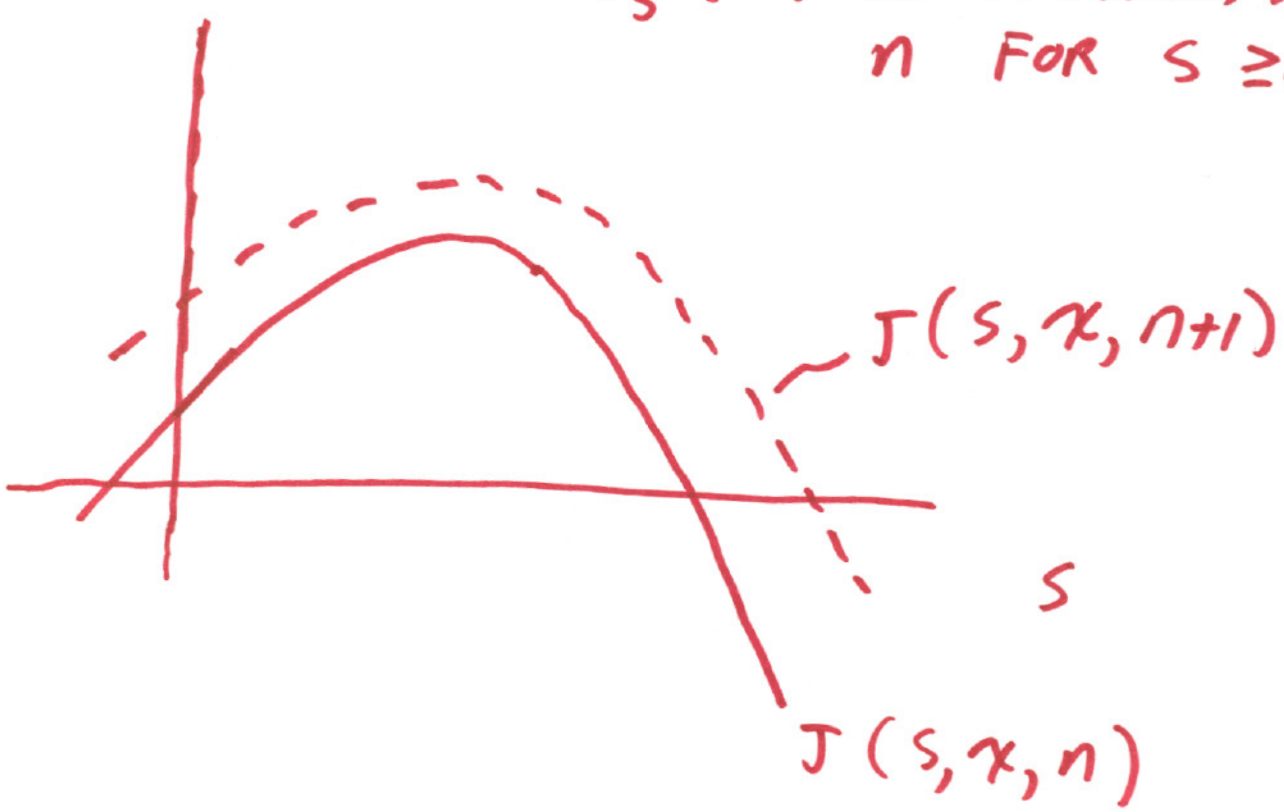
$$J(s, x, n) = \lambda_n(x) + \mu_n(x)\, s + \frac{\rho_n(x)}{2} s^2$$

$$\Rightarrow \quad J_s = \mu_n + \rho_n\, s$$

BASIC RESULT :

$\rho_n(x)$ & $\mu_n(x)$ ARE INCREASING IN n $\Rightarrow$

$J_s(\ )$ INCREASING IN n FOR $s \geq 0$



RESULTS:

LARGER n (i.e. MORE OPPORTUNITY TO LEARN)

- INCREASES (EXPECTED) EMISSIONS
- ~~REDUCES~~ FAVORS TAXES OVER QUOTAS

LOGNORMAL MODEL

$$\text{DAMAGES} = \frac{G^*}{2}(S_t - \bar{S})^2 w_t$$

$$w_t \sim \text{LOGNORMAL}\left(-\frac{\sigma_w^2}{2}, \sigma_w^2\right)$$

$$g^* \equiv \ln(G^*)$$

PRIORS:

$$g^* \sim N(g_t, \sigma_{g,t}^2)$$

1. A higher stock increases the variance of damages. (Some physical scientists believe that an increased variance of climate-related events is the greatest threat of greenhouse gasses.)

2. Even if the regulator currently believes that stock-related damages and marginal damages are moderate, he might discover that they are very high.

3. The subjective variance of G^* (and thus, the variance of damages and marginal damages) is stochastic, and it is possible that unexpected observations make the regulator less certain about the true value of G^*. However, as the number of observations approaches infinity, the subjective distribution collapses to the true value of G^* (Thus, Corollary 1 holds.)

4. Assumption 1 is satisfied: the current estimate of G^* is an unbiased estimate of future estimates, and the regulator chooses emissions to control the stock rather than to learn about the unknown parameter.

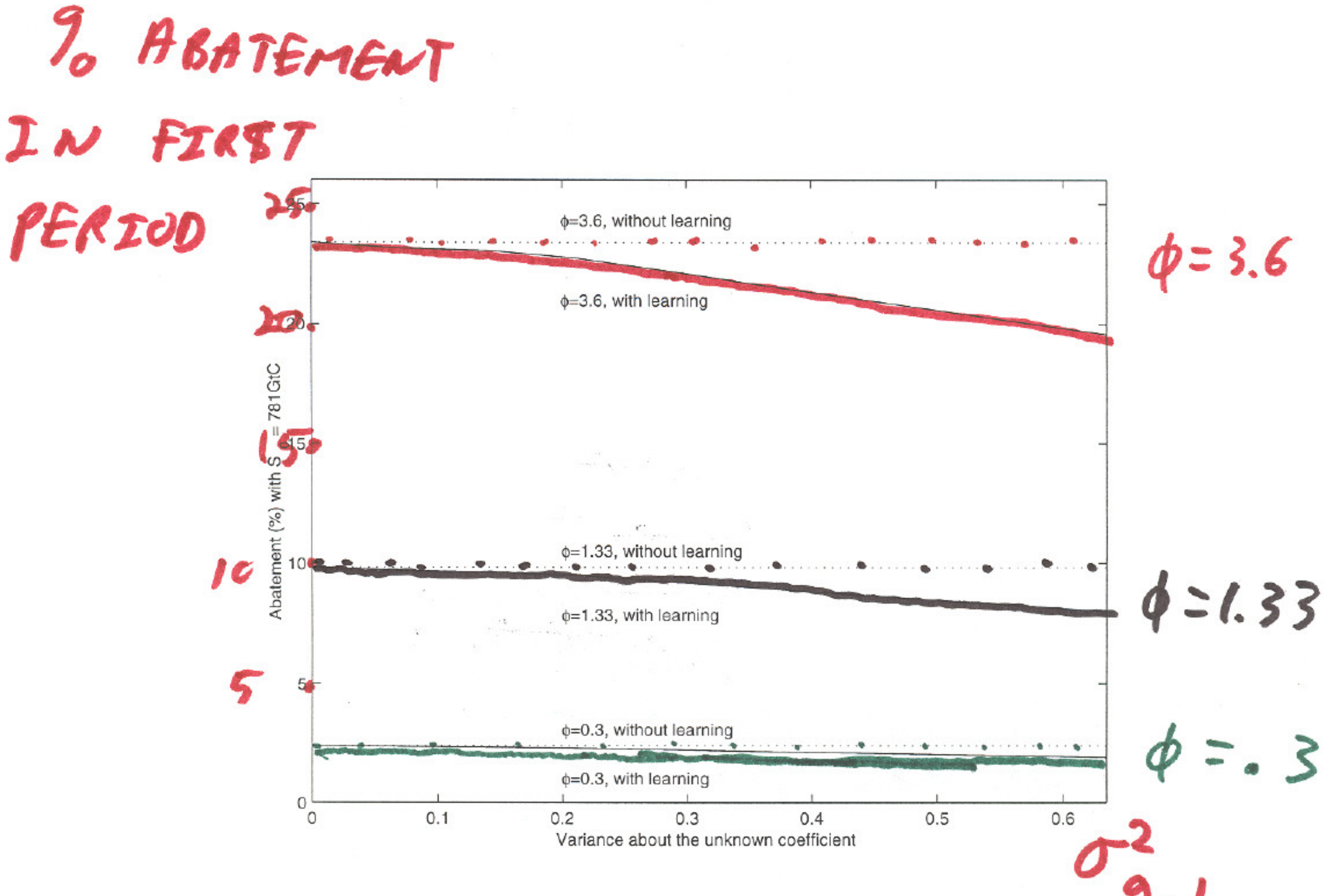


Figure 1: Abatement with learning as a function of initial uncertainty

% of ABATEMENT

FIRST PERIOD

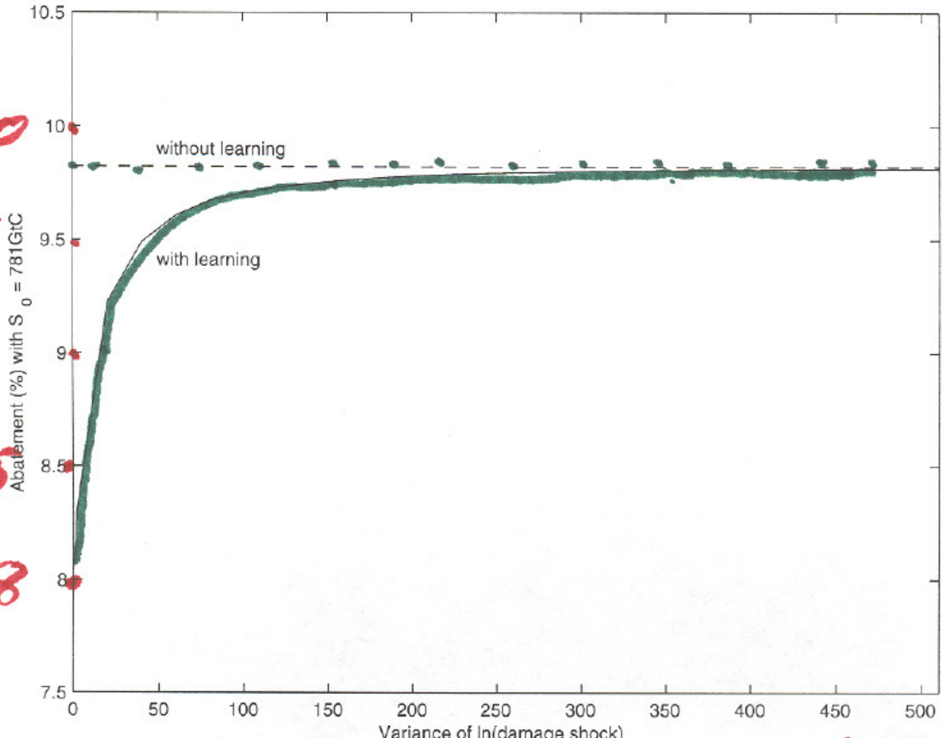
10

9.5

9

8.5

8



VAR(LN(w))

Figure 2: Abatement with learning as a function of the variance of the signal ($\phi = 1.33$, $\sigma_g^2 = 0.63$)

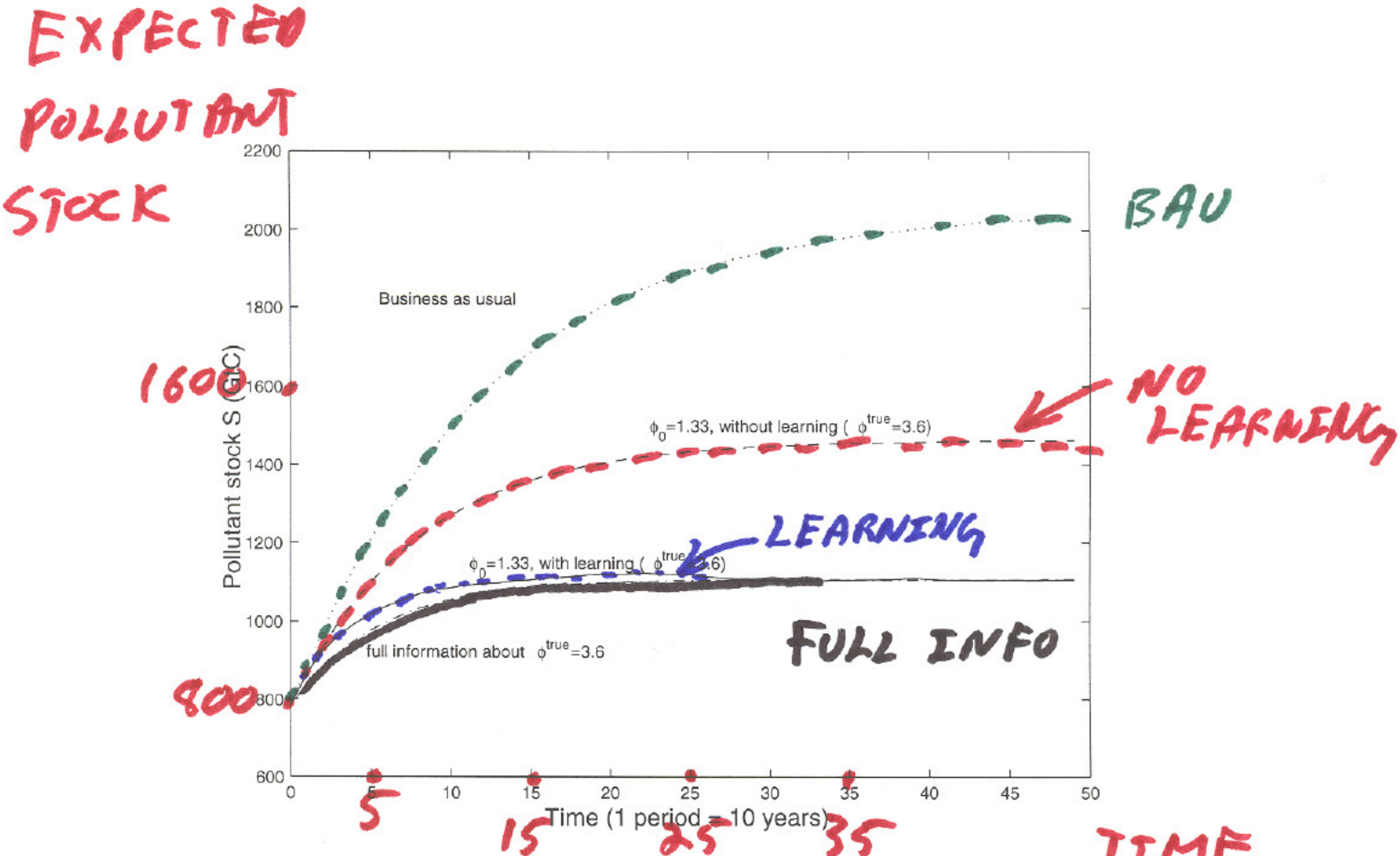


Figure 3: The effect of learning on the stock trajectory

TRUE VALUE : $\phi = 3.6$

PRIOR : $\phi = 1.33$