

# Dynamic taxation of non-point source stock pollution : the waterlogging case \*

Sophie Legras <sup>†</sup>

June 1, 2004

## Abstract

In this paper, our concern is the design of dynamic taxes in a context of non-point source, stock and endogenous externality. Particular taxation schemes have been proposed to address informational issues linked to the non-point source nature of numerous externalities. These are mostly static and consequently unadapted to the management of non-point source pollution which accumulates over time, as is the case with waterlogging in most semi-arid regions of the world. We use a differential game approach to study dynamic taxes, namely an input tax and an ambient tax. We show that it is possible to design optimal taxation schemes.

**keywords:** waterlogging, non-point source pollution, stock externality, input tax, ambient tax.

## 1 Introduction

Our aim is to study a situation combining two notions underlying the most pressing environmental problems, namely stock externality and non-point source pollution. In irrigated districts in Australia, as in most semi-arid regions of the world, over-irrigation has led to the appearance of waterlogging, defined as the rising of the water table over time that leads to the soaking of soils, impeding agricultural production. In this regard, one might consider the water applied in excess of irrigation as a stock pollutant, which accumulates in the underground hydrological system and causes damage to production. Also, the generation of excess water has some characteristics that allow considering it as a non-point source pollution. Indeed, it involves a great number of irrigators and the mechanisms at stake during the percolation of water are stochastic. This means that individual emissions (here,

---

\*I would like to thank, without implicating, Mabel Tidball and Robert Lifran for their supporting help with this work.

<sup>†</sup>INRA-LAMETA and University of Montpellier I, 2 place Pierre Viala, 34060 Montpellier Cedex 2, France. E-mail: legras@ensam.inra.fr

contributions to the watertable rise) are not observable at a reasonable cost. Another interesting characteristic of this situation is the endogenous nature of the externality. Indeed, in standard non-point source or stock pollution problems, the externality does not affect the polluters but some third parties [3]. In the context of waterlogging, the rising of the water-table, resulting from individual irrigation decisions of the agents, has an adverse effect on the production of all the irrigators. The endogenization of the externality induces agents to partially internalize the adverse effect, as they are directly affected by it; it also induces strategic interaction among the agents, as in Cochard et al. [3].

Our purpose is to find optimal taxation schemes to correct the inefficient resource allocation resulting from irrigation decisions by non cooperative agents.

Stock externalities are characterised by the fact that, if their generation is a flow variable at any given time, their impact is a stock variable at the time of the evaluation [16]. Consequently, an analysis of this special category of externalities should not ignore their inherent dynamic characteristics. Stock externalities are of particular relevance in the study of the most stringent environmental problems, such as climate change [11], acid rains [10] or, in semi-arid countries, waterlogging and its twin menace, salinity [4]. A stream of literature has developed around the issue of taxing agents in the context of stock point-source pollution [9][1]. Benchekroun and van Long [1] analyse a linear Markov perfect tax rule, developed by Karp and Livernois [9] for a monopoly exploiting a non-renewable resource, in the context of a polluting oligopoly. This taxation scheme sends to polluters the message that the more they pollute now, the higher their tax liability will be in the future. Benchekroun and van Long [1] show that there is a taxation scheme, independent of time, that induces the agents to attain the optimal pollution and production paths. This tax is linear in the firm's output, and its rate is dependant on the stock of pollution.

Informational issues are at the core of non-point source externalities analysis. When the regulator cannot observe individual emissions of pollution, because the polluters are numerous and spatially distributed, and/or because of stochastic issues related to the pollutant transport, she cannot penalise each polluter at the *pro rata* of his contribution to the total pollution. The regulator should change perspective in looking for a basis for taxation. Candidates for compliance bases have to be observable, enforceable and targetable in space and time [2]. They include inputs correlated with pollution flows and ambient pollution. Most non-point source pollution analyses do not take account of the cumulative process of pollutants and restrict to static settings [5][12][13]. Dynamic analyses of non-point source pollution taxation include [8][14].

Inputs-based instruments have long been considered as potential substitutes to direct taxation on negative externalities [13]. Griffin and Bromley [5] first proposed to apply an input tax in the context of non-point source pollution. An efficient input taxation scheme is highly demanding in information. On the one hand, the regulator is supposed to be able to identify with certainty all the inputs contributing to the generation of the pollution. On the other hand, the incitation being farm-specific, the regulator is supposed to have access to each firm-type, reflecting the differentiation of environmental impacts according to parameters such as the nature of the soils. We circumvent these informational issues by developing a simple model in which the only input contributing to pollution is water, and farms are homogenous by respect to pollution generation. The informational burden is thus reduced. In this context, an adaptation of Benchekroun and van Long [1]'s taxation framework is proposed, based on inputs rather than emissions.

Another way to circumvent the unobservability of emissions is to address non-point source pollution as a group moral hazard problem. Segerson [12], following Holmstrom's [6] analysis of moral hazard in production teams, proposed a method of regulating non-point source pollution, by charging each firm a unit tax based on the aggregate level of pollution. Supposing that the regulator has defined a pollution target, the scheme is such that each firm pays a tax from the moment that the target is not attained, which happens when a single agent deviates. A well-understood limitation to this solution is that polluters have to recognize that their individual emissions have a significant impact on ambient pollution. In some non-point source pollution contexts, it is not reasonable to assume so, for instance when considering exhaust fumes. However, non-point source pollution is often related to pollution of waterbodies. In this case, the scale of analysis allows reducing the number of agents under consideration. In the context of waterlogging, the analysis is driven by hydrological relevance, and agents under consideration are those located above the watertable. Being aware of their contribution to the level of pollution, and consequently to the level of taxation, agents behave strategically between them and with the regulator. They are incited to reduce their individual discharges in reaction to a tax which is not perceived as fixed, but dependant on their irrigation decisions. Karp [8] and Xepapadeas [14] have developed ambient taxes in a dynamic setting. Karp [8] considers the case of a flow pollutant, and designs an optimal dynamic taxation scheme, based on aggregate emissions. Xepapadeas [14] explicitly takes into account the dynamic process of pollutant accumulation, thus the inherent stock nature of most non-point source pollutions. In both contexts of certainty and environmental uncertainty about the natural decay rate, he analyses the design of intertemporal incentive scheme for the management of a non-point source stock pollution. He shows that an efficient scheme

takes the form of a charge per unit deviation between desired and observed pollution accumulation paths, a kind of analogue to the taxation scheme developed by Segerson [12] in a static setting.

Developing a simple model of waterlogging, we propose to study the taxation frameworks addressed above. First, we apply the taxation concept of Benckroun and van Long [1] to non-point source pollution. We develop an input tax, linear in the agents' input use, and dependant on the stock of pollution. Then, we apply the ambient tax analysed by Xepapadeas [14] to the particular setting of waterlogging.

Section 2 presents the model, and analyses cooperative and non-cooperative outcomes. Section 3 analyzes a dynamic input tax. Section 4 develops an ambient tax. Section 5 concludes.

## 2 The basic model

### 2.1 Model description

Consider  $n$  agents, indexed by  $i$ , that produce a homogeneous good from a unique input. Let  $u_i$  be agent  $i$ 's use of the input, irrigation water in our example. The totality of the applied water is not used by plants; some water percolates to reach the watertable where it accumulates. This percolation water, the pollution  $e_i$  that each agent emits, is a function of the quantity of input that has been used:  $e_i = \theta u_i$ <sup>1</sup> where  $\theta$  is a percolation parameter. In order to simplify the notations, by an appropriate choice of units we can set  $\theta = 1$ . If  $e_i$  is a flow variable at each  $t$ , the impact of pollution, namely its accumulation over time  $X$ , is a stock variable at the time of evaluation. The accumulation of the pollutant is described by the following differential equation, where  $\delta > 0$  is a natural discharge rate<sup>2</sup>:

$$\dot{X} = \sum_i u_i - \delta X \tag{1}$$

$$X(0) = X_0 > 0 \tag{2}$$

We consider that the agents are homogeneous and all discount rates are equal. The benefit of any agent at each instant of time can be written as a

---

<sup>1</sup>This simplification is of course very restrictive, as it does not allow taking account of the heterogeneity of transfer coefficients between agents. Furthermore, it implies that the relationship between applied inputs and pollution reaching the stock is determinist. See [2] for an analysis of these questions

<sup>2</sup>We suppose in the first place that  $\delta$  is an exogenously defined parameter. Indeed, we could consider giving a more complex definition to the discharge rate, for instance  $\delta(X)$ . This specification would more in compliance with hydrological models.

function of its input and of the stock of pollution:  $F(u_i, X)$ , with  $\frac{\partial F}{\partial u_i} > 0$  and  $\frac{\partial F}{\partial X} < 0$ . This characterizes the endogenous nature of the externality. In the remainder of this paper, we will often have recourse to a quadratic expression of the benefit function, to facilitate the resolution of some problems. We will consider the following linear quadratic benefit function :

$$F(u_i, X) = a + bu_i - \frac{c}{2}u_i^2 - \frac{d}{2}X^2$$

where  $a, b, c, d$  are positive parameters.

We chose to express the benefit function in a separable form, with independence between the terms in  $u_i$  and  $X$ , by convenience. We assume that the totality of the damages are captured by the endogenous externality, in other words consumers are not affected by the pollution stock.

## 2.2 Cooperative outcome

When external effects are fully internalised, the optimal provision of a stock externality is a cooperative solution involving all agents. We compare this solution to the non cooperative outcomes, either in the open loop or the feedback formulation.

Cooperative agents jointly maximise their benefits.

$$\max_{u_1, \dots, u_n} V = \int_0^\infty \sum_i F(u_i, X) e^{-rt} dt$$

subject to (1) and (2), where  $r$  denotes the agents' discount rate.

The current value Hamiltonian is:

$$H^C(u_i, X, \lambda^*) = nF(u_i, X) + \lambda^*(nu_i - \delta X)$$

where  $\lambda^*$  is interpreted as the dynamic shadow cost of pollutant concentration.

The necessary conditions for optimality are:

$$\lambda^* = -\frac{\partial F}{\partial u_i} \tag{3}$$

$$\dot{\lambda}^* = (r + \delta)\lambda^* - n\frac{\partial F}{\partial X} \tag{4}$$

along with (2) and the transversality condition :  $\lim_{t \rightarrow \infty} e^{-rt} \lambda^*(t) X(t) = 0$

### Optimal steady state

The optimal steady state is characterized by the following equations, obtained by setting  $\dot{\lambda}^* = \dot{X} = 0$ :

$$u_{\infty}^* = \frac{\delta}{n} X_{\infty}^* \quad (5)$$

where  $X_{\infty}^*$  follows the relation:

$$\frac{\partial F}{\partial u_i}(u_{\infty}^*, X_{\infty}^*) = -\lambda_{\infty}^* = -\frac{n}{r + \delta} \frac{\partial F}{\partial X}(u_{\infty}^*, X_{\infty}^*) \quad (6)$$

Thus, at the steady state, the valuation of the individual marginal benefit is equal to the present value of the stream of marginal adverse effect on group production.

With the quadratic expression of the benefit function, the optimal steady state is given by:

$$X_{\infty}^* = \frac{nb(r + \delta)}{c\delta(r + \delta) + n^2d} \quad (7)$$

$$u_{\infty}^* = \frac{\delta}{n} X_{\infty}^* = \frac{\delta b(r + \delta)}{c\delta(r + \delta) + n^2d} \quad (8)$$

$$\lambda_{\infty}^* = -\frac{nd}{r + \delta} X_{\infty}^* = -\frac{n^2db}{c\delta(r + \delta) + n^2d} < 0 \quad (9)$$

### Optimal stock path

It can be shown that the optimal stock path  $X^*(t)$  is the following:

$$X^*(t) = (X_0 - X_{\infty}^*)e^{\rho t} + X_{\infty}^* \quad (10)$$

and consequently the input use optimal path is:

$$u_i^*(t) = \frac{X^*(t)(\delta + \rho) - \rho X_{\infty}^*}{n} \quad (11)$$

where  $\rho$  is a negative root of the quadratic equation:

$$\rho^2 - r\rho - \left[\frac{n^2d}{c} + \delta(r + \delta)\right] = 0 \quad (12)$$

### 2.3 Non cooperative outcomes

We carry out our analysis in the context of an  $n$ -player non cooperative dynamic game. The strategy of the players, defined as a time path  $\{u_i(t)\}$ , will depend on their informational structures.

An open-loop informational structure refers to a situation where the agents' decisions at  $t$  depend on  $t$  and the initial level of the stock. Resulting open-loop Nash equilibrium (OLNE) correspond to an infinite period of commitment : agents commit themselves to a particular emission path and do not respond to observed variations of the state variable. OLNE are questioned for being unrealistic. However, they constitute a "benchmark for discussing the effects of strategic incentives in the closed-loop information structure"<sup>3</sup>.

A feedback informational structure denotes a situation where decisions are conditioned upon  $t$  and the current state of the stock. Feedback Nash equilibria (FBNE) allow studying strategic interaction between agents, as each knows that the others will react in function of the level of the stock. Expectations of the others' reaction can be expressed in various ways. Among Feedback strategies, we consider a particular linear Markov strategy.

#### Open loop case

$$\max_{u_i} V_i = \int_0^{\infty} F(u_i, X) \cdot e^{-rt} \cdot dt \text{ subject to (1) and (2).}$$

The current value Hamiltonian for non cooperative agents is:

$$H^{No}(u_i, X, \lambda^N) = F(u_i, X) + \lambda^N (u_i + \sum_{j \neq i} u_j - \delta X)$$

The necessary conditions for this problem are:

$$\lambda^N = - \frac{\partial F}{\partial u_i} \tag{13}$$

$$\dot{\lambda}^N = (r + \delta)\lambda^N - \frac{\partial F}{\partial X} \tag{14}$$

along with (2) and the transversality condition :  $\lim_{t \rightarrow \infty} e^{-rt} \lambda^N(t) X(t) = 0$

#### Steady state

The steady state for non cooperative agents is characterized by the following equations:

$$u_{\infty}^{No} = \frac{\delta}{n} X_{\infty}^{No} \tag{15}$$

---

<sup>3</sup>Fudenberg and Tirole (1991). *Game Theory*. Cambridge (USA), MIT Press. p. 131

where  $X_\infty^{No}$  follows the relation:

$$-\frac{\partial F}{\partial u_i}(u_\infty^{No}, X_\infty^{No}) = \lambda_\infty^{No} = \frac{1}{r + \delta} \frac{\partial F}{\partial X}(u_\infty^{No}, X_\infty^{No}) \quad (16)$$

Using the quadratic expression of  $F$ , we obtain the steady state stock:

$$X_\infty^{No} = \frac{nb(r + \delta)}{c\delta(r + \delta) + nd} \quad (17)$$

Thus, at the steady state, the valuation of the individual marginal benefit by non cooperative agents is equal to the present value of the stream of marginal adverse effect on his own production. The shadow cost of pollution accumulation perceived by non cooperative agents is lower than the social one :  $|\lambda_\infty^N| < |\lambda^*|$ . Consequently, the optimal steady state is not attained. However, agents operate a partial internalisation of the externality, because it directly affects their benefit function.

### Feedback case

Following Xepapadeas [14] [15] we assume that the conjuncture function of each agent about the emission decisions of the others is of the following form :  $u_j = \tilde{u}_j + \beta X$  where  $\beta < 0$ . This is a reasonable understanding of the situation faced by farmers in irrigation salinity prone areas: the higher the watertable, the less they irrigate. Such a formulation supposes that all agents react to the level of the stock in a similar way.

The current value Hamiltonian for non cooperative agents is:

$$H^{Nf}(u_i, X, \mu^N) = F(u_i, X) + \mu^N [u_i + \sum_{j \neq i} (\tilde{u}_j + \beta X) - \delta X]$$

The necessary conditions are unchanged, except for the dynamics of the shadow cost of pollution :

$$\dot{\mu}^N = [(r + \delta) - \beta(n - 1)]\mu^N - \frac{\partial F}{\partial X} \quad (18)$$

#### *Steady state*

The steady state for non cooperative agents is characterized by the following equations:

$$u_\infty^{Nf} = \frac{\delta}{n} X_\infty^{Nf} \quad (19)$$

where  $X_\infty^{Nf}$  follows the relation:

$$-\frac{\partial F}{\partial u_i}(u_\infty^{Nf}, X_\infty^{Nf}) = \mu_\infty^{Nf} = \frac{1}{[(r + \delta) - \beta(n - 1)]} \frac{\partial F}{\partial X}(u_\infty^{Nf}, X_\infty^{Nf}) \quad (20)$$

Using the quadratic expression of  $F$ , we obtain:

$$X_\infty^{Nf} = \frac{nb[(r + \delta) - \beta(n - 1)]}{c\delta[(r + \delta) - \beta(n - 1)] + nd} \quad (21)$$

$$\mu_\infty^{Nf} = \frac{-dX_\infty^*}{(r + \delta) - \beta(n - 1)} \quad (22)$$

Consequently:  $|\mu_N^\infty| < |\lambda_N^\infty| < |\lambda^*|$ . This reflects the strategic interaction between agents following feedback strategies. Even if they partially internalize the externality, they tend to over-emit because of their expectations that the other agents will respond to a rise in the stock by emitting less. The shadow cost they perceive is lower than the open-loop one, and *a fortiori* the social one.

In the following sections, we study different corrective taxes that induce non cooperative agents to follow the optimal emission path. According to the strategies followed by the agents, the taxation schemes might be different in order to take account of the strategic interaction between agents that appears in the feedback formulation.

### 3 Dynamic input tax

We develop an input tax inspired by the corrective tax in the context of polluting oligopoly analysed by Benchekroun and van Long [1]. As we consider non-point source pollution, individual emissions cannot be a base for taxation. However, input-based instruments have been proposed to circumvent informational constraints inherent to non-point source problems. By reference to [1], we propose a linear Markov input tax rule, which is linear in inputs and dependant on the current level of pollution stock only :  $T_i(u_i, X) = \sigma(X)u_i$ . As each agent knows that his emissions contribute to the accumulation of pollution, by way of this formulation he realizes that his emissions will affect the future tax rate.

Agents faced to this type of tax maximize their benefits in the following way:  $\max_{u_i} V_i = \int_0^\infty [F(u_i, X) - \sigma(X).u_i]e^{-rt}.dt$  subject to (2) and (3).

#### Open loop case

The current value Hamiltonian is :

$$H^{To}(u_i, X, \lambda^T) = F(u_i, X) - \sigma(X).u_i + \lambda^T(u_i + \sum_{j \neq i} u_j - \delta X)$$

The necessary conditions for this problem are:

$$\lambda^T = -\frac{\partial F}{\partial u_i} + \sigma(X) \quad (23)$$

$$\dot{\lambda}^T = (r + \delta)\lambda^T - \frac{\partial F}{\partial X} + \sigma'(X)u_i \quad (24)$$

along with (2) and the transversality condition :  $\lim_{t \rightarrow \infty} e^{-rt}\lambda^T(t)X(t) = 0$

In order to identify the optimal input tax, we compare the first-order conditions for cooperative agents to the ones we have just obtained. From (3) and (23), we get:

$$\lambda^T = \lambda^* + \sigma(X) \quad (25)$$

Deriving this expression:

$$\dot{\lambda}^T = \dot{\lambda}^* + \sigma'(X)\dot{X} \quad (26)$$

Combining (4) and (26) we obtain:

$$\sigma'(X)[\delta X - (n-1)u_i] + \sigma(X)(r + \delta) + (n-1)\frac{\partial F}{\partial X} = 0 \quad (27)$$

#### *Quadratic case*

In order to resolve this first order linear differential equation in  $\sigma(X)$ , we have recourse to the quadratic formulation of the problem. We suppose that the solution is of the following form:  $\sigma(X) = AX + B$ . We replace  $\sigma(X)$  by  $AX + B$  and  $\sigma'(X)$  by  $A$  in (27). Since the differential equation must hold for all  $X > 0$ , we collect the terms in  $X$  and equal their sum to zero. This gives  $A$ . We use this to express  $B$  from the rest of the terms that do not depend on  $X$ .

We find that the optimal input tax is :  $\sigma(X) = A^o X + B^o$  with

$$A^o = \frac{(n-1)d}{r + \rho\frac{1-n}{n} + \delta\frac{n+1}{n}} \geq 0 \quad (28)$$

$$B^o = -\frac{\rho(n-1)X_\infty^*}{n(r + \delta)} A^o \geq 0 \quad (29)$$

The taxation framework we obtain consists of two parts : a component independent of the stock,  $B^o$ , and a stock-induced component,  $A^o X$ . Both  $A^o$  and  $B^o$  are positive when  $n > 1$  and equal to 0 when  $n = 1$ . Thus, this

scheme consists only of a tax <sup>4</sup>. Such a tax tells the polluters that, as their emissions affect the stock, the more they use inputs now, the higher their tax liability will be in the future. Even if their current input use corresponds to optimality, the level of the tax is conditioned upon the current state of the stock, affected by their past actions.

### Feedback case

The resolution mechanism is the same as for open-loop strategies. The optimal input tax is of similar form as in the open loop case, but with different parameter values. Using the quadratic formulation of the benefit function, the parameters of the optimal tax are :

$$A^f = \frac{(n-1)d}{r + \rho \frac{1-n}{n} + \delta \frac{n+1}{n} - \beta(n-1)} \geq 0 \quad (30)$$

$$B^f = -\frac{\rho(n-1)X^*A^f + \lambda^*\beta(n-1)}{n(r+\delta)} \geq 0 \quad (31)$$

Again, the incentive framework consists only of a tax. We notice that  $B^o < B^f$  and  $A^o > A^f$ . This means that at low levels of the pollution stock, feedback strategies are punished more heavily than open-loop ones, by way of the higher stock-independent term. On the contrary, high levels of the stock are more taxed for open-loop strategies, because the stock-induced parameter is higher. Intuitively, the idea is that in the feedback formulation, agents are incited to pollute more now. Indeed, as seen before, they ought to pollute more now as they expect others to lower their emissions as a response to the increase in the stock. This is due to the particular formulation of the feedback strategy we use; indeed emissions are considered as strategic substitutes [14]. A high fixed tax parameter could be a way to induce agents following feedback strategies to lower their emissions from the beginning of the game.

A comparison of the incentive frameworks for open-loop and feedback strategies shows that the informational issues that the regulator has to deal with are not reduced to the standard informational asymmetries that characterize non-point source problems; she also needs to know which strategy is followed by the agents to design the optimal taxation scheme.

---

<sup>4</sup>Benckroun and Van Long studied the case of an oligopoly; a surprising result of their study is that it might be optimal to subsidize polluters even if their laissez faire output is higher than the social one; indeed oligopoly tend to underemit due to market power. In our study, market power are not an issue.

Our model does not allow addressing informational issues related to input taxes. When considering more realistic hypotheses, the informational burden gets higher, encompassing transaction costs. First, we consider homogeneous agents, which induces our input tax to be the same for every agents. With more realistic hypotheses allowing heterogeneity between agents, the tax would be farm-specific, thus more complex to design and implement. Second, even if we only consider a unique input, irrigation water, the observability of individual decisions is subject to high costs, even when agents are organized in irrigation districts. That is why ambient pollution has been proposed as a basis for regulation, as it potentially reduces the informational burden. A measurement of pollution at some receptor point should be enough for the regulator to design an ambient tax framework. Others costs arise, such as the social cost of imposing on individuals a penalty based on the actions of numerous agents. We focus in our analysis on the design of an optimal ambient taxation scheme, without consideration for such potential limitations.

## 4 Dynamic ambient tax

The taxation scheme we study is related to the one developed by Xepapadeas [14], but it allows taking account of the endogenous nature of the externality. We design the tax so that agents are induced to follow the optimal emission path, and we consider that for each  $t$ , the target is the optimal steady state stock. This corresponds to a situation where a target has been set by a regulatory body.

Suppose an ambient tax of the form:  $\tau(z)$  with  $z = X(t) - X_\infty^*$  such that  $\tau \begin{matrix} \geq \\ < \end{matrix} 0$  as  $z \begin{matrix} \geq \\ < \end{matrix} 0$ .

Agents' program is :  $\max_{u_i} V_i = \int_0^\infty [F(u_i, X) - \tau(z)]e^{-rt}.dt$  subject to (2) and (3).

### Open loop case

The current value Hamiltonian is:

$$H^{Xo}(u_i, X, \lambda^X) = F(u_i, X) - \tau(z) + \lambda^X [u_i + \sum_{j \neq i} u_j - \delta X]$$

The necessary conditions are:

$$\lambda^X = -\frac{\partial F}{\partial u_i} = \lambda^* \tag{32}$$

$$\dot{\lambda}^X = (r + \delta)\lambda^X - \frac{\partial F}{\partial X} + \tau'(z) \quad (33)$$

along with (2) and the transversality condition :  $\lim_{t \rightarrow \infty} e^{-rt} = \lambda^X(t)X(t) = 0$

#### *Quadratic case*

In order to define the optimal ambient tax, we develop the preceding equation replacing  $X(t)$  and  $u_i(t)$  by  $X^*(t)$  and  $u_i^*(t)$  as defined in section 2. Indeed, we want non cooperative agents to follow the optimal path. In the quadratic case, this leads to the following expression:

$$\dot{\lambda}^X = (r + \delta)\lambda^* + dX^*(t) + \tau'(z) = cu_i^*(t) \quad (34)$$

Referring to (10) and (11) we have:

$$\tau'(z) = -[(r + \delta)\lambda^* - dX_\infty^*] + z\left[\frac{c}{n}\rho(\delta + \rho) - d\right] \quad (35)$$

After integration, remembering that  $\tau(0) = 0$ , this gives the following result:

$$\tau(z) = z[dX_\infty^* - (r + \delta)\lambda^*] + \frac{z^2}{2}\left[\frac{c}{n}\rho(\delta + \rho) - d\right] \quad (36)$$

which can be written as:

$$\tau(z) = -z\lambda^*(r + \delta) + dz(X_\infty^* - \frac{z}{2}) + \frac{z^2}{2}\left[\frac{c}{n}\rho(\delta + \rho)\right] \quad (37)$$

The first term is analogue to the efficient tax developed by Xepapadeas, which induces agents to attain the optimal steady state.

The second term bears the endogenous externality effect. It reflects the fact that individual agents already take into account a part of the effect of stock accumulation in their private maximization program, they partially internalise the external effect. Indeed, this term is negative and consequently lowers the tax burden for agents when the externality is endogenous.

The third one induces to follow the optimal path.

With such a taxation scheme, once a deviation is measured, every agent pays the same amount. If past emissions have caused the stock to be too high, firms will continue to pay even if their current emissions are optimal.

#### **Feedback case**

The current value Hamiltonian becomes:

$$H^{Xf}(u_i, X, \mu^X) = F(u_i, X) - \tau(z) + \mu^X[u_i + \sum_{j \neq i} (\tilde{u}_j + \beta X) - \delta X]$$

Using the same method, we obtain the optimal ambient tax for feedback agents:

$$\tau(z) = -z\lambda^*[(r + \delta) - (n - 1)\beta] + dz(X_\infty^* - \frac{z}{2}) + \frac{z^2}{2}[\frac{c}{n}\rho(\delta + \rho)] \quad (38)$$

The strategic interaction effect appears through the term in  $\beta$ . Indeed, the resulting tax is higher than the one for the open-loop case,  $\beta$  being negative, as all other terms remain the same. As agents tend to over-emit in reason of the strategic interaction that appears when they follow feedback strategies, the taxing instrument has to be more stringent for the feedback loop formulation.

## 5 Concluding remarks

We have shown that it is possible to design tax rules that guide agents linked through an endogenous externality to achieve the socially optimal time path of pollution accumulation and input use. We have also shown that, according to the informational structure of the agents, the taxation framework differs; feedback strategies are taxed more heavily than open-loop ones.

However, our model has numerous limitations. First, firms are supposed to be homogenous, which renders the taxation frameworks quite simple; in order to attain the first best outcome, if agents were heterogenous then the tax rate would have to be farm-specific. Second, the non-point source nature of the problem is simplified; the recourse to stochastic functions, in particular to link inputs to emissions, would be a more realistic assumption. Third, we do not treat of a known limitation of ambient taxes, which is the generation of large transfers. We restrict our analysis to non-budget balancing incentives; hence the absence of rewards or penalties at each instant in order to equal the total amount paid by dischargers to the social cost of deviation from the optimal path.

Our aim is to extend this analysis by considering a more realistic model of waterlogging, by making the discharge rate a function of the level of accumulation and taking account of uncertainties that contribute to the non-point source nature of waterlogging.

## References

- [1] Benchekroun, H. and N. van Long (1998). "Efficiency inducing taxation for polluting oligopolists." *Journal of Public Economics* 70(325-342).
- [2] Braden, J. B. and K. Segerson (1991). "Information problems in the design of nonpoint source pollution policy". Paper presented to the Workshop on The Management of Nonpoint-source Pollution, Lexington, KY.
- [3] Cochard, F. Willinger, M. and Xepapadeas, A. (2003). "Efficiency of Nonpoint Source Pollution Instruments : An Experimental Study". Paper presented to the 20ièmes Journées de Microéconomie Appliquée, Montpellier 5-6 june 2003.
- [4] Greiner, R. and O. Cacho (2001). "On the efficient use of a catchment's land and water resources: dryland salinization in Australia." *Ecological Economics* 38: 441-458.
- [5] Griffin, R. C. and D. W. Bromley (1982). "Agricultural Runoff as a Nonpoint Externality: A Theoretical Development." *American Journal of Agricultural Economics* 64: 547-542.
- [6] Holmstrom, B. (1982). "Moral Hazard in teams." *The Bell Journal of Economics* 13(324-340).
- [7] Horan, R. D. et al. (1998). "Ambient Taxes When Polluters Have Multiple Choices." *Journal of Environmental Economics and Management* 36: 186-199.
- [8] Karp, L. (2004). "Nonpoint Source Pollution Taxes and Excessive Tax Burden". Working Paper No. 835, revised 2004. Forthcoming, *Environmental and Resource Economics*.
- [9] Karp, L. and J. Livernois (1992). "On efficiency-inducing taxation for a nonrenewable resource monopolist." *Journal of Public Economics* 49: 219-239.
- [10] Mäler, K. G. and A. de Zeeuw (1998). "The acid rain differential game." *Environmental and Resource Economics* 12: 167-184.
- [11] Nordhaus, W. D. and Z. Yang (1996). "A regional dynamic general-equilibrium model of alternative climate-change strategies." *American Economic Review* 86(4): 741-65.
- [12] Segerson, K. (1988). "Uncertainty and Incentives for Non-Point Source Pollution." *Journal of Environmental Economics and Management* 15: 87-98.

- [13] Shortle, J. S. and R. D. Horan (2001). "The Economics of Nonpoint Pollution Control." *Journal of Economic Surveys* 15(3): 255-289.
- [14] Xepapadeas, A. (1992). "Environmental Policy Design and Dynamic Nonpoint Source Pollution." *Journal of Environmental Economics and Management* 23: 22-39.
- [15] Xepapadeas, A. (1997). "Advanced principles in environmental policy." Edward Elgar Publishing, Cheltenham, 321p.
- [16] Yang, Z. (2001). "Time preference, Stock externalities and Strategic Reactions- Policy implications in Climate Change." *Environmental and Resource Economics* 18(233-250).