Preliminary version

Energy Use, Endogenous Technical Change and Economic Growth

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First version: November 2003; this version: March 6th, 2004.

This is a preliminary version. New versions will be posted on our websites (center.uvt.nl/staff/smulders)

Abstract

A model of endogenous growth and non-renewable resource extraction is presented. Resource owners endogenously determine the extraction path and firms endogenously determine the rate and direction of technological change. We explore under what conditions the short-run dynamics of the model can replicate some important trends of last decades' OECD experience. These are, in particular, an increase in per capita energy supply, a decrease in the cost share of energy in GDP, a decrease in energy cost relative to labor cost, and reductions in energy use per unit of GDP. We also study the long-run properties of the model to examine whether current trends are sustainable.

Keywords: non-renewable resources, energy, economic growth, innovation

JEL codes: O41, Q43

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1. Introduction

All developed countries have been dependent on fossil fuels for the past decades. For most sectors of economic activity it is very costly to replace fossil fuels by alternatives sources of energy. Nevertheless, this oil dependence is no direct problem from the point of view of producers. Despite low substitution possibilities, the share of energy in total cost has declined secularly. Although alternative fuels have become cheaper over time, the use of conventional energy has become more attractive at an even faster pace. A possible explanation is that technological change mainly benefits the productivity of conventional fuels, which reduces the need to exploit alternative energy sources. An important indicator of these technological developments is that the productivity of energy, as measured by value added per unit of energy, has steadily increased. Also the supply of conventional energy has not been much of a problem, since supply (both in absolute and in per capita terms) has increased steadily. Yet, the supply of fossil fuels is finite, because it is a non-renewable resource. Ultimately supply must decline. The question is how sustainable current trends are, and how economic growth and innovation is likely to be affected if supply is going to decrease.

Our aims in this paper are threefold. First, we would like to build a dynamic general equilibrium model of growth and energy use that incorporates rational optimizing behavior of individual market participants with respect to extraction of the non-renewable stock of energy resources as well as with respect to investment in new technologies. That is, we want the model to be firmly rooted in dynamic resource theory and in growth and innovation theory. Second, we require the short-run dynamics to be consistent with stylized facts on energy use and growth. That is, we perform a rough calibration check. Third, we use the model to infer what trends are likely to be reversed in the long run, not to make detailed forecasts, but rather to examine the determinants behind the current trends and to study whether these forces are likely to produce the same trends in future. We thus need a small tractable model, rather than a big simulation model, to clearly sort out the different determinants.

The first main building block of our model is the supply of energy, thought of as a non-renewable resource with a fixed initial endowment. The optimal extraction of non-renewable resources has been extensively studied in the economic literature. Most theoretical papers about non-renewable resources predict a declining time pattern for resource extraction (see, for example, Dasgupta and Heal (1974, 1979)), which seems to run against the related empirical evidence. This result arises because the resource stock is finite and agents discount utiltiy at a positive rate, which makes resource owners impatient so that they prefer profits from extracting the resource today over profits from extracting in the future. A notable exception is given by Tahvonen and Salo (2001), who present a model in which non-renewable resource extraction follows an inverted U-shaped time pattern. These authors assume that energy can be provided by both a renewable and a non-renewable resource and abstract from R&D. The two resources have different exploitation (extraction) costs, which change with depletion and production levels. Over time, depletion and decreasing returns change the relative attractiveness of the resources so that the inverted U shape arises. We complement this analysis by focusing on the link between substitution between a single resource and other inputs on the one hand and technological change on the other hand. In our model, the direction of technological change determines whether it is attractive to speed up or slow down depletion over time.

The second main building block of our model is the supply of technology. Three types of technological change are distinguished: labor-saving technological change, energy-saving technological change and declines in extraction costs. We start from the recent formalization of directed technological change by Acemoglu (2002) to model the first two types. We adapt his model, first, to make it applicable to energy use and, second, to allow for inhouse R&D activities. The main value added of the second adaptation is, apart from introducing realism in the model, that it avoids the unrealistic bang-bang dynamics of Acemoglu's model. We are not aware of any other theoretical growth model that links directed technological change and costly non-renewable resource extraction. For example, Grimaud and Rouge (2002) and Schou (2001) assume only one type of technological change and unitary elasticities of substitution. Smulders and de Nooij (2003) allow for directed technological change, but they assume an exogenous supply of energy.

The main stylised facts we use to calibrate our model are summarized in Jones (2002). He presents four stylised facts regarding growth and energy in the US for 1950-1998 (based on EIA 1999). First, energy efficiency (GDP per unit of energy input) has increased at an annual rate of 1.4 per cent on average. Second, per capita energy use has increased at an average annual rate of about 1 percent. Third, the share of energy cost in GDP has declined at an average annual rate of about 1 percent.

Fourth, energy prices per unit of labor cost have declined. Ideally we would like to have detailed knowledge about the rate and direction of technological change. However, since technology cannot be observed directly, stylized facts cannot be easily identified for technology trends. Although there is some important econometric work that estimates the rate and direction of technological change (e.g. by Jorgenson), the number of studies is too small and their results are too mixed to give a clear picture. Instead of relying on data for technology, we will use technology as the unobserved variable that nevertheless is an important driving force. We will identify the path of technological change that generates the stylised facts and then look for the parameters that can generate this once we endogenize technological change.

Our analysis is divided in three stages to clearly disentangle the effects of (i) the presence of technical change per se, (ii) the endogeneity of technological change in the use of energy, and (iii) the endogeneity of technical change in the extraction of energy supplies. In section 2, we present a dynamic general equilibrium model of energy production and use, but we take technology as exogenous. In section 3 we investigate how the model can replicate the stylised facts described above and what is the implied path of technology. In section 4 we introduce induced technological change by modelling how firms choose their innovation projects. In section 5 we endogenize extraction costs by assuming spillovers from R&D to mining technology. We show that the results of section 4 still go through. Section 6 concludes.

2. The model with exogenous technological change

A closed economy produces a homogeneous consumption good, using labor and energy services. In turn, labor (energy) services are produced using labor (energy) and a set of specific intermediates.[1] The supply of labor, denoted by L, is assumed to be exogenous. The supply of energy, denoted by R, results from the endogenous extraction of a non-renewable resource stock. To fix ideas, we will always refer to R as energy, but a broader interpretation (e.g. materials for production) is possible.

[1] Intermediates can be understood as "capital". Nevertheless, for simplicity, we model these inputs as a flow and so, to avoid confusion with the traditional stock of capital, we prefer the denomination "intermediates".

Following Acemoglu (1998), we assume that the productivity of the primary inputs (labor and energy) mainly depends on the quantity and quality of factor-complementary intermediate goods. While final goods producers optimally choose the quantity, the quality is a state variable that increases as a result of R&D effort performed by monopolist firms. In this section we disregard the source of technical change and take the time pattern for the quality of intermediates as exogenously given.

Final goods production

There is a final consumption good (Y) that is produced using labor services (Y_L) and energy services (Y_R) according to the following CES function with elasticity of substitution equal to σ, which is assumed to be smaller than one.

$$Y = A \cdot \left(Y_L^{(\sigma-1)/\sigma} + Y_R^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)} \tag{1}$$

Labor (energy) services are derived from combining raw labor (energy) inputs and a range of specialized intermediate inputs m, each of which is available at a certain quality q. In particular, services of type $i = L, R$ are produced according to the following Cobb-Douglas/Romer (1990) production function:

$$Y_i = S_i^{\beta} \int_0^1 q_{ik} m_{ik}^{1-\beta} dk \tag{2}$$

where S_i is the use of raw input i (i.e. S_L=L and S_R=R), m_{ik} is the use of intermediates of variant k in the production of type i services, and q_{ik} is the associated quality level. The number of different intermediates in each sector is normalized to unity. Note that intermediates m are input-specific.

Final-goods producers take prices as given. Their factor demand (for labor, energy and intermediates) is given by the first-order conditions:

$$p_Y \frac{\partial Y}{\partial Y_i} \frac{\partial Y_i}{\partial S_i} = w_i (1+\tau_i) \quad \Leftrightarrow \quad \frac{\beta \theta_i p_Y Y}{S_i} = w_i (1+\tau_i), \qquad i = R, L \tag{3}$$

$$p_Y \frac{\partial Y}{\partial Y_i} \frac{\partial Y_i}{\partial m_{ik}} = p_{mik} \quad \Leftrightarrow \quad \frac{\theta_i p_Y Y (1-\beta) q_{ik} m_{ik}^{-\beta}}{\int_0^1 q_{ik} m_{ik}^{1-\beta} dk} = p_{mik}, \qquad i = R, L \tag{4}$$

where w_i is the factor price for raw inputs, τ_i is a tax rate on raw input i, p_{mik} is the price of intermediate good m_{ik}, and $\theta_i \equiv (\partial Y / \partial Y_i) Y_i / Y$ is the production elasticity of sector i's services in final gross output Y. From (3), we see that the share of gross revenue $p_Y Y$ that is devoted to remunerate raw inputs (including taxes) equals $\beta(\theta_L + \theta_R)$, which equals β because of constant returns to scale in production; the remaining share $(1 - \beta)$ is spent on intermediates, as can be derived from (4). Hence total factor payments equals $\beta p_Y Y$ and θ_i is the share of factor i in factor income. In the sequel we will refer to θ_R as the energy share.

Intermediate goods production and price setting

The market for intermediates is characterized by monopolistic competition (see Dixit and Stiglitz 1977). Each producer supplies a unique variety and sets a monopoly price. The cost of producing one unit of m_{ik} at quality q_{ik} is q_{ik} units of the final good. Equation (4) reveals that the elasticity of demand for each intermediate good equals $1/\beta$. This fact implies that monopoly prices for intermediates are set as a mark-up over unit costs (qp_Y). As usual, the mark-up is negatively related to the elasticity of demand $1/\beta$:

$$p_{mik} = q_{ik} p_Y /(1-\beta) \qquad i = R, L \qquad (5)$$

Substituting the price in the demand function (4), we find that all intermediate goods producers within the same sector i produce the same level of output m_i:

$$m_{ik} = m_i \equiv \theta_i Y (1-\beta)^2 / Q_i \qquad i = R, L \qquad (6)$$

where Q_L and Q_R denote the average quality of labor-related and energy-related inputs, defined as

$$Q_i \equiv \int_0^1 q_{ik} dk \qquad i = R, L \qquad (7)$$

Static goods market equilibrium

The static goods market equilibrium can be characterized in terms of primary inputs (R and L) and the state of technology (Q_L and Q_R). Substituting equilibrium quantities of intermediate inputs from (6) into the production and demand functions, we first solve for relative factor shares, relative supply of intermediates, and relative input prices:[2]

$$\theta^B = \left(S^B Q^B\right)^{-(1-\nu)/\nu} \tag{8}$$

$$m^B = \left(Q^B\right)^{-1/\nu} \left(S^B\right)^{-(1-\nu)/\nu} \tag{9}$$

$$w^B = \frac{1+\tau_L}{1+\tau_R}\left(Q^B\right)^{-(1-\nu)/\nu} \left(S^B\right)^{-1/\nu} \tag{10}$$

where $\nu = 1-\beta(1-\sigma)$. The superscript B, which stands for "bias", denotes ratios of energy to labor variables, e.g. $\theta^B \equiv \theta_R/\theta_L$, $S^B \equiv R/L$, $w^B \equiv w_R/w_L$ and so on. Since $\theta_L + \theta_R = 1$ and $\theta^B = \theta_R/\theta_L$, we can solve (8) for each of the factor shares:

$$\theta_R = 1-\theta_L = \left[(Q^B S^B)^{(1-\nu)/\nu} + 1\right]^{-1} \tag{14}$$

From (10) and (14) we can see that poor substitution ($\sigma < 1$ so that $\nu < 1$) means that an increase in the bias of technology Q^B implies a fall in energy prices and in the energy share. An increase in Q^B therefore has the interpretation of energy-saving (or labor-biased) technical change.

We next solve for aggregate variables. We express aggregate output as a function of technology and factor inputs by taking into account that the equilibrium level of intermediates use (m_i) depends on output, technology and factor inputs according to (6). In particular, we combine (1), (2), (6), and (14) to arrive at the following expression:[3]

$$Y = \left((Q_L L)^{(\nu-1)/\nu} + (Q_R R)^{(\nu-1)/\nu}\right)^{\nu/(\nu-1)} \tag{11}$$

This equation shows that, at equilibrium levels of intermediates use, Q_i act as factor augmentation levels. The elasticity of substitution between *effective* labor input ($Q_L L$) and *effective* energy input ($Q_R R$) is $\nu = (1-\beta)+\beta\sigma$. This elasticity differs from σ, since any change in relative factor use not only has direct effects on the

[2] Differentiating the production function (1) and using the definitions $\theta_i = (\partial Y/\partial Y_i)Y_i/Y$ and $Y^B = Y_R/Y_L$, we find $\theta^B = \left(Y^B\right)^{\frac{\sigma-1}{\sigma}}$; using (2) and (6), we get (8). From (6) and (3) we get (9) and (10) after using (8) to eliminate θ^B.

[3] We have chosen units of Y in (1) such that $A = (1-\beta)^{-2(1-\beta)}$, so that the scale constant in (11) becomes unity.

relative productivity of resources (as measured by σ), but also indirect effects through a change in the relative use of intermediates (of which the share is $1 - \beta$).

From (11), we can directly write output per capita (y) and its growth rate (g) as

$$y \equiv Y / L = Q_L (1-\theta_R)^{\nu/(1-\nu)} \tag{12}$$

$$g \equiv \hat{Y} - \hat{L} = \hat{Q}_L + \theta_R (\hat{S}^B + \hat{Q}^B) \tag{13}$$

where hats denote growth rates. Equation (13) shows three sources of growth: growing per capita energy inputs, changes in the level of (labor-related) technology, and changes in the bias of technical change.

The production of Y serves as intermediates or as final consumption goods (C_Y), so that goods market equilibrium requires $Y = C_Y + \sum_i \int_0^1 q_{ik} m_{ik} dk$. After substituting (6) and (7), we find that net output C_Y is a constant fraction of gross output Y:[4]

$$C_Y = [1-(1-\beta)^2]Y \tag{11'}$$

Consumer behavior

The representative consumer maximizes intertemporal utility, specified as $\int_0^\infty [\alpha \ln c_H + (1-\alpha)\ln c_Y] \exp(-\rho t) dt$, subject to his or her budget constraint,[5] where ρ is the utility discount rate, and c_Y and c_H is per capita consumption of the Y-good and another consumption good, which we will discuss below, respectively. The Cobb-Douglas utility specification implies that a fixed fraction of income, $1-\alpha$, is spent on C_Y-goods and α on C_H-goods, so that $(c_H p_H)/(c_Y p_Y) = \alpha/(1-\alpha)$, which means that $c_H p_H$ and $c_Y p_Y$ grow at the same rate. Furthermore, the logarithmic form of the intertemporal utility function implies that the consumer chooses a consumption path

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[4] Note that $p_Y C_Y = [1-(1-\beta)^2] p_Y Y = (2-\beta)\beta p_Y Y$ is the value of net output, while $w_R R + w_L L = \beta p_Y Y$ net factor income (net of intermediates but including taxes). The excess of net production over net factor payments, $(1-\beta)\beta p_Y Y$, is the monopoly rent accruing to intermediate goods suppliers.

[5] We assume that all tax revenue that the government collects are rebated in a lump-sum fashion to the households.

along which total spending grows with the difference between the nominal interest rate r and the utility discount rate ρ:

$$(1-\alpha)(\hat{c}_Y + \hat{p}_Y) + \alpha(\hat{c}_H + \hat{p}_H) = r - \rho \qquad (15)$$

Using the fact that $c_H p_H$ and $c_Y p_Y$ grow at the same rate, using (11') and assuming that population and labor supply grow at the same rate $\hat{L}$ so that $\hat{c}_Y = \hat{C}_Y - \hat{L} = \hat{Y} - \hat{L}$, we may write (15) as:

$$\hat{p}_Y + \hat{Y} - \hat{L} = r - \rho \qquad (18)$$

Natural resource extraction and energy use

The energy inpur R is produced from a non-renewable resource stock E. To produce one uit of energy, 1+μ units of the resource stock have to be extracted. Hence, the resource stock changes over time according to the following differential equation

$$\dot{E} = -(1+\mu)R \qquad (16)$$

where a dot over a variable denotes derivation with respect to time. We can interpret μ as the unit resource cost to produce energy. Alternatively, it is a unit extraction or mining cost. Of each unit extracted from the resource stock, a fraction $\mu/(1+\mu)$ is lost in the mining process and only $1/(1+\mu)$ arrives at the market. This specification is similar to the so-called *iceberg costs* in trade literature (introduced by Samuelson, 1954).[6] We focus on the impact of technical change on the efficiency of the extraction and energy production technology, by allowing unit extraction costs μ to decrease over time. Evidence for declining extraction costs is documented by, for example, Chermak and Patrick (1995) and Fagan (1997). The decline in extraction costs can be the result of some learning by doing process (see, for example, Tahvonen and Salo 2001). As a first step, we take the rate of decrease in extraction costs as given; in

[6] In the partial equilibrium literature, it is common to assume that extraction involves a monetary cost (to pay for the inputs in the mining process) rather than a loss of marketable resources and to assume that extraction costs decrease with the remaining stock of resource (stock effects), and decrease with some technology indicator (technological change). For the sake of analytical tractability, we model extraction costs in terms of resources energy rather than in terms of output. Essentially, this assumption separates the extraction process from the goods market clearing conditions (if extraction required inputs from the Y-sector, (11') would become more complicated; if extraction required inputs from the H-sector, (27) would become more complicated). We abstract from stock effects to avoid that the resource stock variable enters the dynamics of the model as a third state variable.

section 5 we endogenize the cost reductions by allowing for technological spillovers from R&D activities in the economy.

The resource owners decide the extraction path in order to maximize their discounted profit, $\int_0^\infty R\, w_R e^{-rt} dt$, subject to (16), where r is the interest rate. The solution to this problem results in the modified Hotelling rule

$$r = \hat{w}_R - \frac{\dot{\mu}}{1+\mu} \tag{17}$$

Equation (17) is the arbitrage rule for which resource owners are indifferent between, on the one hand, extracting the resource today, selling it, and putting the net revenue in the bank at interest rate r, and, on the other hand, extracting and selling it later, thus benefiting from higher prices and lower mining costs but forgoing the interest payments. The faster mining costs μ decline, the more profitable it is to wait extracting, so the slower prices w_R have to rise or the lower the interest rate r has to be to make the resource owner indifferent. If technological change is fast enough, such that $0 < r < -\dot{\mu}/(1+\mu)$, the energy price w_R decreases because of the cost reduction effect. However, this situation cannot continue forever: whenever μ is non-increasing, $\dot{\mu}$ approaches zero in the long run (since μ is non-negative).[7] Then, as time goes to infinity, equation (17) collapses to the standard Hotelling rule, which implies that in the long run the energy price increases at a rate equal to the interest rate.

3. The role of technological change

In this section we take as given the changes in technology variables Q_L, Q_R, and μ. Our purpose is to examine whether the model can replicate some important stylized facts on growth and energy use, and what pattern of technological change is consistent with these stylised facts. In particular, we want the model to be able to generate over time a growing per capita energy supply, declining cost share of energy in GDP, declining energy cost relative to labor cost, and declining energy use per unit of GDP.

[7] To get some of the results in the paper, we need that $\dot{\mu}/(1+\mu)$ is high enough in the short run and low enough in the long run. A possible specification for μ to get this shape is $\mu = \mu_0 e^{-\gamma t}$, where μ_0 represents unit extraction cost at time zero and γ is the (constant) decay rate.

3.1. A necessary condition to match the stylised facts

We first summarize equilibrium in the energy and capital market. Using the time derivative of the demand for energy, equation (3), to eliminate energy prices, and using the Ramsey rule, equation (18), to eliminate the interest rate, we can write the modified Hotelling rule (17) as:

$$\underbrace{\widehat{R/L}}_{\equiv \hat{S}^B} = \hat{\theta}_R + \underbrace{\frac{-\dot{\tau}_R}{1+\tau_R}}_{\equiv \pi} + \underbrace{\frac{-\dot{\mu}}{1+\mu}}_{\equiv X} - \rho \tag{19}$$

where $X \equiv -\dot{\mu}/(1+\mu)$ and $\pi \equiv -\dot{\tau}_R/(1+\tau_R)$. Equation (19) represents simultaneous equilibrium in the capital market [the required rate of return as demanded by consumers, see (18), equals the realized rate of return on investment in resource stock, see (17)] and in the energy market [demand, see (3) meets supply, see (18)].

According to equation (19), the higher the rate of utility discount or the smaller mining cost reductions or resource tax reductions, the lower is the rate of growth of relative energy supply. The reason is that these three factors makes postponing extraction less profitable for resource owners. Therefore, the term $\rho - \pi - X$, which will frequently reappear in the sequel, could be labelled the effective impatience factor in the extraction of the resource. The more patient the market effectively is, the more extraction is postponed, which makes it more likely that energy supply grows instead of declines over time. We also see from (19) that the faster the energy share declines over time, the smaller the rate of growth of energy supply is. The reason is that the energy share θ_R is also the production elasticity of energy, so that a declining share indicates declining productivity of marginal increases in energy use. Such declines also make postponement of extraction (i.e. non-decreasing energy supply) less attractive, since it is profitable to use energy more intensively at the beginning of the planning horizon (when it is relatively more productive) and decrease its intensity as time goes on.

According to the stylised facts reported by Jones (2002), during the period 1950-1998 S^B was increasing on average and θ_R was decreasing on average. If $\hat{\theta}_R < 0$, $\hat{S}^B > 0$ requires $\pi + X > \rho$, according to equation (19). Hence, **a necessary condition for the model to be consistent with these two stylised facts is $\pi + X > \rho$.** This condition says that the energy tax or the extraction cost should decrease sufficiently fast to create a 'patience' incentive large enough to compensate the impatience incentive introduced by the discount rate. In other words, the effective impatience factor $\rho - \pi - X$ has to be negative.

Before we go on to find a sufficient - rather than necessary - condition to generate the stylised facts, we discuss how plausible and feasible it is that reductions in taxes and mining costs render the effective impatience factor negative. On the one hand, mining and energy production cost reductions can be important to explain a negative effective impatience factor only in the short run. The reason is that once extraction costs are small, further reductions are necessarily small: as noted above, whenever μ is non-increasing, $X = -\dot{\mu}/(1+\mu)$ approaches zero in the long run (since μ is non-negative).

On the other hand, steady tax reductions, such that π remains large, are feasible. Note that π is the rate of decline of the tax factor $1+\tau_R$. When we allow for subsidies (negative tax rates), this tax factor is constrained to be non-negative, $0 \leq 1+\tau_R$, and a constant rate of decline π is feasible.[8] However, in reality, energy taxes are not steadily declining. Although energy is traditionally highly subsidized and we have not seen many successful attempts to reduce these subsidies and start taxing energy (all of which would suggest $\pi < 0$), we cannot argue for the opposite (π large and positive) either, since there has not been a steady decrease in energy taxation or a steady rise in energy subsidies over time.

Hence, in the sequel we will assume the following. First, π is 'low' ($\pi < \rho$) and constant for simplicity. Second, the downward trend in extraction costs, X, is sufficiently large to render the effective impatience factor negative and to act as the driving force behind increases in per capita energy supply S^B initially. Necessarily X declines to zero over time.

3.2. Matching the stylised facts and the dynamics of the model

In our model the energy share θ_R is an endogenous variable. Hence, to determine under which circumstances the model can generate both stylized facts (S^B increasing

[8] Note that, as shown by Sinclair (1992), a constant tax on resource use (in our setting, $\pi = 0$) is irrelevant for the dynamic behavior of economic decisions. Sinclair claims that the tax rate on fossil fuels should be decreasing to cut harmful emissions.

and θ_R decreasing) as an endogenous result, we have to find out what drives changes in the energy share. Differentiating (14) with respect to time, we find an expression for $\hat{\theta}_R$, which we can combine with (19) to solve for the growth rates of energy supply and energy share:

$$\hat{S}^B = \left(\frac{\nu}{\nu + (1-\theta_R)(1-\nu)}\right)(X + \pi - \rho) - \left(\frac{(1-\theta_R)(1-\nu)}{\nu + (1-\theta_R)(1-\nu)}\right)\hat{Q}^B \tag{20}$$

$$\hat{\theta}_R = -\left(\frac{(1-\theta_R)(1-\nu)}{\nu + (1-\theta_R)(1-\nu)}\right)(\hat{Q}^B + X + \pi - \rho) \tag{21}$$

Equations (20) and (21) solve for the evolution of energy supply and energy share for a given pattern of technological change and tax rate changes. The technology variables that drive changes in energy supply and energy share in particular are, first, the bias of technological change ($\hat{Q}^B \equiv \hat{Q}_R - \hat{Q}_L$), second, the rate of change in extraction costs, $X \equiv -\dot{\mu}/(1+\mu)$, and, third, the rate of change in taxes $\pi \equiv -\dot{\tau}_R/(1+\tau_R)$. We assume that energy and labor services are poor substitutes ($\sigma <$ 1, which implies ν < 1) from now on; the opposite case can be investigated in a similar way, but seems less in line with empirical evidence (Neumayer, 2003). Then, an increase in Q^B, i.e. $\hat{Q}^B > 0$, implies that technological change is biased and energy-saving (since, ceteris paribus, it reduces the energy share).

If there is no bias in technological change ($\hat{Q}^B \equiv \hat{Q}_R - \hat{Q}_L = 0$), the model replicates the stylized facts of increasing energy supply and declining energy share provided the effective impatience factor is negative ($\pi + X > \rho$). Under this condition, tax reductions and extraction cost reductions make postponement of extraction attractive as explained above. This makes energy supply grow over time, so that it is relatively less and less scarce over time, which makes the energy share falls over time if energy is a poor substitute for labor.

A small bias in technological change (in either direction) in combination with a negative effective impatience factor still replicates the two stylized facts. The reason the bias cannot be allowed to be large, is that $\hat{S}^B$ and $\hat{\theta}_R$ both increase with $\hat{Q}^B$, while the stylized facts require that $\hat{S}^B$ and $\hat{\theta}_R$ are of opposite signs (positive and negative, respectively). The bias in technological change enters equations (20) and (21) because it affects the abundance of energy versus labor services for given raw factor supplies. In particular, when $\hat{Q}^B \equiv \hat{Q}_R - \hat{Q}_L > 0$, technological change makes a given supply of energy per unit of labor relatively more abundant in effective terms, which makes energy's marginal contribution fall over time (the production elasticity θ_R tends to fall over time if $\nu < 1$) and which makes postponement of extraction less attractive (S^B tends to fall over time).

We now check whether the dynamics of our model can replicate the stylized fact about the cost of energy, relative to labor cost. We obtain the following result after differentiating (10) and using (20):

$$\hat{w}^B = \frac{\dot{\tau}_L}{1+\tau_L} - \frac{1}{\nu + (1-\theta_R)(1-\nu)}\left[(1-\nu)\theta_R\left(\hat{Q}^B + \pi\right) + X - \rho\right] \tag{22}$$

Relative energy cost w^B turns out to be decreasing (as we see in the stylized facts) under the conditions we identified above, i.e. if $\hat{Q}^B$ is close enough to zero and $\rho - \pi - X < 0$. The economic interpretation of this condition is straightforward: as technology becomes more strongly energy-biased, energy becomes less scarce as compared to labor; since labor and energy are poor substitutes, the remuneration of energy to falls relative to that of labor. Comparing (22) to (21) we conclude that, if the condition for $\hat{\theta}_R < 0$ holds, it is also guaranteed that $\hat{w}^B < 0$ holds.

We now check whether the dynamics of our model can replicate the stylized fact about per capita output and energy productivity. From (20) and (13) we derive:

$$\hat{Y} - \hat{L} = \hat{Q}_L + \frac{\nu\theta_R}{\nu + (1-\theta_R)(1-\nu)}\left(\hat{Q}^B + \pi + X - \rho\right) \tag{23}$$

$$\hat{Y} - \hat{R} = \hat{Q}_L + \frac{\nu + (1-\theta_R)}{\nu + (1-\theta_R)(1-\nu)}\hat{Q}^B - \frac{\nu(1-\theta_R)}{\nu + (1-\theta_R)(1-\nu)}\left(\pi + X - \rho\right) \tag{23'}$$

The stylized facts report that both per capita output and energy productivity grow at a positive rate. The equations show that this is possible under the conditions identified above (small bias and negative effective impatience), provided labor augmenting technological change is large enough.

3.3. A long-run solution of the model and an assessment of sustainability

We now use equations (20)-(23’) to examine whether the current trends in energy supply, energy share, energy price, energy productivity and economic growth are sustainable. Note that in the previous subsection we have discussed the conditions for which the *dynamics* of the model generate the stylized facts. We now use the *steady state* of the model to draw conclusions about the long-run trends in the variables just mentioned. In the long run, X approaches zero and π is assumed to be a small constant, so the effective impatience factor becomes positive ($\rho - \pi - X > 0$).

Without energy-saving technological change ($\hat{Q}^B = 0$), the long-run trends of energy supply and the energy share are the opposite of the trends in the stylized facts: energy supply grows at a negative rate and the energy share grows steadily, approaching its limit value of unity in the long run. The latter implies that energy cost asymptotically absorbs all factor payments.

Energy-saving technological change is needed to prevent the energy share to reach its upper limit. If energy-saving technological change is sufficiently fast ($\hat{Q}^B > \rho - \pi$), we know from (21) that θ_R goes to 0 and θ_L goes to 1, so that (23) collapses to $g = \hat{Q}_L$ and, in the long run, only the growth of quality in the labor-intensive sector matters for growth. On the contrary, if $\hat{Q}^B < \rho - \pi - X$, we know from (21) that θ_R goes to 1 and θ_L goes to 0, so that (23) collapses to $g = \hat{Q}_R + \pi - \rho$ and, in the long run, only the growth of quality in the energy-intensive sector matters for growth. In the latter case, the condition for the economy to grow in the long run is $\hat{Q}_R > \rho - \pi > 0$, so that, the energy-complementary technological change has to be large enough to offset the decreasing trend coming from the exhaustion of the resource.

4. Endogenous technical change in production

We now model the economic mechanisms driving technological change by assuming that each intermediate goods producer improves the quality of his or her good by investing in in-house research and development activities. This allows us to investigate the interactions between energy use, the bias of technology and total innovation efforts in the economy.

4.1. Modeling endogenous technological change

The investment technology is:

$$\dot{q}_{ik} = [\xi_i Q_i D_i^{1-\omega_i}] D_{ik}^{\omega_i} \tag{24}$$

where D_{ik} represents the amount of resources spent on development by firm k in sector i. Apart from the scaling parameter ξ_i the productivity in development activities depends on two types of spillovers. First, an individual firm builds on the knowledge accumulated in the past by all firms in the sector (see Popp (2002) for evidence with respect to energy-related research). This knowledge stock is proxied by the current aggregate quality level Q_i. The firm takes it as given and neglects that its own current development efforts expand the knowledge stock on which future development builds. Thus, intertemporal spillovers arise, which play an important role in preventing the returns to innovation to fall over time. Since production costs rise with the quality level of the product, the return on subsequent innovation tends to fall. However, intertemporal spillovers reduce the cost of innovation, which boosts the rate of return. Under the present specification, both forces exactly offset each other in the long run and rates of return can be sustained.

Second, quality development efforts become more productive when other firms are more active. This instantaneous intrasectoral research spillover is captured by parameter $1 - \omega_i$. Whereas $1 - \omega_i$ reflects the returns to innovation that leak to other firms, its complement ω_i reflects the share of returns to innovation that accrue to the inventing firm. We therefore label ω_i as the appropriability parameter. A higher value implies that innovators can better appropriate the returns to R&D, which increases the marginal incentives to innovate.

Firms choose innovation efforts D_{ik} in order to maximize the net present value of the firm. The attached optimal control problem results in the following no-arbitrage equation:[9]

$$r = \left(\frac{\beta\omega_i\xi_i}{1-\beta}\right)\frac{p_Y m_i Q_i}{w_D}\left(\frac{D_i}{D_{ik}}\right)^{1-\omega_i} + \hat{w}_D - \hat{Q}_i - (1-\omega_i)(\hat{D}_i - \hat{D}_{ik}) \equiv r_{ik} \tag{25}$$

[9] The firm maximizes the net present value of profits, which are given by $p_{ik} m_{ik} - p_{ik} m_{ik} p_Y - w_D D_{ik}$, subject to (4) and (24) and using m_{ik} and D_{ik} as controls, taking as given all prices, aggregate variables and quantities of other firms.

where w_D is the cost of development D, which can be understood as the salary of a researcher. Equation (25) states that the firm invests until the marginal returns from investment (r_{ik}) equal the cost of capital r. The first term on the right-hand side is the direct return from higher quality. Profits rise with quality in proportion to its sales m_i. The other terms equal the expected rate of change in the shadow price of quality improvements. Fast quality growth in the economy (captured by $\hat{Q}$) implies large spillovers and cheaper development in the future, which provides an incentive to postpone innovation and reduces the current rate of return. A higher future cost of development (captured by $\hat{w}_D$) has an opposite effect.

In equilibrium, all firms active in development should earn the same marginal return. Equation (25) shows that this requires $D_{ik} = D_i$, that is, all firms within a sector choose the same level of development efforts. Moreover, the marginal return across the sectors is equalized ($r_{Rk} = r_{Lk} = r$). After substituting (6) to eliminate $m_i Q_i$, we may write (25) as:

$$r - \hat{w}_D = \left(\frac{\beta(1-\beta) p_Y Y}{w_D} \right) \omega_L \xi_L \theta_L - \hat{Q}_L = \left(\frac{\beta(1-\beta) p_Y Y}{w_D} \right) \omega_R \xi_R \theta_R - \hat{Q}_R \qquad (26)$$

To determine the labour cost of research and development activities w_D, we turn to the allocation of skilled workers. Skilled workers can decide to work on producing good C_H (see section 2) or doing research. One unit of skilled labor produces one unit of C_H, which is sold under perfect competition. Therefore total supply of skilled labor, H, is divided over research, $D = D_L + D_R$, and production of C_H, that is, $C_H = H – D$. In equilibrium, skilled workers must be indifferent between working as researcher and producing the C_H -good. Hence, the price of the C_H -good equals the wage they earn in research, that is, $p_H = w_D$.

Consumers spend a fraction α and (1 – α) on Y-goods and H-goods, respectively. In equilibrium, the quantities supplied of consumption goods are $C_Y = [1-(1-\beta)^2]Y$ and $C_H = H - D$ respectively (see (11’)), while the prices are p_Y and w_D. Goods market equilibrium therefore implies the following expression for the wage of skilled workers:

$$\frac{1-\alpha}{\alpha} = \frac{p_Y Y[1-(1-\beta)^2]}{w_D (H-D)} \qquad (27)$$

4.2. The dynamics with endogenous technological change

We now analyse the general equilibrium dynamics of the model with endogenous technological change. To simplify, we assume $\beta(1-\beta)/[1-(1-\beta)^2]=\alpha/(1-\alpha)$.[10] We first derive an expression for the bias of technological change: using (27) to eliminate w_D from the second equality of (26), we find (note that $\hat{Q}^B=\hat{Q}_R-\hat{Q}_L$):

$$\hat{Q}^B=[\omega_L\xi_L+\omega_R\xi_R](H-D)(\theta_R-\overline{\theta}) \tag{28}$$

where

$$\overline{\theta}\equiv\frac{\omega_L\xi_L}{\omega_L\xi_L+\omega_R\xi_R}$$

From (28) we conclude that there is no bias in innovation if $\theta_R=\overline{\theta}$, which implies that the share of energy in GDP is high enough to offset relatively low research productivity and/or appropriability in energy-related technology. For higher energy shares, innovation becomes biased to energy.

To derive an expression for the rate of return, we substitute (27) and $\hat{Q}_i=\xi_i D_i$ into (26). The latter expression follows from $D_{ik}=D_i$ (all firms in a sector choose the same amount of research), (7), and (24). Together with the total research constraint $D_L+D_R=D$, we find:

$$r-\hat{w}_D=\zeta\left\{[\theta_R\omega_R+(1-\theta_R)\omega_L](H-D)-D\right\} \tag{26'}$$

Differentiating (27) with respect to time and using the result to eliminate $\hat{p}_Y+\hat{Y}$ in (18), we find:

$$r-\hat{w}_D=\rho-\hat{L}-\frac{D}{H-D}\hat{D} \tag{30}$$

While (30) represents households' supply of funds on the capital market, (26') represents firms' demand for funds. We can characterise equilibrium in the capital market by combining the two equations, which gives:

$$\hat{D}=\frac{H-D}{D}\left\{\rho-\hat{L}+\zeta D-\zeta[\omega_L-(\omega_L-\omega_R)\theta_R](H-D)\right\}. \tag{31}$$

[10] Since we are not interested in comparative statics on β or α, this assumption is innocuous. The assumption amounts to a rescaling of ξ, i.e. all ξ_i's that appear in the expressions that follow should be multiplied by $(1-\alpha)(1-\beta)/(\alpha(2-\beta))$ to find the expressions without the simplifying assumption.

This differential equation reveals how total research effort (D) changes over time in order to ensure that the rate of return that firms realize on their innovation efforts equal the rate of return that households require on their savings.

Using (28) in (21) we get

$$\hat{\theta}_R = -\frac{(1-\theta_R)(1-\nu)}{\nu+(1-\theta_R)(1-\nu)}\left\{(H-D)\left[\omega_L\xi_L+\omega_R\xi_R\right](\theta_R-\bar{\theta})+X+\pi-\rho\right\} \qquad (32)$$

Equations (31) and (32) constitute a dynamic system in two variables, D and θ_R. It is saddlepoint stable. From (31), we see that the $\hat{D}=0$ locus slopes up (down) if $\omega_L > \omega_R (\omega_L < \omega_R)$; from (32) we obtain that the $\hat{\theta}_R = 0$ locus has an asymptote at $\theta_R = \bar{\theta}$. Furthermore, because it depends on X, which is time-varying, the $\hat{\theta}_R = 0$ locus is shifting while time goes on, as illustrated in figure 1. Specifically, in the short run, we have $X+\pi>\rho$ and the slope of this locus is negative. In the long run, this inequality reverses and the slope of the locus becomes positive. This mechanism can result in an overshooting for θ_R and D, in such a way that both variables may reach a value above its steady state, so that, their evolution is non-monotonic (as illustrated in figure 2).

INSERT FIGURES 1 AND 2 ABOUT HERE

Assuming $lim\ X = 0$ and setting $\hat{D}=\hat{\theta}_R=0$ in (31)-(32), we can characterize the steady state:

$$D(\infty)=\frac{\Omega H-(\rho-\hat{L})/\zeta-(\omega_L-\omega_R)(\rho-\pi)\left[\omega_L\xi_L+\omega_R\xi_R\right]^{-1}}{1+\Omega} \qquad (33)$$

$$\theta_R(\infty)=\bar{\theta}+\frac{(\rho-\pi)(1+\Omega)}{\left[\omega_L\xi_L+\omega_R\xi_R\right](H+\rho/\zeta)+(\rho-\pi)(\omega_L-\omega_R)} \qquad (34)$$

where $0<\Omega\equiv\omega_L\omega_R(\xi_L+\xi_R)/(\omega_L\xi_L+\omega_R\xi_R)<1$ and the ∞-index is used to denote long-run values. As is common in endogenous growth models, the equilibrium amount of R&D increases if the average productivity of research (ζ) increases, if the total supply of potential researchers (H) increases, or if the discount rate falls. The last term in the numerator shows that a reduction in energy taxation ($\pi > 0$) contributes to reduce long run R&D when appropriability in energy-related innovation is relatively good ($\omega_R > \omega_L$). Intuitively, a larger value of π contributes to slow down the extraction of the resource, resulting in a higher steady state value of $\hat{S}^B$. This, in turn, goes with a lower value of $\hat{Q}^B$,[11] which means that research activity tends to shift to labor-related knowledge in the long run. The fact that the appropriability in this sector is lower provides a negative incentive to overall research effort.

The long-run bias in technological change and long-run growth of energy supply can be derived from (20)-(21) and the fact that θ_R is constant and $X = 0$ in the long run. This gives:

$$\hat{Q}^B(\infty) = \rho - \pi > 0 \tag{34'}$$

$$\hat{S}^B(\infty) = \pi - \rho < 0 \tag{34''}$$

Taking limits in (22) and using $\hat{Q}^B(\infty) = \rho - \pi$ we conclude that the long run value for the growth rate of the relative price of energy, turns out to be positive, so that the model predicts a U shape for the evolution of w^B.

We can write the growth rate of the energy efficiency as $\hat{Y} - \hat{R} \equiv g - \hat{S}^B$. Using (13), we conclude that energy efficiency is increasing (as it is observed in the stylized facts) if total R&D effort is large enough. Taking limits and using $\hat{Q}^B(\infty) = \rho - \pi$ we obtain that the long run value of $\hat{Y} - \hat{R}$ equals $\zeta D(\infty) + (\rho - \pi)\xi_R / (\xi_L + \xi_R)$, which is unambiguously positive.

Substituting (34') into (23) we obtain $g = \hat{Q}_L$, meaning that, in the long run, only labor-augmenting technical change matters for economic growth.

5. Endogenous extraction costs

In the previous sections we have kept the assumption that extraction costs (μ) are exogenously decreasing. In this section we propose a way to rationalize this trend of extraction costs as the result of technological spillovers stemming from R&D in the economy. Assume that the extraction costs are given by the following equation:

$$\mu = \mu_0 Q_R^{-\eta} Q_L^{-(1-\eta)} \tag{35}$$

[11] Recall that, in steady state, $\hat{R} - \hat{L} + \hat{Q}_R - \hat{Q}_L = 0$ must hold.

where μ_0 is a positive constant and $\eta \in [0,1]$ is a parameter measuring the impact of the level of energy-complementary technology on extraction costs and 1-η measures the impact of labor-complementary technology. Since Q_R and Q_L are increasing, μ is decreasing and eventually falls to zero. Using (26), (27) and (26') to solve for $\hat{Q}_i$, and substituting the results in the time derivative of (35), and we get the following equation for the evolution of μ:

$$\hat{\mu} = -\zeta D - \frac{\eta \xi_R - (1-\eta)\xi_L}{\xi_R + \xi_L}(H - D)\left[\omega_L \xi_L + \omega_R \xi_R\right](\theta_R - \bar{\theta}) \tag{36}$$

Since we can substitute the identity $X = -\hat{\mu}\,\mu / (1+\mu)$ in (32), the dynamics of the whole model are characterized by the three-dimensional system with variables D, θ_R, μ and equations (31), (32), (36). If we add the simplifying assumption $\omega_L = \omega_R \equiv \omega$, then equation (31) becomes one-dimensional so that D does not depend on any other variables in the model. Furthermore, (31) turns out to be unstable, so that the only plausible possibility is that D remains for ever on its equilibrium value $\bar{D} = [\omega H - (\rho - \hat{L})/\zeta]/(1+\omega)$, which is assumed to be positive, and the dynamics of the model can be fully characterized by equations (31) and (36), with variables θ_R and μ, where D is substituted by $\bar{D}$. Noting that $\mu(\infty) = 0$, we get the long-run value for the energy share, $\theta_R(\infty) = \bar{\theta} + (1+\omega)(\rho - \pi)/\left[\omega\left(H + \rho/\zeta\right)\left(\xi_R + \xi_L\right)\right]$ and using (20), (22) and (28) we conclude that the long run values for $\hat{S}^B$, $\hat{w}^B$ and $\hat{Q}^B$ are the same as in the case with exogenous extraction costs. Figure 3 shows the joint evolution of θ_R and μ.[12]

INSERT FIGURE 3 ABOUT HERE

[12] The diagram is plotted under the assumption $\xi_L \xi_R \bar{D} + (\rho - \pi)[\eta \xi_R + (\eta - 1)\xi_L] > 0$, which makes the $\dot{\theta}_R = 0$ locus to be downward sloping.

6. Conclusions

We have developed an endogenous growth model that connects the rate and direction of technical change to energy use and optimal resource extraction within a dynamic general equilibrium framework. The novel element is that both technical change and resource use are assumed to be endogenous variables resulting from the interactions between rational optimizing agents.

This model turns out to be compatible with some key stylized facts about economic growth and energy use, when suitably calibrated. Specifically, we show that when resource extraction costs and energy taxes steadily decline, the model generates steadily increasing per capita energy use and decreasing energy share in GDP.

We also arrived at the conclusion that some important economic trends on energy use and prices, which we observe in reality, are likely to be reversed in the future. First, the model predicts that per capita energy use will start declining at some point in the future, and will keep declining in the long run. Apart from the theoretical foundation for this inverted-U result, it is also reasonable from a sustainability perspective, taking into account that, by definition, the stock of nonrenewable resources is finite. The increased scarcity of resources shifts technical change progressively towards energy-saving technological change. This comes at the cost of total factor productivity growth, which is the driving force behind per capita income growth in the long run.

The effect of efficiency gains in resource-extraction technologies is likely to become progressively smaller as compared to the exhaustion effect, which means that the observed decrease in energy prices relative to labor prices is likely to be reversed in the long run as the energy price evolution converges to the standard Hotelling rule. In a similar way, the evolution of energy share is also likely to experience an overshooting phenomenon, in such a way that the current decreasing trend may be reversed in the future resulting in a increasing behavior to converge to its steady state value.

For the sake of simplicity, we have derived the key results in the paper while keeping the simplifying assumption that resource extraction costs exogenously decrease. Nevertheless, we have also shown a way to endogenize this process as a spillover effect from R&D performed in the economy. To keep the model tractable, we have deliberately chosen some other simplifications that do not crucially affect the purpose of our study; in future work we could address capital accumulation and endogenous population growth.

References

Acemoglu, D. (1998) “Why do new technologies complement skills? Directed technical change and wage inequality,” *Quarterly Journal of Economics*, 113, 1055–1089.

Acemoglu, D. (2002) Directed Technical Change, *Review of Economic Studies*

Chermak, J. and R.H. Patrick (1995) “A Well-Based Cost Function and the Economics of Exhaustible Resources: The Case of Natural Gas”. *Journal of Environmental Economics and Management* 28, 174-189.

Dasgupta, P.S. and G.E. Heal (1974) “The Optimal Depletion of Exhaustible Resources”. *Reviewof Economic Studies (Symposium)*, 3-28.

Dasgupta, P.S. and G.E. Heal (1979) *The Economics of Exhaustible Resources*, Cambridge: Cambridge University Press.

Dixit A.K. and J.E. Stiglitz (1977) “Monopolistic Competition and Optimum Product Diversity,” *American Economic Review* 67, 297–308.

Fagan, M.N. (1997) “Resource Depletion and Technical Change: Effects on U.S. Crude Oil Finding Costs from 1977 to 1994” *The Energy Journal* 18, 91-106.

Grimaud, A. and L. Rougé (2001) "Non Renewable Resources and Growth with Vertical Innovations: Optimum, Equilibrium and Economic Policies", forthcoming in Journal of Environmental Economics and Management.

Groth, C. and P. Schou (2003) “Capital Taxation, Growth, and Non-renewable Resources”. Working paper.

Jones, C.I. (2002) *Introduction to Economic Growth*, Second Edition, New York: W.W. Norton.

OECD (1999) International Sectoral Database 1998, Paris.

Popp, D. (2002) “Induced Innovation and Energy Prices”, *American Economic Review* **92**, 160-180.

Neumayer, E. (2003). *Weak versus Strong Sustainability; exploring the limits if two opposing paradigms*. Cheltenham: Edward Elgar.

Romer, P.M. (1990) “Endogenous Technological Change”, *Journal of Political Economy*, 98, S71–103.

Simon, J. (1996) *The Ultimate Resource 2*. Princeton: Princeton University Press.

Sinclair, P. (1992) “High Does Nothing and Rising is Worse: Carbon Taxes Should Keep Declining to Cut Harmful Emissions”, *The Manchester School* 60, 41-52.

Samuelson, P.A. (1954) “The transfer problem and transport costs II: Analysis of the effects of trade impediments” *Economic Journal* 64, 147-162.

Smulders, S. and M. de Nooij (2003) “The Impact of Energy Conservation on Technology and Economic Growth”, *Resource and Energy Economics* 25, 59-79.

Tahvonen, O. and S. Salo (2001) “Economic Growth and Transition Between Renewable and Nonrenewable Energy Resources” *European Economic Review* 45, 1379-1398.

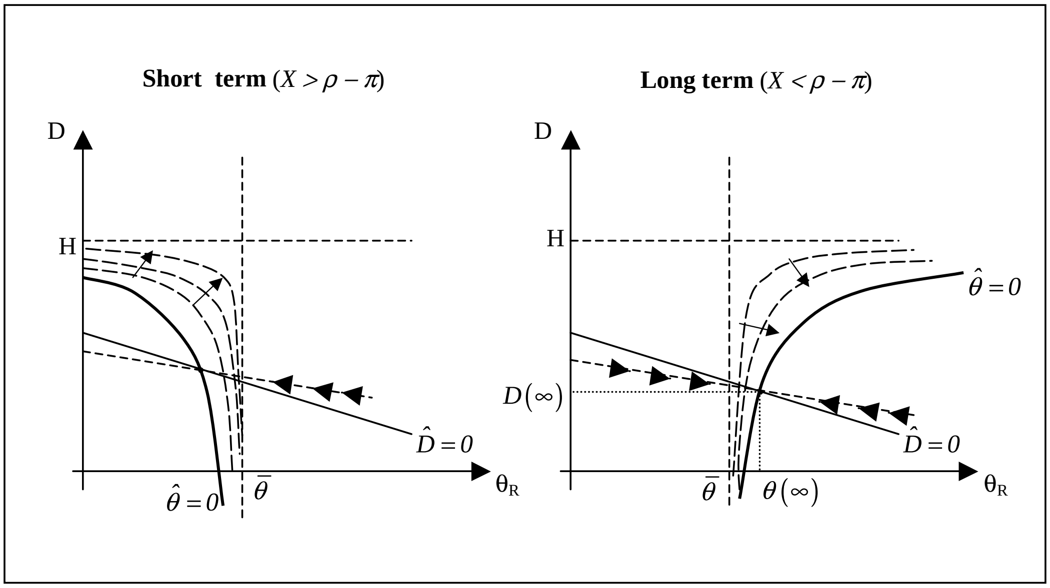

Figure 1. Phase diagram

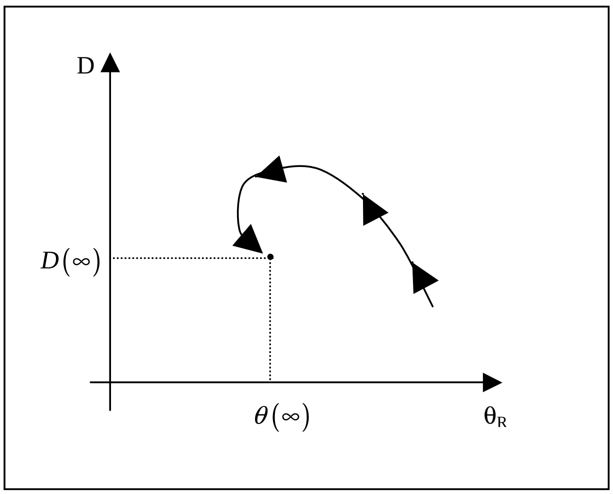

Figure 2. Joint evolution of θ_R and D.

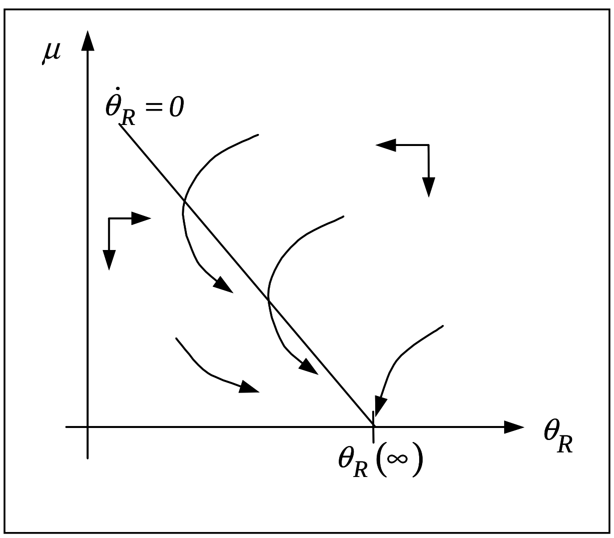

Figure 3. Joint evolution of θ_R and μ with endogenous μ.