Environmental Externality and Capital Accumulation in an Overlapping Generations Model[¤]

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Abstract

We study an overlapping generations model a' la Diamond in which agents care about environmental quality in old age. At any period, the environmental quality is negatively a¤ected by the saving decision of the previous generation, and this creates an intergenerational externality over time. Young agents can invest in private capital and/or in environmental preservation. We show that in such a framework, the assumption that private saving depends positively on the interest rate is not su¢cient to guarantee that capital accumulates over time. Furthermore, we show that the global stability of the steady state equilibrium depends on the relative e¤ects of the capital and environmental quality on the investment functions of the individuals. Moreover, in our model, the decentralised solution is ine¢cient, compared with the one chosen by a social planner, and thus, dynamic ine¢ciency can arise because the presence of an environmental externality creates an overinvestment in private capital. Furthermore the e¤ects of the introduction of a pay-as-you-go security system in our economy is considered.

Keywords: overlapping generations, environmental externality.

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1 Introduction

A vast amount of economic literature has dealt with the relation between economic activity and environmental externalities over the years. The traditional neoclassical point of view explains the root of environmental problems as a result from market failures. In the case of collective goods these failures are due to di¢culties in establishing markets, while in the presence of negative externalities, the failures are due to a lack of well-de…ned property rights. The solution proposed by Coase (1960) to environmental problems would be to determine property rights as a basis for negotiations between involved parties, but because of transaction costs and several other real world problems this would only rarely be applicable. Suggestions have thus mainly concentrated on the design of environmental regulation able to realized a socially optimal level of pollution through the use of either tari¤s or tradeable permits. The main feature of this traditional analysis is the evaluation of the costs and bene…ts of existing and proposed regulations[1]. Following John at al.(1995) we can say that such analysis, being implicitly static, ignores two important aspects related to environmental problems. First, since environment is an asset which is passed on to future generations, environmental externalities are intra- as well as intergenerational: actions taken today a¤ect the welfare of future generations. Such external e¤ects are di¢cult to internalise and their existence alters the set of policies that are socially desirable. Second, the macroeconomic perspective is missing. Actions that a¤ect the environment both in‡uence and respond to macroeconomic variables, and environmental policy decisions have implications for economic growth and capital accumulation as showed by John and Pecchenino (1994) and Stokey (1998) among others.

In recent years, researchers have investigated the con‡ict between environmental preservation and economic growth in a dynamic setting. Examples of such analysis are John and Pecchenino (1994), Ono (1996), Bovemberg and De Mooj (1997) among the others.[2] A common result that arises in these models is that, from a welfare perspective, there is too much environmental degradation and economic growth is too high in an unregulated market economy, since economic agents do not take into account the environmental externality. Thus government, evaluating the e¤ect of negative externality in the optimization process, can design an optimal environmental policy using …scal instruments in order to reach a better intertemporal allocation of resources. However, the trade-o¤ between environmental quality and economic growth needs not to follow a monotonic path. Stokey (1998) has shown that under particular conditions on the intertemporal elasticity of consumption, the relation between output growth and environmental quality can follow a “U”-shaped curve.

[1] See Van Der Straaten (1998) for a critical analysis of the traditional theory of environmental policy

[2] For example, Bovemberg and De Mooj (1997) consider the e¤ects of environmental taxes on growth in a model with pre-existing distortionary taxes.

This implies that pollution tends to increase in the early stage of growth while it decreases once output is higher enough. The main limitation of Stokey's analysis can be found in the assumption of a representative agent economy. By assuming that the life span of individuals and the economy are the same, all these models restricted themselves to the analysis of the intragenerational con‡ict given by the existence of the well-understood free-rider problems within a generation. However, once dynamics is introduced in models of environmental policy, intergenerational issues become a predominant part of the analysis as pointed out by Solow (1986) which considers these problems in the …eld of economics of exhaustible natural resources. In fact, the overlapping generations approach allows intertemporal aspects to be disentangled from intergenerational considerations. In this paper we consider a discrete time overlapping generations model in which individuals care about environmental quality when they are old. When young, individuals can divide their income between consumption, investment in private capital and investment in environmental maintenance. Our main goal is to analyse the e¤ects of the presence of an environmental externality on capital accumulation and thus on long-run growth. Our model is closely related with the model of John and Pecchenino (1994). However, our analysis di¤ers from their one in several aspects. First of all, we consider a model in which individuals consume in both periods they are alive, while the analysis in John and Pecchenino (1994) abstracts from the consumption-saving decisions, since generations do not get any utility from consumption in the …rst period. Secondly, in our model, environmental quality is negatively a¤ected by production, and thus, by the saving decisions of previous generations. This creates an interegnerational externality that has important e¤ects on the capital accumulation of the economy under analysis. Moreover, these two elements a¤ect the design of an optimal environmental policy, since a policy derived in our framework will have di¤erent features with respect the one derived from the structure of John and Pecchenino's model.[3] We show that in a model with an environmental externality the dynamics of capital accumulation becomes richer and more complex than in standard overlapping generations models without externalities. In particular, the fact that the saving in private capital depends positively on the interest rate is not su¢cient to guarantee a positive capital growth. This is the case in an economy in which environmental quality is extremely low. More generally, a positive capital accumulation depends on the relative e¤ects that capital and environmental quality have on the capital investment and on the environmental preservation investment. Furthermore, these relative e¤ects on the saving functions will a¤ect the global stability of the steady state allocation. We show also that in our economy, the decentralised equilibrium tends to be ine¢cient, and thus, dynamic ine¢ciency can arise in our model in two ways. First, there is the possibility the individuals over invest in private capital or they can over invest in environmental preservation. In

[3] For an example of such a policy in the framework of John and Pecchenino's model, see Ono (1996), while Ono (2003) derive an optimal environmental policy using a model in which is production that deteriorates the environment.

both cases, a decrease in such investments can increase the consumption, and thus the welfare, of each generation. The paper is organised as follows. In Section 2 we present and discuss the main features of the model. In Section 3, we derive the main properties of a competitive equilibrium of our economy. In Section 4 we analyse in detail the steady state equilibrium and we derive the conditions for its global stability. Finally, in Section 4, we perform some welfare analysis in order to compare the equilibrium of the decentralised economy with the solution that would be attained by a social planner.

2 The Model

We consider a discrete time overlapping generations model as in Diamond (1965) in which at each period t a new generation is born. We assume no population growth and thus, we normalize the size of each generation to one agent[4]. Each generation lives for two periods. Preferences of each generation are defined over consumption in period t, c_{1t}, where c_{1t} denotes the consumption in young age, over consumption in old age, c_{2t+1}, and over an index of environmental quality in old age, E_{t+1}. These preferences are given by the following time-separable utility function: $U(c_{1t}) + \delta U(C_{2t+1}, E_{t+1})$, where $\delta \geq 0$ represents the discount factor.

A possible justification for the fact that agents care about environment when they are old could be found in the possible relationship between pollution and health costs, as in Williams (2002) and Gutierrez (2003). In their models, an increase in pollution will deteriorate consumers' health. Thus, in their models, consumers care indirectly about the environmental quality through the health costs that enter in the consumers' budget constraints. Agents born in period t are endowed with one unit of labour that they supply inelastically to firms and they receive a competitive wage w_t. They divide the wage into saving, s_t, consumption, c_t and investment in environmental maintenance m_t. In period $t+1$ they retire and supply their saving (s_t) to firms and earn the gross return $(1 + r_{t+1})$[5].

Assumption 1. The utility function $U(c_{1t}) + \delta U(C_{2t+1}, E_{t+1})$ is twice continuously differentiable with: $U'_{c_{1t}}(\cdot) > 0, U'_{c_{2t+1}}(\cdot) > 0, U'_{E_{t+1}}(\cdot) > 0$ and $U''_{c_{1t}}(\cdot) < 0, U''_{c_{2t+1}}(\cdot) < 0, U''_{E_{t+1}}(\cdot) < 0$, and $U_{c_{1t+1}, E_{t+1}} \geq 0$. Furthermore, we assume that $\lim_{c \to 0} [U(c) + \delta U(c, E)] = \infty$ and $\lim_{E \to 0} [U(c) + \delta U(c, E)] = \infty$;

The firms are perfectly competitive and have access at the same technology given by the following production function: $F_t(K_t, L_t)$, where K_t is the stock of capital and L_t is the labour supply. We assume that $F_t(\cdot)$ displays constant return to scale,

[4] Since we are interested in intergenerational issues, as in John and Pecchenino (1994), we abstract from the well-known intragenerational free-rider problem.

[5] In our model we are implicitly assuming that there exists a generation of old in period 0 that is endowed with k_0 units of capital that is supplied indelastically to firms.

thus, we can rewrite it as $f_t(k_t)$, where k_t is the usual capital/labour ratio. Capital fully depreciate after one period.

Assumption 2. The production function is twice continuously differentiable with: $f' > 0, f'' \leq 0$ and $kf''(k) + f'(k) > 0$. Furthermore, we have that $\lim_{k \to \infty} f'(k) = 0, \lim_{k \to 0} f'(k) = \infty$ and $f(0) = 0$;

Environmental quality is a public good that is affected negatively by human activity. In particular, we assume that the environmental quality is a decreasing function of the production activity of the previous period. However, each generation can decide to invest in maintenance or improvement of the environment when they are young and this affects positively the environmental quality.

Following John and Pecchenino (1994), we consider the evolution of the environmental quality as a first-order difference equation given by[6]:

$$E_{t+1} = \alpha E_t - \beta f(k_t) + \gamma m_t \tag{1}$$

where $\beta, \gamma > 0$ are exogenous parameters that measure the effects on the environmental quality of the production activity and of the investment in maintenance respectively. While $\alpha \in (0, 1)$ measures the degree of persistence of the environmental quality.[7] The initial level $E_0 > 0$ is given. If $\beta < \gamma$, then the investment in maintenance is efficient. Possible interpretations of E_t can be the quality of soil, the quantity of national parks or the inverse of the concentration in the atmosphere of greenhouse gases.

Differently from John and Pecchenino (1994), we consider the case in which environmental quality is negatively correlated with production. We use this different specification because we are interested on the particular interaction between capital accumulation and the evolution of the environmental externality.

2.1 Saving Decisions and Profit Maximization

Each generation maximises the intertemporal utility function $U(c_{1t}) + \delta U(C_{2t+1}, E_{t+1})$, subject to the evolution of environmental quality given by 1) and of the following constraints:

$$\begin{aligned} c_{1t} &= w_t - s_t - m_t \\ c_{2t+1} &= (1 + r_{t+1}) s_t \\ c_{1t}, c_{2t+1}, s_t, m_t &\geq 0 \end{aligned} \tag{2}$$

[6] This specification for the evolution of the environmental quality has been widely used in the recent literature. See for example, Ono (1996, 2003), Jouvet et al. (2000) and Gutierrez (2003) among the others.

[7] Since $\alpha \in (0, 1)$ if there is no human activity, the environmental quality tends to a an autonomous level in which $E = 0$, and α measures the speed of this natural process.

Given assumption 1, the problem above admits a solution and the first order conditions are:

$$U'_{c_{1t}}(c_{1t})/U'_{c_{2t+1}}(c_{2t+1}, E_{t+1}) = \delta(1 + r_{t+1}) \tag{3}$$
$$U'_{c_{1t}}(c_{1t})/U'_{E_{t+1}}(c_{2t+1}, E_{t+1}) = \delta\gamma$$

Supposing that $s_t, m_t > 0$, these two conditions give us a simple arbitrage condition between the rate of return on the private saving, $(1 + r_{t+1})$, and the rate of return on the investment in environmental improvement, γ, that is $U'_{c_{2t+1}}(\cdot)/U'_{E_{t+1}}(\cdot) = \gamma/(1 + r_{t+1})$. This condition says that consumers choose s_t and m_t in order to equate the marginal rate of substitution between consumption and environmental quality with the marginal rate of transformation.[8] In our model we impose that $s_t > 0$, implying that $c_{t+1} > 0$, while we maintain the possibility that $m_t \geq 0$. Conditions 3) implicitly define an optimal aggregate saving function $S_t^* \equiv S(w_t, r_{t+1}, E_t, f(k_t)) = s_t^* + m_t^*$ as a function of the wage rate, w_t, the rate of return on private saving, r_{t+1}, the environmental quality E_t and the capital level at time t through the production function $f(\cdot)$. Note that $S(\cdot) \in (0, w_t]$. Suppose we can discriminate between s_t and m_t inside the function S_t^*, and denote the optimal private saving function s_t^* and the optimal investment in environmental maintenance m_t^* implied by conditions 3), by:

$$s_t^* = s(w_t, r_{t+1}, E_t, f(k_t)) \tag{4}$$
$$m_t^* = m(w_t, r_{t+1}, E_t, f(k_t))$$

We can notice that first order conditions 3) give us a system of implicit functions of the form $s_t = G(w_t, r_{t+1}, E_t, f(k_t), m_t)$ and $m_t = H(w_t, r_{t+1}, E_t, f(k_t), s_t)$ from which we derived equations 4). we can show that the saving functions in 4) have the following properties:[9]

$$s_w \equiv \frac{\partial s_t}{\partial w_t} > 0; s_r \equiv \frac{\partial s_t}{\partial r_{t+1}} \gtrless 0; s_E \equiv \frac{\partial s_t}{\partial E_t} > 0; s_{f(k)} \equiv \frac{\partial s_t}{\partial f(k_t)} < 0; \tag{5}$$
$$m_w \equiv \frac{\partial m_t}{\partial w_t} > 0; m_r \equiv \frac{\partial m_t}{\partial r_{t+1}} \gtrless 0; m_E \equiv \frac{\partial m_t}{\partial E_t} < 0; m_{f(k)} \equiv \frac{\partial m_t}{\partial f(k_t)} > 0;$$

[8] This is the same condition found in John and Pecchenino (1994). Note that the Samuelson condition for the optimal provision of public good is satisfied in our framework, since the size of each generation has been normalised to one.

[9] Using Assumption 1 and 2, and deriving implicitly conditions 3), we can show that the functions $G(\cdot)$ and $H(\cdot)$ have the following properties:

$G_w \in (0,1); G_r \gtrless 0; G_E > 0; G_{f(k)} < 0$ and $G_m \gtrless 0$ and $G_m \in (-1,1)$;

$H_w \in (0,1); H_r \gtrless 0; H_E < 0; H_{f(k)} > 0$ and $H_s \gtrless 0$ and $H_s \in (-1,1)$;

Using these facts, and assuming that $G_m < 0$ and $H_s < 0$ into conditions 3) and applaying again the Implicit Function Theorem, we get the properties in the article.

The …rst two comparative statics properties for each function stated above are standard in overlapping generations models. The aggregate saving function is increasing in the wage rate, and furthermore $s_w, m_w \in (0,1)$. Moreover, the e¤ects of the rate of return r_{t+1} is uncertain, since it depends on the substitution and income e¤ects caused by a change in r_{t+1}. If the intertemporal elasticity of substitution between c_t and c_{t+1} is greater than one, then $S_r > 0$.[10] The private saving depends positively on the environmental quality, while it depends negatively on the production level in period t. Private saving depends positively on the environmental quality because higher is the environmental quality and lower will be the investment in maintenance, and this implies an higher level for the private saving. On the other hand, private saving depends negatively on the production in period t because an higher production implies a degradation of the environment and thus an higher investment in maintenance.[11] The investment in maintenance m_t is a decreasing function of the environmental quality. This is because when environmental quality is higher, there is no need to invest many resources in its maintenance. Finally, m_t is increasing in the production at period t, since higher is the production level and worse will be the environmental quality, and then, higher should be the investment in maintenance.

Firms are identical and perfectly competitive. The production function is $f_t(k_t)$, where k_t is the usual capital/labour ratio. Maximization of the pro…ts, together with Assumption 2, imply the following …rst order conditions:

$$\begin{aligned} r_t &= f'(k_t) \\ w_t &= f(k_t) - f'(k_t)k_t \end{aligned} \tag{6}$$

Conditions 6) are the standard conditions stating that …rms hire capital and labour until their marginal products equal their factor prices.

3 The Competitive Equilibrium

In the previous section we have analysed the behaviour of …rms and consumers and we have derived the optimal saving functions and the conditions for pro…t maximization. In this section we de…ne the competitive equilibrium of our model and we shall analyse in more details the dynamic of capital accumulation and environmental quality along an equilibrium path.

[10] This is the necessary and su¢cient condition for $S_r > 0$ in standard OLG models. See Blanchard and Fisher (1989), Ch. 3, for details on this point.

[11] Note that $\frac{\partial s_t}{\partial E_{t+1}} > 0$ by implict di¤erentiation of the …rst order conditions in 3). Of course the fact that $\partial s_t/\partial f(k_t) < 0$ it does not imply that $\partial s_t/\partial k_t < 0$.

The goods market clears when the capital stock at time $t+1$, k_{t+1}, equals the private saving decision taken at time t, s_t^*, that is:[12]

$$k_{t+1} = s\left(w_t, r_{t+1}, E_t, f(k_t)\right) \tag{7}$$

While equations 6) give us the equilibrium conditions for the market of productive factors.

Given these market clearing conditions, we can define a competitive equilibrium for our economy as follows

Definition 1 A competitive equilibrium for the economy under analysis is a sequence $\{c_{1t}^*, c_{2t}^*, r_t^*, w_t^*, s_t^*, m_t^*, k_t^*, E_t^*\}_{t=0}^{\infty}$ such that: i) firms maximize profits; ii) consumers maximize their utility function; iii) markets clear given the initial conditions on the state variables $\{k_0, E_0\}$.

The competitive equilibrium of our model can be summarised by the following set of equations:

$$\begin{aligned}
c_{1t}^* &= w_t^* - S\left(w_t^*, r_{t+1}^*, E_t^*, f(k_t^*)\right) \\
c_{2t+1}^* &= (1 + r_{t+1}^*) s(w_t^*, r_{t+1}^*, E_{t+1}^*) \\
S\left(w_t^*, r_{t+1}^*, E_t^*, f(k_t^*)\right) &= s(w_t^*, r_{t+1}^*, E_t^*, f(k_t^*)) + m\left(w_t, r_{t+1}, E_t, f(k_t^*)\right) \\
s(w_t^*, r_{t+1}^*, E_t^*, f(k_t^*)) &= k_{t+1} \\
w_t^* &= f(k_t) - f'(k_t)k_t \\
r_{t+1}^* &= f'(k_{t+1}) \\
E_{t+1}^* &= \alpha E_t^* - \beta f(k_t^*) + \gamma m\left(w_t^*, r_{t+1}^*, E_t^*, f(k_t^*)\right)
\end{aligned} \tag{8}$$

Now we start studying the dynamic of the capital accumulation and of the environmental quality.

Substituting conditions 6) into equation 7) we obtain that the capital stock in period $t+1$ evolves according to:

$$k_{t+1} = s\left(f(k_t) - f'(k_t)k_t, f'(k_{t+1}), E_t, f(k_t)\right) \tag{9}$$

which is a non-linear dynamic equation that define implicitly k_{t+1} as a function of k_t and E_t, with k_0 and E_0 given. The complete dynamic of the model is closed considering the evolution of the other state variable that is E. The evolution of E is given by:

$$E_{t+1} = \alpha E_t - \beta f(k_t) + \gamma m\left(f(k_t) - f'(k_t)k_t, f'(k_{t+1}), E_t, f(k_t)\right) \tag{10}$$

[12] Since we have normalised the size of each generation to one, the per-capita capital stock and the aggregate capital stock at each period coincide.

Equations 9)-10) form a system of non-linear first order difference equations that describe the dynamic of the capital accumulation and the evolution of the environmental quality along the competitive equilibrium path of the model.

Equation 10) defines E_{t+1} as a function of E_t, k_{t+1} and k_t. Define this function as:

$$E_{t+1} = \Psi(E_t, k_{t+1}, k_t) \tag{11}$$

The following proposition states the main properties of the function $\Psi(\cdot)$:

Proposition 1 The function $\Psi(\cdot)$ has the following properties: i) $\Psi_{E_t} \gtreqless 0$ if and only if $\alpha \gtreqless -\gamma m_E$; ii) $\Psi_{k_t} \lesseqgtr 0$ if and only if $\gamma\left(m_{f(k)}f'(k_t) - m_w f''(k_t)k_t\right) \lesseqgtr \beta f'(k_t)$.

Proof. We simply differentiate equation 10), taking into account Assumption 2 and the properties of the function m derived previously. ∎

The first result in Proposition 1 is quite intuitive. It says that environmental quality increases over time only if the parameter that measure its natural speed α is higher than the negative impact of the investment in maintenance, since we know that m_E is negative. That condition simply implies that $|\gamma m_E| < \alpha$. The second result says that E_{t+1} increases in k_t only if the marginal negative effect of k_t on E_{t+1}, given by $\beta f'(k_t)$, is less than the marginal benefits that k_t has on E_{t+1} through the maintenance investment, given by $\gamma\left(m_{f(k)}f'(k_t) - m_w f''(k_t)k_t\right)$.[13] The third results simply state that environmental quality is increasing in the saving decisions of generation t only if the maintenance investment is decreasing in the interest rate.

The law of capital accumulation 8) can be written as:

$$k_{t+1} = s\left(f(k_t) - f'(k_t)k_t, f'(k_{t+1}), E_t, f(k_t)\right) \equiv \Phi(E_t, k_t) \tag{12}$$

The properties of the function $\Phi(\cdot)$ are stated in the following proposition:

Proposition 2 Suppose that $s_r > 0$, then the function $\Phi(\cdot)$ has the following properties: i) $\Phi_{k_t} > 0$ if and only if $(-s_w f''(k_t)k_t + s_{f(k_t)}f'(k_t)) > 0$; ii) $\Phi_{E_t} > 0$.

Proof. To prove result i) we need to differentiate implicitly equation 12) and then, using Assumption 2 and the properties of the private saving function s_t. The derivative we are looking for is

$\partial k_{t+1}/\partial k_t = (-s_w f''(k_t)k_t + s_{f(k_t)}f'(k_t))/(1 - s_r f''(k_{t+1}))$. The denominator is positive if $s_r > 0$, while the numerator is positive if and only if $-s_w f''(k_t)k_t + s_{f(k_t)}f'(k_t) > 0$, where we have that $s_{f(k_t)} < 0$ and $f''(k_t) < 0$, then, the result follows. To prove the result ii) we follow the same steps, and we obtain $\partial k_{t+1}/\partial E_t = s_E/(1 - s_r f''(k_{t+1}))$ that is clearly positive. ∎

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[13] Note that $-m_w f''(k_t)k_t$ is a positive number, since $f''(k_t) < 0$. Note also that $-m_w f''(k_t)k_t$ is the equilibrium effect of a change in k_t on m_w. The same holds for $m_{f(k)}f'(k_t)$.

The first result in Proposition 2 is very important because it says under which condition our economy accumulates capital over time. In a standard overlapping generations model without an environmental externality, the condition $s_r > 0$ is sufficient to guarantee that the economy accumulates capital over time and thus, that each new generation is better off with respect the previous one, because this capital accumulation implies higher wages at each period[14]. On the other hand, when there is an environmental externality that depends negatively on the lagged value of the production, there are more possible scenarios for the evolution of capital even if we maintain the assumption that $s_r > 0$. In fact, Proposition 2 says that capital accumulation is positive (ª$_{k_t} > 0$) if and only if the positive marginal effect of a change in k_t (that is decided by the previous generation) on the saving function s_t of the present generation, given by the effect of k_t on the wage rate w_t, is greater than the marginal negative effect of k_t on s_t through the environmental externality ($s_{f(k_t)} f'(k_t)$).[15] However, if the initial level of capital is high while the level of environmental quality is low, it is possible that capital decreases over time until the level of environmental is high enough to stimulate private saving. In order to have a convergent growth of capital we need also to assume that $(-s_w f''(k_t)k_t + s_{f(k_t)} f'(k_t)) < 1 - s_r f''(k_{t+1})$, because only in that case we have $0 <$ ª$_{k_t} < 1$. The second result of Proposition 2 is related with the interaction between capital accumulation and the evolution of the environmental externality. We have that higher is the environmental quality and higher will be the level of capital at each period of time. This fact is consistent with observations that relatively poor countries experience higher environmental degradation than developed countries. Thus, in our model, an higher level of output can be associated with an high level of environmental quality because countries with higher output can invest more resources in environmental improvement. This result is also consistent with the idea that the relationship between per-capita income and environmental quality follows a "U"-shaped curve, in which there is environmental degradation in the first part of the growing path of the economy, but when capital is sufficiently high, environmental quality tends to increase.[16] Furthermore,this result confirms the result of John and Pecchenino (1994) even if we are considering the case in which production, and not consumption, affects negatively the environmental quality but differs from the one found in Gutierrez (2003). This is because, in our model, as in John and Pecchenino (1994), consumers invest in maintenance when they are young, therefore an increase in environmental quality increases their private

[14] See Blanchard and Fisher (1989), Ch.3, for a detailed analysis of the standard overlapping generation models.

[15] We can see that the same inequality holds if $s_w / s_{f(k_t)} < f'(k_t) / (k_t f''(k_t))$. The term $f'(k_t) / (k_t f''(k_t))$ reflects the reciprocal of the curvature of the production function. We can notice that if $f(k_t)$ is quasi linear, meanig that the marginal productivity of capital is nearly constant, the term $f'(k_t) / (k_t f''(k_t))$ tends to be very high, so if $s_{f(k_t)} \neq 0$, the economy will accumulate capital over time.

[16] For a theoretical model that implies an "U"-shaped curve for environmental quality see Stokey (1998).

saving and thus future capital. In contrast, in Gutierrez (2003), higher environmental quality decreases health costs for old people, and thus, consumers can decrease saving when they are young.[17]

4 The Steady State

In this section we shall analyse in detail the properties of the dynamic system given by 8) and 9). We start with the derivation of the steady state equilibrium of our model, that is an allocation in which capital and environmental quality remain constant over time. Define with $\{\overline{k}, \overline{E}\}$ the steady state levels of the capital and the environmental quality, then in a stationary equilibrium path, the system given by 7) and 8) becomes:

$$\overline{k} = s\left(f(\overline{k}) - f'(\overline{k})\overline{k}, f'\left(\overline{k}\right), \overline{E}, f(k)\right) \tag{13}$$

$$\overline{E} = \frac{\gamma}{(1-\alpha)} m\left(f(\overline{k}) - f'(\overline{k})\overline{k}, f'\left(\overline{k}\right), \overline{E}, f(k)\right) - \frac{\beta f(\overline{k})}{(1-\alpha)} \tag{14}$$

We need to analyse the stability properties of the system 13)-14). In particular, we want to show under which conditions that system is globally stable, that is, it converges towards a steady state equilibrium independently of the initial conditions. The following proposition states the conditions for global stability of the system 13)-14):

Proposition 3 Suppose that $0 < \Psi_{\overline{k}} < 1$ and $\Phi_{\overline{E}} > 0$, then: i) when $\Phi_{\overline{k}} > 0$, the steady-state equilibrium $\{\overline{k}, \overline{E}\}$ of 13)-14) is asymptotically stable if $\Psi_{\overline{k}}/\Phi_{\overline{k}} > \Psi_{\overline{E}}/\Phi_{\overline{E}}$; ii) the steady-state has the following comparative statics features: $\partial\overline{k}/\partial\beta < 0$; $\partial\overline{k}/\partial\gamma > 0$; $\partial\overline{E}/\partial\beta < 0$ and $\partial\overline{E}/\partial\gamma > 0$.

Proof. We need to linearize the system 10)-12) around the steady-state solution $\{\overline{k}, \overline{E}\}$. The linearized system becomes

$$\begin{bmatrix} k_{t+1} - \overline{k} \\ E_{t+1} - \overline{E} \end{bmatrix} = P \begin{bmatrix} k_t - \overline{k} \\ E_t - \overline{E} \end{bmatrix}$$

where $P = \begin{bmatrix} \frac{-s_w f''(\overline{k})\overline{k} + s_{f(\overline{k})} f'(\overline{k})}{A} & \frac{s_{\overline{E}}}{A} \\ \gamma\left(m_{f(k)} f'(k_t) - m_w f''(k_t) k_t\right) - \beta f'(k_t) & \alpha + \gamma m_E \end{bmatrix}$

and $A = 1 - s_r f''(\overline{k}) > 0$ if $s_r > 0$.

A steady state $\{\overline{k}, \overline{E}\}$ is asymptotically stable if the following inequalities

[17] Furthermore, in Gutierrez (2003), is the production at time $t+1$ that affects the environmental quality at time $t+1$, while in our model is the productionat time t that affects the environment. This creates a more complex externality process respect to her model.

hold:[18] a) $|\det(P)| < 1$ and b) $|\det(P) + 1| < Tr(P)$; where $Tr(P)$ is the trace of P.

Proof of i): Suppose $\Psi_{\overline{k}} > 0$, the determinant of P is given by $\frac{\left(-s_w f''(\overline{k}) + s_{f(\overline{k})} f'(\overline{k})\right)}{A}(\alpha + \gamma m_E) - \frac{s_E\left(\gamma m_{f(k)} f'(k_t) - \gamma m_w f''(k_t)k_t - \beta f'(k_t)\right)}{A}$.

Since $1/A$ is positive under the assumption that $s_r > 0$,we can show that if $\Phi_{\overline{k}}/\Psi_{\overline{k}} > \Phi_{\overline{E}}/\Psi_{\overline{E}}$, that determinant is always between 0 and 1. The determinant of P is positive if $\frac{\left(-s_w f''(\overline{k}) + s_{f(\overline{k})} f'(\overline{k})\right)}{\left(\gamma m_{f(k)} f'(k_t) - \gamma m_w f''(k_t)k_t - \beta f'(k_t)\right)} > \frac{s_E}{(\alpha+\gamma m_E)}$ that is exactly $\Phi_{\overline{k}}/\Psi_{\overline{k}} > \Phi_{\overline{E}}/\Psi_{\overline{E}}$. Given that the determinant is positive, in order to have condition a) satisfied, we need $\frac{\left(-s_w f''(\overline{k}) + s_{f(\overline{k})} f'(\overline{k})\right)}{A}(\alpha + \gamma m_E) < 1 + \frac{s_E\left(\gamma m_{f(k)} f'(k_t) - \gamma m_w f''(k_t)k_t - \beta f'(k_t)\right)}{A}$ but given the assumptions stated in the Propositions, we have that $\frac{\left(-s_w f''(\overline{k}) + s_{f(\overline{k})} f'(\overline{k})\right)}{A} < 1$ and $(\alpha + \gamma m_E) < 1$, their product has to be less than one as well, therefore we have that $0 < \det(P) < 1$.

To prove condition b), we proceed in two steps, since that condition implies: $|\det(P)| + Tr(P) + 1 > 0$ and $|\det(P)| - Tr(P) + 1 > 0$. Since we know that $|\det(P)|$ is positive, to prove the first statement we just need to show that $Tr(P) > 0$. Since $Tr(P) = \frac{\left(-s_w f''(\overline{k}) + s_{f(\overline{k})} f'(\overline{k})\right)}{A} + (\alpha + \gamma m_E)$, this expression is positive under the assumptions stated in the Proposition. Finally we show that $|\det(P)| - Tr(P) + 1 > 0$. Under our assumptions we can check that $-1 < \det(P) - Tr(P) < 0$, and therefore $\det(P) - Tr(P) + 1 > 0$.

Proof of ii) We implicitly differentiate equations 13) and 14), and thus, under the assumptions $s_r > 0, m > 0, f(\overline{k}) > 0$, $\Psi_{k_t} > 0$ and $\Phi_{k_t} > 0$, we obtain:

$$\frac{\partial \overline{k}}{\partial \beta} = \frac{-s_E f(\overline{k})}{1 - s_w f''(\overline{k})\overline{k} - s_r f''(\overline{k}) - s_{f(\overline{k})} f'(\overline{k})} < 0;$$

$$\frac{\partial \overline{k}}{\partial \gamma} = \frac{s_E m}{1 - s_w f''(\overline{k})\overline{k} - s_r f''(\overline{k}) - s_{f(\overline{k})} f'(\overline{k})} > 0;$$

$$\frac{\partial \overline{E}}{\partial \beta} = \frac{\frac{\gamma}{1-\alpha} m_E f(\overline{k})}{\frac{(1-\alpha) - \gamma m_E \alpha}{1-\alpha}} < 0; \frac{\partial \overline{E}}{\partial \gamma} = \frac{-\frac{\gamma}{1-\alpha} m_E f(\overline{k})}{\frac{(1-\alpha) - \gamma m_E \alpha}{1-\alpha}} > 0$$

[18] We can notice that those inequalities are simply the discrete time version of the Ruth-Hurwicz conditions for the case of a 2×2 dynamic system. Those conditions imply that the two eigenvalues associated with the matrix P are both inside the unit circle.See for example Azariadis (1993, pp. 62-67) and Gutierrez (2003).

where, given the assumption above and the comparative statics properties of functions s and m, we have that

$1 - s_w f''(\bar{k})\bar{k} - s_r f''(\bar{k}) - s_{f(\bar{k})} f'(\bar{k}) > 0$ and $\frac{(1-\alpha) - \gamma m_E \alpha}{1-\alpha} > 0$ ■

Proposition 3 gives us the condition under which we can have a steady state equilibrium that is globally stable. Given the assumptions in Proposition 3, the stable steady state equilibrium is the one with high capital level and with high environmental quality. The condition for global stability is $\Psi_{\bar{k}}/\Phi_{\bar{k}} > \Psi_{\bar{E}}/\Phi_{\bar{E}}$. The ratio $\Psi_{\bar{k}}/\Phi_{\bar{k}}$ measures the relative effect of a change in the capital level on the saving functions $s(\cdot)$ and $m(\cdot)$ in steady state, while the ratio $\Psi_{\bar{E}}/\Phi_{\bar{E}}$ measures the relative effect of a change in the environmental quality on the same two saving functions. That condition simply states that the relative effect of a change in the capital level has to be greater than the relative effect of a change in the environmental quality on the saving functions in equilibrium.[19] If this is the case the steady state equilibrium with a high level of capital and a high level of environmental quality is asymptotically stable. The intuition behind this result is again that, if the saving function $s(\cdot)$ is more sensitive to capital accumulation than to the environmental quality, the economy will accumulate more resources in each period, but this will imply that there will be more resources available also to improve the environment as well. We can see that this stability condition implies the one in John and Pecchenino (1994). Indeed, in their model the steady state is stable when $\Psi_{\bar{k}} > \Phi_{\bar{k}}$. Finally, we did not say anything about multiplicity of equilibria that normally arise in overlapping generations models. In fact, the dynamic system is given by a couple of nonlinear equations, thus, multiple equilibria can easily arise in our analysis. Indeed, the dynamic of our model can be much complex as showed by Zhang (1999). However, if the assumptions in Proposition 3 hold, we can say that there is at least one stable steady state equilibrium towards which the economy tends to move and that this steady-state is characterised by high levels of capital and environmental quality.[20]

5 Welfare Analysis

In the previous section we have derived the steady-state properties of the decentralised competitive equilibrium. It is well known from the seminal paper of Samuelson (1965) that generically, the decentralised solution in overlapping generations models without externalities is not Pareto-optimal. The reason is that there is the possibility that consumers invest more than the level associated with the golden rule, implying that

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[19] We remark that to derive this condition we assume $\Phi_{\bar{k}} > 0$, that is $\gamma \left(m_{f(k)} f'(k_t) - m_w f''(k_t) k_t \right) > \beta f'(k_t)$.

[20] Zhang (1999), using the same specification of John and Pecchenino (1994), showed that if the maintenance efficiency relative to environmental degradation is not sufficiently high, cyclically or chaotically fluctuating equilibria are more likely to exists. This implies that the transition towards an environmentally sustainable state is not trivial.

each generation can be better off by reducing the level of capital and thus, increasing consumption. Economies in such a situation are characterised by dynamic inefficiency.[21] In order to find if our economy can be dynamically inefficient, we need to compare the decentralised equilibrium with the equilibrium that would be chosen by a social planner, since we know that such a solution will be Pareto-efficient.[22] The objective function of the social planner is:

$$U^S = \delta U(c_{20}, E_0) + \sum_{t=1} (1 + R)^{-t-1} [U(c_{1t}) + \delta U(c_{2t+1}, E_{t+1})] \tag{15}$$

where $R \geq 0$ is the discount rate of the social planner.[23] The constraints the social planner faces in period t are:

$$f(k_t) + k_t = c_{1t} + c_{2t} + k_{t+1} + m_t \tag{16}$$

$$E_t = \alpha E_{t-1} - \beta f(k_{t-1}) + \gamma m_{t-1} \tag{17}$$

Equation 16) is the resource constraint that says that the supply of goods at time t should be allocated to the consumption of young and old, maintenance in environmental quality and providing the capital in period $t+1$. Equation 17) is simply the evolution of the environmental quality. Social planner maximizes U^S with respect c_{1t}, c_{2t}, k_t and m_t, subject to the constraints 16) and 17). The first order conditions for this problem are:

$$\begin{gathered} \delta U_{c_{2t}}(c_{2t}, E_t) = (1 + R)^{-1} U_{c_{1t}}(c_{1t}) \\ \gamma \delta U_{E_{t+1}}(C_{2t+1}, E_{t+1}) = U_{c_{1t}}(c_{1t}) \\ U_{c_{1t-1}}(c_{1t-1}) = \delta \left(1 + f'(k_t)\right) U_{c_{2t}}(c_{2t}, E_t) - \beta f'(k_t)(1 + R)^{-1} U_{E_{t+1}}(c_{2t+1}, E_{t+1}) \end{gathered} \tag{18}$$

The first condition in 18) is standard in overlapping generation models, it simply states that optimal allocation of resources between young and old alive in the same period.[24] The second condition says that the social planner chooses the investment in environmental maintenance in order to equate the marginal rate of substitution

[21] See Blanchard and Fisher (1989), Ch.3 for an introduction to this problem.

[22] See Diamond (1965) on the reasons why the social planner solution is Pareto-optimal in overlapping generations economies.

[23] If $R = 0$ then the social planner treats all the generations symmetrically, while if $R > 0$ then, it cares less about future generations. Notice that with $R = 0$ the sum in the objective function needs not be finite, thus, it does not converge. Note also that we have included in the objective function the initial generation of old.

[24] In general, this condition implies that the marginal rate of substitution between consumption of young and old should be equal to the marginal rate of transformation $(1 + n)$, where n is the growth rate of population.

between consumption and environmental quality with the term $\gamma\delta$. As we can see this is exactly the same first order condition in 3). Thus, the social planner behaves exactly like each consumer in deciding the optimal amount of m_t. The last condition is the most interesting. It gives us the optimal condition for the allocation of capital from the social planner point of view. We can notice that condition differs from the one we found in the decentralised context, and the difference is given by the term $-\beta f'(k_t)(1+R)^{-1}U_{E_{t+1}}(c_{2t+1}, E_{t+1})$. Thus, the social planner in deciding the optimal allocation of capital takes into account the negative effect of the production on the environmental quality, and thus, on the utility function of the next generation. This means that in a decentralised economy, each generation tends to accumulate more capital than the social desirable level implied by the last condition in 18). In order to find the golden rule for capital accumulation, we need to consider the optimal allocation implied by 18) in steady state, where $c_{2t} = c_{2t+1} = c_2, c_{1t-1} = c_{1t} = c_1, E_t = E_{t-1} = E$, and $k_t = k$, when the discount factor of the social planner is equal to zero. Conditions 18) become:

$$\begin{aligned} \delta U_{c_2}(c_2, E) &= U_{c_1}(c_1) \\ \gamma\delta U_E(C_2, E) &= U_{c1}(c_1) \\ U_{c_1}(c_1) &= \left(1 + f'(k)\right) U_{c1}(c_1) - \beta f'(k) U_E(c_2, E) \end{aligned} \tag{18}$$

where in the last condition we have used the fact that $\delta U_{c_2}(c_2, E) = U_{c_1}(c_1)$. Using the second condition as well in the last one, we have:

$$0 = f'(k)\left(1 - \frac{\beta}{\gamma\delta}\right) \tag{19}$$

Equation 19) give us the optimal steady state level of capital. Without the environmental externality, equation 19) is simply the golden rule of a standard overlapping generations model when there is no population growth and capital fully depreciate after each period.[25] As in standard overlapping generations model, dynamic inefficiency can be a feature of our model, since it is possible that individuals accumulate more capital in steady state than the level implied by the golden rule. However, in an economy with investment in environmental quality, there is another source of dynamic inefficiency, that is, individuals may invest too much in environmental preservation, implying that they can be better off by maintaining less and consuming more. When we are in such a situation, Pareto-improving policies are possible.[26] However, comparing the first order conditions of the decentralised economy with the ones of the

[25] In standard overlapping generations models, the golden rule for the optimal steady state capital is given by $f'(k_t) = n$, where n is the growth rate of population, that in our model is zero. Furthermore, without population growth, but with capital that depreciates at the rate λ, the golden rule becomes $f'(k_t) = \lambda$. See Blanchard and Fisher (1989) Ch. 3.

[26] See John and Pecchenino (1994) for a detailed analysis of the dynamic innefficiency problem.

social planner, we can see that the optimal decision about the level of maintenance is the same in both cases.[27] This implies that the only source of dynamic ine¢ciency in our framework is given by the optimal level of investment in private capital. Thus, only policies that a¤ects the level of private investment can be Pareto improving.

Recent literature has focused on optimal tax policies that internalise environmental externalities and attain an e¢cient allocation of resources in the decentralised economy. For example, Ono (1996), using a model in which environment is negatively a¤ected by consumption of previous generations, has shown that there are di¤erent …scal schemes that can allow us to reach the social optimum. In Ono (2003), a similar analysis is performed on a model in which environmental degradation is caused by production.

6 Introducing a Pay-as-you-go Security System

In the previous section we demonstrated that in our model each generation tends to invest more in capital goods than the desirable social level. This is because each generation, in deciding the optimal level of investment, does not take into account the negative e¤ect that production has on the future environmental quality, and thus, on the utility function of future generations. Thus, Pareto improving policies are possible in our framework. Given the fact that the problem is an over-investment in productive capital, there can be di¤erent possibilities to intervene in such a situation. We consider a simple policy based on the introduction of a social security system in our economy. In particular, we introduce a simple pay-as-you-go security system.

The link between social security systems and the investment in environmental preservation has been recently considered by Rangel (2003). Using an overlapping generations model and the concepts of forward and backward intergenerational goods, Rangel showed that is possible to improve the environmental quality for future generations by creating a link between the investment in environmental maintenance by present generations and a pay-as-you-go security system. The main reason why this intergenerational exchange can work is that the contributions to the social security system can be used by future generations as a threat to force present generations in investing in environmental quality.[28] Di¤erently, in our framework, there is no need to create a direct link between a social security system ad the investment in environmental maintenance, since we considered a model in which each generation cares about environmental quality in the second period of life.[29] However, the introduction of a pay-as-you-go system can mitigate the e¤ects of the environmental

[27] Note that this my not be true in the model of John and Peccheino (1994) where the environmental damage is caused by the consumption avtivity and not by the production.

[28] In particular, Rangel (2003) showed that this kind of trigger strategy is a subgame perfect equilibrium of his model.

[29] Rangel (2003) analysed a situation in which present generations do not care about the quality

externality that we are considering, because it affects the private saving. Since the presence of an environmental externality in our model has the main effect to create an over-investment in capital goods, the introduction of a pay-as-you-go security system in economies that are dynamically inefficient represents a Pareto-improving policy.[30]

In order to introduce a social security system we also introduce a rate of growth for the population.[31] The number of people alive in period t is denoted by N_t and the population growth at the rate n. Thus, we have that in period t, $N_t = (1+n)^t N_0$, where N_0 is the initial number of people alive in period 0. We denote by b_t the contribution to the social security system by the young at time t. The rate of return on the pay-as-you-go system is simply the gross rate of growth of the population $(1+n)$. The budget constraints faced by each generations become

$$c_{1t} = w_t - s_t - m_t - b_t \tag{20}$$

$$c_{2t+1} = (1 + r_{t+1})s_t + (1+n)b_{t+1} \tag{21}$$

The evolution of the environmental quality now becomes:

$$E_{t+1} = \alpha E_t - \beta F(k_t) + \gamma(1+n)m_t \tag{22}$$

where $F(k_t)$ is the aggregate production as a function of the capital/labour ratio. The first order conditions for this problem are

$$\begin{aligned} U'_{c_{1t}}(c_{1t})/U'_{c_{2t+1}}(c_{2t+1}, E_{t+1}) &= \delta(1 + r_{t+1}) \\ U'_{c_{1t}}(c_{1t})/U'_{E_{t+1}}(c_{2t+1}, E_{t+1}) &= \delta\gamma(1+n) \end{aligned} \tag{26}$$

Conditions 26) give us the optimal saving function s_t^* and the optimal level of environmental maintenance m_t^*. The equilibrium condition on the capital market is now given by $(1+n)k_{t+1} = s_t^* \ \forall t$. In order to see the effect of introducing a pay-as-you-go security system in our model we need to differentiate conditions 26) with respect the social contribution b, assuming that $b_t = b_{t+1} = b$.

The derivatives are:

$$\frac{\partial s_t^*}{\partial b} = -\frac{U''_{c_{1t}}(\cdot) + \delta(1 + r_{t+1})(1+n)U''_{c_{2t+1}}(\cdot)}{U''_{c_{1t}}(\cdot) + \delta(1 + r_{t+1})^2 U''_{c_{2t+1}}(\cdot)} \tag{27}$$

$$\frac{\partial m_t^*}{\partial b} = -\frac{U''_{c_{1t}}(\cdot) + \delta(1 + r_{t+1})(1+n)U''_{c_{2t+1}}(\cdot)}{U''_{c_{1t}}(\cdot) + \delta\gamma^2(1+n)^2 U''_{E_{t+1}}(\cdot)} \tag{28}$$

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of the environment while future generations do. The problem is to create a system of incentives to force present generations to invest in the well-being of future generations even if there is no altruism.

[30] See Blanchard and Fischer (1989) pp. 110-114.

[31] The introduction of a positive rate of growth for the population does not change qualitatively the main results obtained in previous sections.

Note that those partial derivatives can only describe partial equilibrium e¤ects, since all the other variables are considered constants. Given the assumptions we made on the utility function it is clear that we have $\frac{\partial s_t^{¤}}{\partial b} < 0$ and $\frac{\partial m_t^{¤}}{\partial b} < 0$. As we expect the introduction of a pay-as-you-go security system has a negative e¤ect on the private saving, and also a negative e¤ect on the investment in environmental maintenance. The main reason is that the contributions for the social security system represent a lum sum tax that reduces the amount of resources available to invest in capital goods and in environmental quality. Thus, in our model, the introduction of a social security system in the form of a pay-as-you-go represents a Pareto-improvement for each generation. However, we cannot say if the introduction of the social security system allows us to reach the optimal capital level implied by the golden rule 19). As far as the environmental quality is concern, we do not know exactly what is the total e¤ect of the social security system on the environmental quality. Indeed, there is a possible reduction in the investment in maintenance that has a negative e¤ect on the environmental quality, but on the other hand, there is also a reduction in s that has a positive e¤ect on the environmental level since it creates a contraction in the production activity. Depending on which of these two e¤ects is dominant, the introduction of a pay-as-you-go security system can have a positive e¤ect on the environmental quality level. However, if we maintain the assumptions that $©_{\overline{k}} > 0$, $©_{\overline{E}} > 0$ and $0 < ª_{\overline{k}} < 1$, then the stable equilibrium is the one implying a high capital level and a high environmental quality level.

7 Conclusion

We studied a discrete time overlapping generations model in which individuals live for two periods and care about environmental quality when they are old. Young individuals can invest in private capital in order to consume when they are old, and they can invest in environmental preservation. The environmental quality at each period is a¤ected negatively by production in the previous period. This creates an intergenerational externality between di¤erent generations that a¤ects the capital accumulation process of our economy. For example, the fact that saving in private capital is an increasing function of the interest rate is not su¢cient for a positive capital growth. Indeed, in our model, capital a¤ects in two di¤erent ways the private investment. There is a positive e¤ect through the wage that individuals received, since an increase in capital increases the wage, and thus, the saving. On the other hand there is a negative e¤ect that passes through the environmental quality. an increase in capital a¤ects negatively environmental quality and this implies that individuals will invest more in maintenance and less in private capital. Thus, capital accumulation is positive in our framework only if the …rst e¤ect dominates the second one. In cases in which the environmental quality is very low, capital can decrease over time until a point in which environmental quality is high enough to restore a positive accumulation process. However, in general, we have that capital accumulation is associated with high environmental quality, even if production a¤ects negatively environmental quality, a similar result as in John and Pecchenino (1994). This is because, in our model as in John and Pecchenino (1994), young individuals can invest in environmental preservation. Thus, an higher level of per capita output can be associated with an high level of environmental quality because individuals have with higher per capita output can invest more resources in environmental improvement. This fact is consistent with observations that relatively poor countries experience higher environmental degradation than developed countries. This result is also consistent with the idea that the relationship between per-capita income and environmental quality follows a "U"-shaped curve, in which there is environmental degradation in the …rst part of the growing path of the economy, but when capital is su¢ciently high, environmental quality tends to increase. Analysing the steady state properties of our model we showed under which conditions a steady state equilibrium with a high capital level and a high environmental quality is asymptotically stable. In particular, we showed that this is true if the relative e¤ect of a change in the capital level on the private saving is greater than the e¤ect that environmental quality has on the same saving function. The intuition behind this result is again that, if private saving is more sensitive to capital accumulation than on the environmental quality, the economy will accumulate more resources in each period, but this will imply that there will be more resources available also to improve the environment. If the investment in environmental preservation are e¢cient, then, we will reach the steady state we described above. Furthermore, we showed that in our economy, the decentralised equilibrium tends to be ine¢cient with respect the allocation that a social planner would choose. Dynamic ine¢ciency can arise in our model because individuals tend to over invest in private capital. The reason is that each generation, in deciding the optimal level of investment, does not take into account the negative e¤ect that production has on the future environmental quality, and thus, on the utility function of future generationsand. In this case a decrease in such investments can increase the consumption, and thus the welfare, of each generation. Finally, given the fact that our economy displays this kind of dynamic ine¢ciency, we considered the e¤ects of the introduction of a pay-as-you-go social security system, that in our framework represents a Pareto-improving policy.

The main limitation of our analysis is that we did not consider the possible solutions of the ine¢ciency problem through the design of an optimal tax policy that try to equate the …rst order conditions of the decentralised solution with the ones of the social planner, since in this case, it should be possible to reach the capital level implied by the golden rule. Furthermore, it could be worth to look in more details at the dynamic of the model in the transition to a particular steady state, given the fact that the dynamic under analysis is highly non-linear.

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