Non-renewable Resources and Economic Growth

The Classics
versus
New models of endogenous technology

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Main questions of the literature on non-renewable resource:

- How does depletion of essential non-renewable resources impose a *drag on growth*?
- How can investment in *physical capital* offset this drag on growth?
- How can investment in *new technologies* offset the resource drag?
- How does resource depletion affect the incentives to invest in capital or new technologies?

It is all about the interaction between

- Substitution
- Technological change
- Investment

This paper:

- How much does substitution and technological change matter for long-run growth?
- Are the old workhorse models (the 1974 classics) still relevant, once we depart from Cobb-Douglas and exogenous technological change?

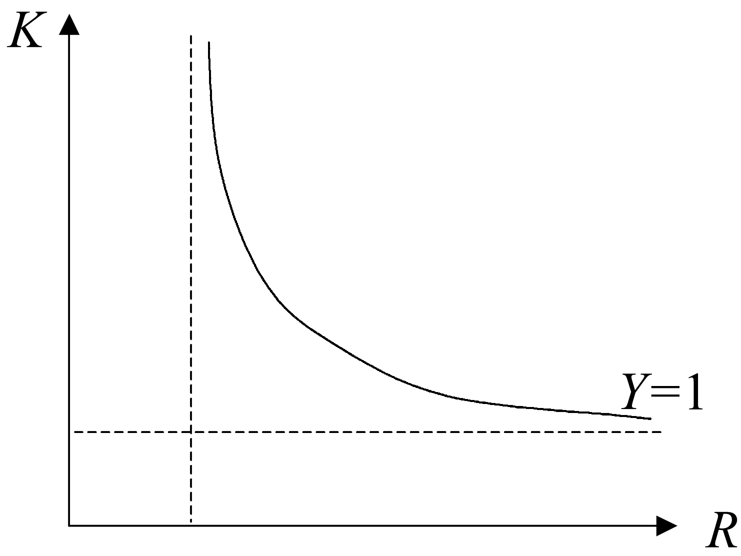
Key Concepts

Finite resources: cumulated extraction is bounded.

Necessary resources: no production without resource use.

Cumulative production bounded?
Not necessarily thanks to substitution and technical change.

Substitution: movements along the isoquant.



Poor substitution ($\sigma_Y < 1$): minimum requirement of input factor.
Technical progress: reduces minimum requirements.

- resource augmenting
- capital augmenting

$$Y = F(A_K K, A_R R)$$

Growth with finite resources:

- R must steadily decline
- Offset by increases in A_R
  (resource-augmenting technical progress)

What are the classics? What is hot?

<table>
<tr><th></th><th>poor substitution
$\sigma_Y < 1$</th><th>$\sigma_Y = 1$</th><th>good substitution
$\sigma_Y > 1$</th></tr>
<tr><td>no technical change</td><td>Club of Rome

doomsday</td><td>Solow 1974

constant production if $\alpha_K > \alpha_R$</td><td rowspan="4">growth

(resource is not necessary)</td></tr>
<tr><td>exogenous technical change</td><td>Dasgupta/Heal 1979

growth if resource-augmenting techn change</td><td>Stiglitz 1974

growth if rate of techn change > discount rate</td></tr>
<tr><td>no techn change IRS</td><td></td><td>Groth/Schou 2002
Groth/Schou 2003

growth if population grows sufficiently fast</td></tr>
<tr><td>endogenous technical change</td><td>André/Smulders 2004
Bretschger/Smulders 2004
This lecture!

growth if
• high technolog opportunity
• poor subst in traditional sector</td><td>Barbier 1999
Schou 1999
Scholz/Ziemes 1999
Grimaud/Rougé 2003

growth if high technological opportunity</td></tr>
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Outline

0. General model

1. Good substitutes (Cobb Douglas)

    Only resource

    Capital accumulation

        optimum versus market and policies

    Endogenous technology

        optimum versus market and policies

2. Poor substitutes

    Exogenous technology

    Endogenous technology

    Endogenous growth

Notation

Growth rate of a variable $X(t)$ is denoted by

$$\frac{dX(t)/dt}{X} = \frac{\dot{X}}{X} = \frac{d\ln X(t)}{dt} \equiv \hat{X}(t)$$

The time index *t* is omitted where no confusion arises.

A	technology level / knowledge stock / # of blueprints
C	consumption
F	production function
G	knowledge accumulation function (research technology)
g	balanced growth rate
K	capital
L	labour in production
L_A	labour in research
L^S	labour supply
n	rate of population growth
m	intermediates
p	price
r	interest rate
S	resource stock
s	savings rate
t	time
u	depletion rate
V	wealth
w	input price
X	factor input
Y	output
α	production elasticity labour
β	production elasticity capital

γ	elasticity of intermediates production with respect to labour input
ε	elasticity of substitution among intermediates
θ_i	production elasticity of factor i (i=K,L,R)
η	production elasticity of knowledge
λ	elasticity of research output with respect to labour input
ν	production elasticity resource
ρ	utility discount rate
σ	intertemporal substitution elasticity
σ_Y	elastictity of substitution in production
τ	tax rate
φ	spillover parameter
ξ	research productivity

The general model

Technology: $Y = F(R, K, L, A_T, A_R, A_L) = C + \dot{K}$

Resource is necessary: $F(0, K, A, L, t) = 0$

Resource dynamics: $\dot{S} = -R \leq 0, \quad S \geq 0$

Investment technology $\dot{A}_i = G_i(A_i, L_{Ai})$

Exogenous growing factors:

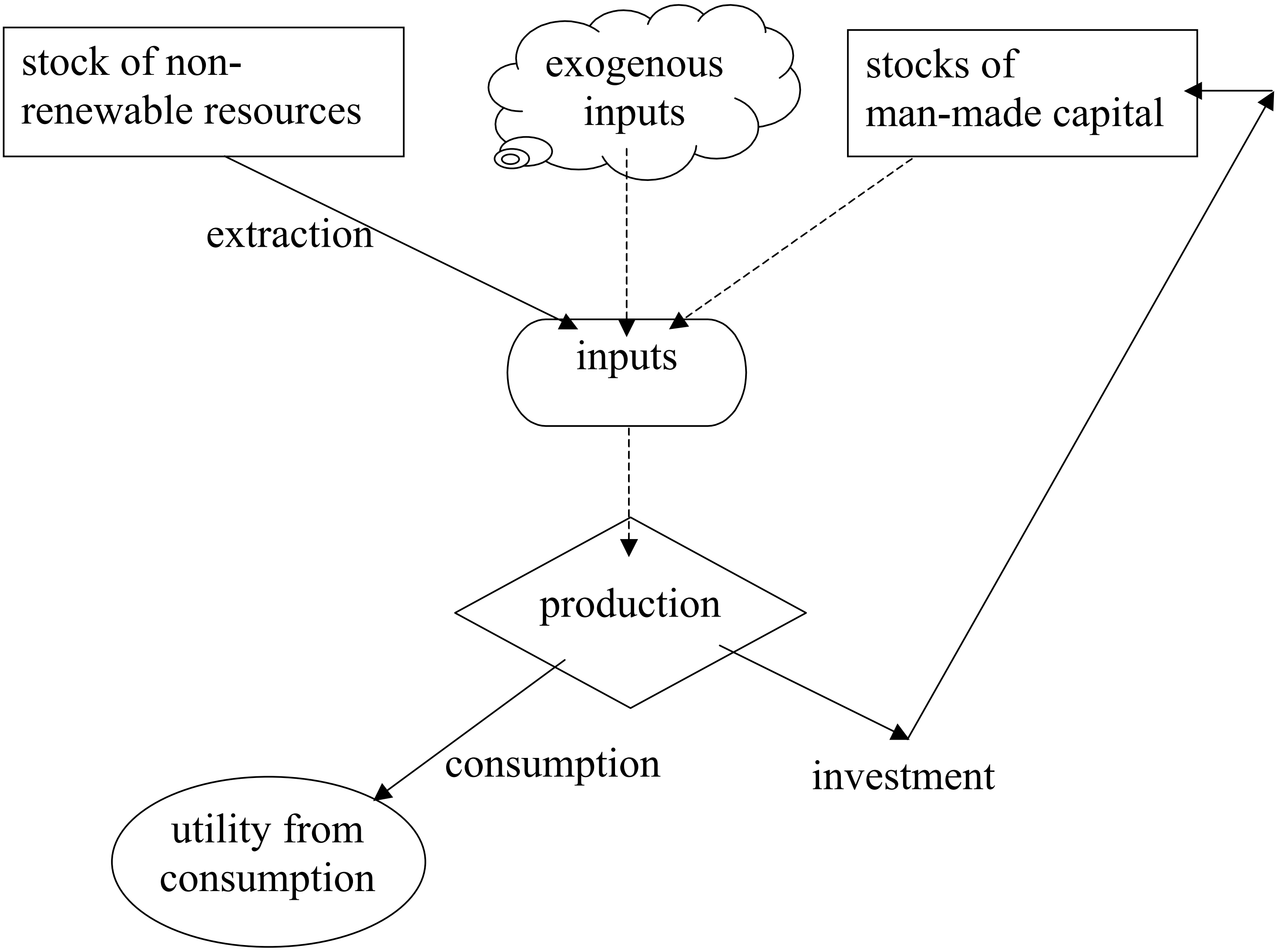
- Exogenous technology $\hat{A}_i$ given
- Labor $\hat{L} = n$ given

Welfare $W(0) = \int_0^\infty \left(\frac{1}{1 - 1/\sigma} C(t)^{1-1/\sigma} \right) e^{-\rho t} dt$

To account for population growth:

$$\rho \equiv \rho_{n=0} + (1 - 1/\sigma)n$$

The general model



Concepts / Definitions

1. No-doomsday path:
Path of consumption such that consumption never falls to zero.
Since the resource is necessary, this requires

$$\int_0^\infty R(\tau)d\tau \leq S(0)$$

2. Constant (per capita) consumption path
Path along which $\hat{C} = n$.
If in addition per capita consumption is maximum, we satisfy the Rawlsian criterion (some claim that this is *the* sustainability criterion): all generations are equally well of

$$W(0) = \max\left\{\min\{U(C(t))\}\right\}$$

3. Balanced growth path (BGP)
All variables grow at constant rates. A feasible BGP requires:

$$\hat{Y} = \hat{C} = \hat{K} = g$$

$$\hat{R} = \hat{S} = -u < 0$$

$$\hat{L} = \hat{L}_{Ai} = n$$

Often, a BGP arises only in the long run (after transitional dynamics).

4. Optimal growth path
The path of consumption that maximizes the welfare criterion subject to the resource and technology constraints.

5. Market equilibrium
The path for which all markets are in equilibrium.

Some notes on resource depletion

$\dot{S} = -R \leq 0$ with

- R resource flow
  $\hat{R} < 0$ on a no-doomsday growth path

- R/S rate of depletion
  $\widehat{R/S} = \hat{R} - \hat{S} = \hat{R} - \frac{-R}{S} = \hat{R} + R/S$
  on a BGP: $-\hat{R} = R/S > 0$ and constant (rate of depletion)

Notation

g balanced growth rate

u rate of extraction along the balanced growth path
($-\hat{R} = -\hat{S} = R/S > 0$)

Optimal growth – General formulation

Max $W(0) = \int_0^\infty U(C(t))e^{-\rho t}dt$

subject to

Technology $\dot{K} = F(R, K, L, A_T, A_R, A_L) - C$

Resource dynamics: $\dot{S} = -R \leq 0, \quad S \geq 0$

Investment technology $\dot{A}_i = G_i(A_i, L_{Ai})$

Labour market constraint $L^S = L + \Sigma L_{Ai}$

Optimality conditions:

wrt R $\rho - \hat{U}_C = \hat{F}_R$

wrt K $\rho - \hat{U}_C = F_K$

wrt L_{Ai} $\rho - \hat{U}_C = \frac{F_{Ai}}{F_L / G_{L_{Ai}}} + G_{Ai} + \widehat{F_L / G_{L_{Ai}}}$

Interested in different specifications for

- F(.), production technology;
    - Cobb-Douglas versus CES
    - Cake versus Capital accumulation
    - CRS versus IRS
- G(.), research technology,
    - exogenous technological change
    - semi-endogenous growth (DRS wrt A)
    - endogenous growth (CRS wrt A)

Interested in

- Feasible growth
- Constant consumption path
- Optimal growth
- Market equilibrium

Constant elasticities – BGP Results

Specify:

$$Y = R^{\nu} K^{\beta} L^{\alpha} A^{\eta}; \qquad \dot{A} = \xi A^{\varphi} L_A^{\lambda}$$

Closed form solution:

$$\hat{Y} = g = \frac{\sigma(a - \psi\rho)}{\psi + (1-\psi)\sigma} \quad ; \qquad -\hat{R} = u = \frac{(1-\sigma)a + \sigma\rho}{(1-\psi)\sigma + \psi}$$

	ψ	a
Exog. technology $\xi = 0$	$\frac{\nu}{1-\beta}$	$\frac{1}{1-\beta}(\eta\hat{A} + \alpha n)$
Endog. technology small spillovers $\xi > 0, \varphi < 1$	$\frac{\nu}{1-\beta}$	$\frac{1}{1-\beta}(\eta\frac{\lambda n}{1-\varphi} + \alpha n)$
Endog. technology large spillovers $\xi > 0, \varphi = 1,\ n = 0$	$\frac{\alpha+\nu}{1-\beta}$	$\frac{1}{1-\beta}(\eta\xi L^S)$

Comparative statics (Optimal growth path).

Variable (i)	Growth (g)	Depletion (u)
$\partial i / \partial\rho$	$-\psi\sigma / D < 0$	$\sigma / D > 0$
$\partial i / \partial\sigma$	$\psi g / \sigma D$	$-g / \sigma D$
$\partial i / \partial a$	$\sigma / D > 0$	$(1-\sigma)/D$
$\partial i / \partial\psi$	$-\sigma u / D < 0$	$-u(1-\sigma)/D$

$D \equiv (1-\psi)\sigma + \psi > 0$

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Cake Eating model (CE)

Technology assumptions ($\alpha = \beta = 0$):

Cake eating

$$C(t) = Y(t) = R(t) \qquad \hat{Y} = \hat{R}$$

Cake + party

$$C(t) = Y(t) = e^{at} R(t) \qquad \hat{Y} = a + \hat{R}$$

Cake + party + hangover

$$C(t) = Y(t) = e^{at} R(t)^{\Psi} \qquad \hat{Y} = a + \psi \hat{R}$$

Constant consumption path:

$$n = \hat{C} = \hat{Y} = a + \psi \hat{R} \quad \Rightarrow \quad -\hat{R} = \frac{a - n}{\psi} > 0$$

Solve optimal growth path

(i) Production function: $\hat{Y} = a + \psi\hat{R}$

(ii) Optimality condition: $\rho + \overbrace{(1/\sigma)\hat{Y}}^{-\hat{U}_C} = \overbrace{\hat{Y} - \hat{R}}^{\hat{Y}_R}$

Note that (ii) holds if

- Both the production and utility function are iso-elastic
- Consumption and output grow at the same rate (BG).

Closed form solution:

$$\hat{Y} = g = \frac{\sigma(a - \psi\rho)}{\psi + (1-\psi)\sigma} \quad ; \quad -\hat{R} = u = \frac{(1-\sigma)a + \sigma\rho}{(1-\psi)\sigma + \psi}$$

Comparative statics (Optimal growth path).

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$\partial i / \partial \rho$	$-\psi\sigma / D < 0$	$\sigma / D > 0$
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$\partial i / \partial a$	$\sigma / D > 0$	$(1-\sigma)/D$
$\partial i / \partial \psi$	$-\sigma u / D < 0$	$-u(1-\sigma)/D$

$D \equiv (1-\psi)\sigma + \psi > 0$

Implications

- Optimal growth can be negative, even though positive growth is feasible (high discount rate, high resource share).
- Higher resource share implies lower growth.
- Growth is affected by preferences (“endogenous”), ...
- ... but requires exogenous technological progress.
- No transition dynamics. $R(t) = uS(t)$

Isomorphy Cake eating model

Define

$K_S = e^{at} S^{\psi}$

stock of resources measured in consumption equivalents.
We then have:

$$\hat{K}_S$$

$$= a + \psi \hat{S}$$

$$= a + \psi \frac{-R}{S}$$

$$= a - \psi \left(\frac{C / e^{at}}{K_S / e^{at}} \right)^{1/\psi}$$

$$= a - \psi \left(C / K_S \right)^{1/\psi}$$

or

$$\dot{K}_S = a K_S - \left[\psi \left(\frac{C}{K_S} \right)^{(1-\psi)/\psi} \right] C$$

Isomorphy:

$\psi = 1$ AKmodel

$\psi < 1$ AK-like-model

Interesting...

- Reinterpretation in terms of endogenous growth
- Interpretation of Groth/Schou (2002) model (concavity).
- New version of AK-model (concave transformation curve).

Capital Accumulation (KA)

Part of output can be turned into a durable input

$$Y = F(R, K, L, t) = R^{\nu} K^{\beta} L^{\alpha} A_{TFP}$$
$$\dot{K} = Y - C$$

Sustainable growth (KA)
Write the production function in growth rates:

$$\hat{Y} = \beta \hat{K} + \nu \hat{R} + \alpha \hat{L} + \hat{A}_{TFP}$$

Along a BGP with $\hat{C} = \hat{Y} = \hat{K} = g$, this boils down to:

$$\hat{Y} = \underbrace{\frac{\nu}{1-\beta}}_{\psi} \hat{R} + \underbrace{\frac{\hat{A}_{TFP} + \alpha \hat{L}}{1-\beta}}_{a}$$

Back to decreasing-returns-cake-eating-model!

Optimal growth (KA)
Optimality: first order condition

$$\rho - \hat{U}_C \qquad = \hat{F}_R \quad = F_K$$
$$\rho + (1/\sigma)\hat{C} = \hat{Y} - \hat{R} = \beta Y / K$$

First equality is the same as in the cake-eating model.
Second equality is DHSS efficiency condition.
Nothing changes... only reinterpretation needed.

Implications capital accumulation

- Savings rate no influence on long-run growth
- Capital accumulation in itself cannot drive growth ...

  (because of decreasing returns:
  K/L and K/S rises, so MPK falls)

  ...unless
  - TFP growth (Stiglity 1974)
  - Population growth and IRS ($\alpha + \beta > 1$) (Groth/Schou 2002)
  - IRS with respect to capital $\beta > 1$ (but this is unstable) (Groth/Schou 2002)

Market equilibrium (KA)

Suppose:

- CRS $\alpha + \beta + \nu = 1$
- Property rights
- Rational expectations

... then the market equilibrium coincides with the social optimum.

Groth/Schou (2003): IRS and Marshallian externality.

Market Equilibrium

Market prices:
w_L wage rate
w_R resource price
r interest rate

Next-to-simplest case:

- Price taking in all markets, but IRS.
  Requires non-IRS at firm level (merger argument).
  Marshallian externality
- Various tax instruments (τ)

Households
maximize utility s.t. dynamic budget constraint:

$$\max W(0) = \int_0^\infty \left(\frac{1}{1-1/\sigma} C(t)^{1-1/\sigma} \right) e^{-\rho t} dt$$

$$\text{s.t. } \dot{V} = (1-\tau_r) r V + w_L L^S - (1+\tau_c) C - \tau$$

$\Rightarrow (1-\tau_r) r = \widehat{1+\tau_c} + \rho + (1/\sigma)\hat{C}$ Keynes-Ramsey Rule

Resource owners
maximize NPV of resource income

$$\max \int_0^\infty w_R(t) R(t) \exp(-\int_0^t r(s) ds) dt$$

$\text{s.t. } \dot{S}(t) = -(1+\mu) R(t)$ μ: mining cost

$\Rightarrow r = \widehat{w_R/(1+\mu)} = \hat{w}_R - \frac{\dot{\mu}}{1+\mu}$ Hotelling Rule

Final goods producers
maximize profits

$$A K_i^{\beta_f} L_i^{\alpha} R_i^{\nu_f} - r(1+\tau_K) K_i - w_L L_i - w_R (1+\tau_u) R_i$$

$$\beta_f + \alpha + \nu_f = 1$$

$$A = A_{TFP} K^{\beta - \beta_f} R^{-(\nu_f - \nu)}$$

Externality with respect to K (learning) and R (pollution).
(exercise: externalities with respect to L).

$$\beta_f Y / K = r(1+\tau_K)$$

$$\nu Y / R = w_R (1+\tau_R)$$

$$\alpha Y / L = w_L$$

Policies

Market equilibrium:

$$\widehat{1+\tau_C}+\rho+\frac{1}{\sigma}\hat{C}=(1-\tau_V)\left(\hat{Y}-\hat{R}-\widehat{1+\tau_R}-\widehat{1+\mu}\right)=(1-\tau_V)\left(\frac{\beta_f}{1+\tau_K}\right)\frac{Y}{K}$$

cf. Optimum:

$$\rho+\frac{1}{\sigma}\hat{C} \quad = \quad \hat{Y}-\hat{R}-\widehat{1+\mu} \quad = \quad \beta\frac{Y}{K}$$

Optimal policies:

- Internalize learning externality:
  $$\beta_f/(1+\tau_K)=\beta \;\Rightarrow\; \tau_K=-(\beta-\beta_f)/\beta$$
- Internalize pollution externality:
  $$\nu_f/(1+\tau_R)=\nu \;\Rightarrow\; \tau_R=(\nu_f-\nu)/\nu$$

Other policies:

- Sustainability policy: make society more patient
  $$\widehat{1+\tau_C}<0$$
- Conservation policy:
  $$\widehat{1+\tau_R}<0$$

If $\tau_V=0,\ \beta_f/(1+\tau_K)=\beta$, we are left with

$$\underbrace{\widehat{1+\tau_C}+\widehat{1+\tau_R}+\widehat{1+\mu}+\rho}_{\equiv\rho_\tau}+\frac{1}{\sigma}\hat{C}=\hat{Y}-\hat{R}=\beta\frac{Y}{K}$$

So taxes can lower the effective discount rate. See previous comparative statics:
Lower discount rate means

- higher long-run growth rate
- lower long-run depletion rate

Endogenous technological change (ET)

1. Marshall/Arrow/Romer:

$$Y = R^{\nu} K^{\beta_1} L^{\alpha} A_{TFP}$$

$$\alpha + \beta_1 + \nu = 1 \qquad \Rightarrow \qquad Y = R^{\nu} K^{\beta_1 + \beta_2} L^{\alpha}$$

$$\dot{K} = Y - C \qquad\qquad \alpha + \beta_1 + \beta_1 + \nu > 1$$

$$A_{TFP} = K^{\beta_2}$$

Back to KA... back to CE...

2. Two-sector model ("semi-endogenous growth")

Investment in new technology is fundamentally different from investment in physical capital:

- Innovation builds on experience and knowledge (spillovers)
- No resources needed.

Generalized Romer model (Jones 1995):

$$Y = R^{\nu} K^{\beta} L^{\alpha} A^{\eta}$$

$$\dot{K} = Y - C$$

$$\dot{A} = \xi A^{\varphi} (L^S - L)^{\lambda}$$

φ knowledge spillovers ($\varphi \leq 1$ for stability)

Long run:

$$\hat{A} = \frac{\xi L_A^{\ \lambda}}{A^{1-\varphi}} \text{constant if } \lambda \hat{L}_A = (1-\varphi)\hat{A} \Leftrightarrow \frac{\lambda n}{1-\varphi} = \hat{A}$$

Combine

$$\hat{Y} = \psi \hat{R} + a$$

$$\psi = \nu/(1-\beta),$$

$$a = (\hat{A}_{TFP} + \alpha n)/(1-\beta) \quad \Rightarrow a = (\frac{\eta\lambda}{1-\varphi} + \alpha) n/(1-\beta)$$

$$\rho + (1/\sigma) g = g - \hat{R}$$

Similar results as CE

- population growth drives growth,
- role of population growth more important (IRS).

Semi-endogenous growth (Jones 1995), but...
Now the discount rate affects growth (through depletion).

3. **Endogenous growth**

As above but now $\varphi = 1, n = 0$ (and for simplicity $\lambda = 1$):

$$\eta\xi(L^S - L)^\lambda = \eta\hat{A} \quad (= \hat{A}_{TFP})$$

Constant labour effort in R&D gives constant rate of technological progress!

Optimality condition

$$\rho + \frac{1}{\sigma} g = g - \hat{R} = \beta \frac{Y}{K} = \frac{\eta\xi}{\alpha} L + g$$

ηL: market size
α/ξ: cost of innovation
g: increase in the opportunity cost of innovation (wage growth)

To eliminate L: combine with balanced growth path

$$\left.\begin{aligned} \eta\xi(L^S - L)^\lambda &= \eta\hat{A} \\ g &= \frac{\eta}{1-\beta}\hat{A} + \frac{\nu}{1-\beta}\hat{R} \end{aligned}\right\} \Rightarrow \eta\xi L = \eta\xi L^S - (1-\beta)g + \nu\hat{R}$$

This gives four equations in four unknowns: $g, \hat{R}, Y/K, L$

$$\hat{Y} = g = \frac{\sigma(a - \psi\rho)}{\psi + (1-\psi)\sigma} \quad ; \qquad -\hat{R} = u = \frac{(1-\sigma)a + \sigma\rho}{(1-\psi)\sigma + \psi}$$

as before, but now:

$$\psi = \frac{\alpha + \nu}{1 - \beta} \quad \text{rather than} \quad \frac{\nu}{1 - \beta}$$

$$a = \frac{\eta \xi L^S}{1 - \beta} \quad \text{rather than} \quad \frac{\hat{A}_{TFP} + \alpha n}{1 - \beta}$$

Separate technology and preferences

$$\rho + (1/\sigma)g = \xi L^S$$

$$\rho + (1/\sigma)g - g = u$$

Endogenous growth – Market economy

Crucial: modeling incentives to come up with new technologies. Appropriability problem.
Romer's solution:

- New technologies are embodied in new "capital components" (intermediates).
- Patent protection for each component
- Imperfect substitution between components

The result is monopolistic competition among the suppliers of capital components. Monopoly profits are the reward for innovation.

Tractability:

- Dixit Stiglitz approach to monopolistic competition
- symmetry among component producers
- $\varepsilon=1/(1-\beta)$

Production: $Y = K_E^{\ \beta} L^{\alpha} R^{\nu}$

Services from capital: $K_E = \left(\int_0^A m(k)^{(\varepsilon-1)/\varepsilon} dk \right)^{\varepsilon/(\varepsilon-1)}$

Production of capital: $K = \int_0^A m(k) dk$

Under symmetry and $\varepsilon = 1/(1-\beta)$:

$$K = Am$$

$$K_E = A^{(1-\beta)/\beta}(Am) = A^{(1-\beta)/\beta} K$$

$$Y = (A^{(1-\beta)/\beta} K)^{\beta} L^{\alpha} R^{\nu} = A^{1-\beta} K^{\beta} L^{\alpha} R^{\nu}$$

Final goods producers:

$$\max\ L^{\alpha} R^{\nu} \int_0^A m(k)^{\beta} dk - w_L L - w_R R - \int_0^A p_m(k) m(k) dk$$

$\nu Y / R = w_R$ $\qquad$ $\nu Y / R = w_R$

$\alpha Y / K = w_L$ $\qquad$ $\alpha Y / K = w_L$

$\beta L^{\alpha} R^{\nu} m(k)^{\beta - 1} = p_m(k)$ $\qquad$ $\beta Y / A = p_m m$

Capital producers:

$\max\ p_m(k) m(k) - rK(k)$

s.t. downward sloping demand function

$p_m = r / \beta$ $\qquad$ $\beta Y / K = r / \beta$

$\pi = (1 - \beta) p_m m$ $\qquad$ $\pi = (1 - \beta) Y / A$

Innovation:

- Researchers invent blueprints for new components,
- sell the patent rights to use the blueprint;
- capital producers buy the blueprints (price p_A).

A is the number of components/blueprints.

$$\dot{A} = \xi A L_A$$

Free entry in R&D: workers are willing to work as a researcher if they earn at least the opportunity wage:

$$p_A \frac{\dot{A}}{L_A} = p_A \xi A = w$$

The price of a patent is the WTP of intermediate goods producer: NPV of profits:

$$rp_A = \pi + \dot{p}_A$$

Solve for steady state

$$\frac{\pi}{p_A} = \frac{(1-\beta)\beta Y / A}{(\alpha Y / L)/\xi A} = \frac{(1-\beta)\beta}{\alpha}\xi L$$

$$\hat{p}_A = \hat{w}_L - \hat{A} = \hat{Y} - \hat{A}$$

$$r = \frac{\pi}{p_A} + \hat{p}_A = \frac{(1-\beta)\beta}{\alpha}\xi L + \hat{Y} - \hat{A}$$

Market equilibrium:

$$\rho + \frac{1}{\sigma}\hat{C} = \hat{Y} - \hat{R} = \beta^2 \frac{Y}{K} = \beta\left(\frac{1-\beta}{\alpha}\right)\xi L + \hat{Y} - \hat{A}$$

cf. optimum

$$\rho + \frac{1}{\sigma}\hat{C} = \hat{Y} - \hat{R} = \beta \frac{Y}{K} = \left(\frac{1-\beta}{\alpha}\right)\xi L + \hat{Y}$$

Growth is too low:

- Knowledge spillover (researchers firms anticipate cheaper research and wait)
- Monopoly distortion (intermediates are sold above marginal cost)

Poor substitution

how to model poor substitution

$$Y = F(R_E, K, L_E, t)$$

$$\hat{Y} = \theta_R \hat{R}_E + \theta_K \hat{K} + \theta_L \hat{L}_E + (\partial F / \partial t) / F$$

$$\theta_R = \frac{\partial F(R_E, K, L_E, t)}{\partial R} \frac{R_E}{F}$$

Up to now: constant production elasticities (Cobb Douglas).

- Knife edge
- empirics
- One-drop-of-oil fairy tale
- No factor bias

Introducing substitution towards other production factors

Poor substitution, Strict definition

Factor i is a poor substitute for R if

$$\lim_{R_E \to 0} \theta_i(R_E, K, L_E, t) = 0 \qquad i \neq R_E$$

CES formulation

$$Y = F(R_E, K, L_E) = \left(\nu R_E^{(\sigma_Y - 1)/\sigma_Y} + \beta K^{(\sigma_Y - 1)/\sigma} + \alpha L_E^{(\sigma_Y - 1)/\sigma_Y} \right)^{\sigma_Y/(\sigma_Y - 1)}$$

$$\theta_R = \nu (Y / R_E)^{(1-\sigma_Y)/\sigma_Y} \ldots$$

$$R_E = A_R R$$

$$L_E = A_L L$$

The factors are poor substitutes if $\sigma_Y < 1$.

$$\lim_{R_E \to 0} \theta_R(R_E, K, L_E) = 1$$

$$\lim_{R_E \to 0} \theta_K(R_E, K, L_E) = 0$$

$$\lim_{R_E \to 0} \theta_L(R_E, K, L_E) = 0$$

Factor augmentation:

Technology determines effective resource and labor input

$$R_E = A_R R$$

$$L_E = A_L L$$

From now on: assume $\sigma_Y < 1$.

Balanced growth
Constant growth rate...
... requires constant production elasticities
... requires balanced input growth

$$(\hat{A}_R + \hat{R}) = \hat{K} = (\hat{A}_L + \hat{L}) = \hat{Y}$$

Back to Cake eating

$$a = \hat{A}_R \quad \Rightarrow \quad \hat{Y} = a + \hat{R}$$

Optimal growth
Optimality condition

$$\rho - \hat{U}_C \quad = \quad \hat{F}_R \quad = \quad F_K$$

$$\rho + \frac{1}{\sigma}\hat{C} = \frac{1}{\sigma_Y}(\hat{Y} - \hat{R}) + \left(1 - \frac{1}{\sigma_Y}\right)\hat{A}_R = \theta_K Y / K$$

(plus two conditions for the optimal investment in technology, if both A_R and A_L are endogenous) .

Combine balanced growth and optimality:

$$\rho + \frac{1}{\sigma}g = \hat{A}_R = \beta\left(\frac{Y}{K}\right)^{1/\sigma_Y}$$

$$-\hat{R} = \hat{A}_R - g$$

So, *if balanced growth applies*, σ_Y no direct effect on g and u.
(Maybe indirect effect if technical change is endogenous…)
But balanced growth only by coincidence.

- Knife edge:
  - if technological change and population growth are exogenous, a BGP arises by coincidence only.
  - If technological change is "semi-endogenous", again BGP by coincidence only.

    $$\hat{A}_i = \frac{\lambda_i}{1-\varphi_i} n, \quad i = R, L$$

Exogenous technology

Key result for long-run optimal growth:

$$g = \min\left\{\sigma(\hat{A}_R - \rho), (\hat{A}_L + \hat{L})\right\}$$

Cake versus Solow

Case i. $\sigma(\hat{A}_R - \rho) = (\hat{A}_L + \hat{L})$
Balanced growth and Cake-solutions

$$g = \sigma(\hat{A}_R - \rho) = (\hat{A}_L + \hat{L})$$

$$u = (1-\sigma)\hat{A}_R + \sigma\rho$$

Case ii. $\sigma(\hat{A}_R - \rho) < (\hat{A}_L + \hat{L})$
Unbalanced growth and Cake-solutions

$$g = \sigma(\hat{A}_R - \rho) < (\hat{A}_L + \hat{L})$$

$$u = (1-\sigma)\hat{A}_R + \sigma\rho$$

$$\theta_L = 0$$

Case iii. $\sigma(\hat{A}_R - \rho) > (\hat{A}_L + \hat{L})$
Unbalanced growth and Solow-solutions

$$g = (\hat{A}_L + \hat{L}) < \sigma(\hat{A}_R - \rho)$$

$$u = \left[(1-\sigma)\hat{A}_R + \sigma\rho\right] + \left(1 - \frac{\sigma_Y}{\sigma}\right)\left[\sigma(\hat{A}_R - \rho) - (\hat{A}_L + \hat{L})\right]$$

$$\theta_R = 0$$

Empirically relevant?

Semi-endogenous growth

$$\dot{A}_R = \xi_R (A_R)^{1-\varphi_R} (L_R)^{\lambda_R}, \quad \dot{A}_L = \xi_L (A_L)^{1-\varphi_L} (L_L)^{\lambda_L}$$

$$\hat{A}_R = \frac{\lambda_R}{1-\varphi_R} n, \qquad \hat{A}_L = \frac{\lambda_L}{1-\varphi_L} n$$

similar three cases.

Endogenous growth

We need: $\varphi_i = 1, n = 0$ (i=R,L)

(For simplicity we assume $\lambda_i = 1$ in addition)
Optimality conditions:

$$\rho + \frac{1}{\sigma} g = g - \hat{R} = \theta_K \frac{Y}{K} = \xi_L L + g = \left(\frac{\theta_R}{\theta_L} \right) \xi_R L + g$$

Long-run solution

$$g = \sigma(\xi L^S - \rho)$$

$$u = \sigma\rho + (1-\sigma)\xi L^S$$

$$\xi \equiv \left(\frac{\xi_L \xi_R}{\xi_L + \xi_R} \right)$$

$$\theta_R / \theta_L = \xi_L / \xi_R$$

Implications

- Elasticity of substitution does not matter (not in engine of growth).
- Scale effect.

Directed technological change – market

Smulders and De Nooij (REE 2003)
André and Smulders (wp 2004)

Main differences with above framework:

- Intermediates are flow variables (no capital stock)
- Inhouse R&D for quality improvements (no entry)

Challenge:
Directed technological change with market incentives

Solution:
Acemoglu (1998, 2002): multi-sector Romer model.

- several (two) sectors
- technological change in each sector: innovation projects improve quality or variety of the intermediates used in the sector
- each sector produces inputs for the final goods sector
- sectors differ in factor intensity: L-intensive versus R-intensive (take extreme position).
- If R-intensive sector innovates more than the L-intensive sector, innovation is resource-saving (provided substitution is poor) at the macroeconomic level.

Notation:

Venice	SdN2003	AS2004	
γ	$1-\beta$		
A	Q		
X	S		
		S	E

Production structure (tree)

Final goods production:

$$Y = a_0 \left(Y_R^{(\sigma_F - 1)/\sigma_F} + Y_L^{(\sigma_F - 1)/\sigma_F} \right)^{\sigma_F /(\sigma_F - 1)}$$

Sectoral production: $Y_R = R^{1-\gamma} K_{ER}{}^{\gamma}; \quad Y_L = L^{1-\gamma} K_{EL}{}^{\gamma}$

Services from capital: $K_{Ei} = \left(\int_0^1 q_i(k) \cdot m_i(k)^{1-\gamma} dk \right)^{1/(1-\gamma)}$

Production of capital: $K_i = \int_0^1 q_i(k) \cdot m_i(k) dk$

Goods market constraint: $Y = C + K_L + K_R$

Producer behaviour

Final goods producers

$$\max\ a_0\left(Y_R^{(\sigma_F-1)/\sigma_F}+Y_L^{(\sigma_F-1)/\sigma_F}\right)^{\sigma_F/(\sigma_F-1)}-p_LY_L-p_RY_R$$

$$\theta_L Y/Y_L = p_L$$
$$\theta_R Y/Y_R = p_R$$

Sectoral goods producers: $i = L, R;\quad X_L \equiv L;\ X_R \equiv R$

$$\max\ p_i X_i^{1-\gamma}\int_0^1 q_i(k)m_i(k)^\gamma dk - w_i X_i - \int_0^1 p_{mi}(k)m_i(k)dk$$

$$(1-\gamma)Y_L/L=(1+\tau_L)w_L$$
$$(1-\gamma)Y_R/R=(1+\tau_R)w_R$$
$$\gamma X_i^{1-\gamma}m_i(k)^{\gamma-1}=p_{mi}(k)$$

Intermediate goods producers:

max $p_{mi}(k)m_i(k)-q_i(k)K_i(k)$
s.t. downward sloping demand function

$$p_{mi}(k)=q_i(k)/\gamma$$

Static equilibrium

Consumption: $C = (1-\gamma^2)Y$

Production: $Y = \left((A_R R)^{(\sigma_Y - 1)/\sigma_Y} + (A_L L)^{(\sigma_Y - 1)/\sigma_Y} \right)^{\sigma_Y/(\sigma_Y - 1)}$

Substitution: $\sigma_Y = \gamma + (1-\gamma)\sigma_F$

Sectoral shares: $\frac{\theta_R}{\theta_L} = \left(\frac{A_R R}{A_L L} \right)^{-(1-\sigma_Y)/\sigma_Y}$

Real energy price: $\frac{w_R}{w_L} = \left(\frac{A_R}{A_L} \right)^{-(1-\sigma_Y)/\sigma_Y} \left(\frac{R}{L} \right)^{-1/\sigma_Y}$

Stories to tell

Stylized facts
(US 1950-1998, see Jones 2002):

SF1 Increasing per capita energy use	(R/L)
SF2 Increasing energy efficiency	(Y/R)
SF3 Declining energy share	(θ_R)
SF4 Declining real energy prices	(w_R/w_L)

Can we match the model to the stylized facts?

Are the trends suggested by the stylized facts sustainable?

Policy experiment
What are the effects of energy conservation on growth and innovation?

Exogenous supply, exogenous technology

Assume R/L increasing over time.
Assume A_R and A_L to change over time exogenously.
No investment – series of static equilibria.

Cake versus Solow (without Capital, though).

Matching stylized facts (SF2-SF4) requires:

Endogenous supply, exogenous technology

Needed: Hotelling and Ramsey rule

$$r = \hat{w}_R - \frac{\dot{\mu}}{1+\mu}$$

$$r = \rho + \hat{C}$$

Producers:

$$(1-\gamma)Y_R / R = (1+\tau_R)w_R \quad \Rightarrow \quad \hat{\theta}_R + \hat{Y} - \hat{R} = \frac{\dot{\tau}_R}{1+\tau_R} + \hat{w}_R$$

$$C = (1-\gamma^2)Y \quad \Rightarrow \quad \hat{C} = \hat{Y}$$

Combining:

$$\hat{\theta}_R - \left[\rho + \frac{\dot{\tau}_R}{1+\tau_R} + \frac{\dot{\mu}}{1+\mu}\right] = \hat{R}$$

To match SF1 & SF2 we need negative effective discount rate.

$$\hat{R} = -\left(\frac{\sigma_Y \rho_\tau + (1-\theta_R)(1-\sigma_Y)(\hat{A}_R - \hat{A}_L)}{\sigma_Y + (1-\theta_R)(1-\sigma_Y)}\right) \qquad (20)$$

$$\hat{\theta}_R = -\left(\frac{(1-\theta_R)(1-\sigma_Y)}{\sigma_Y + (1-\theta_R)(1-\sigma_Y)}\right)(\hat{A}_R - \hat{A}_L - \rho_\tau) \qquad (21)$$

Endogenous supply, endogenous technology

Needed: research technology and incentives.

$$\dot{q}_{ik} = [\xi_i A_i L_{Ai}{}^{1-\omega_i}] L_{Aik}{}^{\omega_i} \qquad (24)$$

$\omega_i < 1$ avoids bang-bang dynamics.
$\varphi_i = 1$ Strong spillovers, endogenous growth

symmetry is the outcome:

$$L_{Aik} = L_{Ai} \quad \Rightarrow \quad \dot{A}_i = \int_0^1 \dot{q}_{ik} dk = \xi_i A_i L_{Ai}$$

To make the model even more symmetric, a second type of labour is introduced which is not used in Y production, but only in research and a second consumption good.

Intermediate goods producers in both sectors improve their own product if this is profitable.

$$\begin{aligned} r - \hat{w}_D &= \left(\frac{(1-\gamma)\gamma Y}{w_D} \right) \omega_L \xi_L \theta_L - \hat{A}_L \\ &= \left(\frac{(1-\gamma)\gamma Y}{w_D} \right) \omega_R \xi_R \theta_R - \hat{A}_R \end{aligned} \qquad (26)$$

Dynamic system:

$$\hat{D} = \frac{H-D}{D}\left\{\rho + \zeta D - \zeta\,[\omega_L - (\omega_L - \omega_R)\theta_R](H-D)\right\} \tag{31}$$

$$\hat{\theta}_R = -\frac{(1-\theta_R)(1-\sigma_Y)}{\sigma_Y + (1-\theta_R)(1-\sigma_Y)}\left\{(H-D)\left[\omega_L\xi_L + \omega_R\xi_R\right](\theta_R - \bar{\theta}) - \rho_\tau\right\} \tag{32}$$