Instabilities and Robust Control in Fisheries*

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Abstract

Demand and supply analysis in fisheries often indicates the presence of instabilities and multiple equilibria, both in open access conditions and in the socially optimal solution. The associated management problems are further intensified by uncertainty on the evolution of the resource stock or on demand conditions. In this paper the fishery management problem is handled using robust optimal control, where the objective is to choose a harvesting rule that will work, in the sense of preventing instabilities and overfishing, under a range of admissible specifications for the stock-recruitment equation. The paper derives robust harvesting rules, leading to a unique equilibrium, which could be used to design policy instruments such as robust quota systems.

Keywords: Fishery Management, Multiple Equilibria, Instabilities, Robust Control, Robust Harvesting Rules.

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1 Introduction

Demand and supply analysis in fisheries has been associated with instabilities and multiple equilibria, both in the context of an open access fishery and a socially optimal fishery.[1] The source of instability is the emergence of a backward bending supply curve which is the consequence of biological overfishing that occurs when effort expands beyond the level corresponding to the maximum sustainable yield. The combination of a standard downward sloping demand curve with the backward bending supply curve can produce an odd number of interchanging locally stable and locally unstable market equilibria in open access fisheries. There exist locally stable equilibria corresponding to high price and low harvesting, which can be seen as an indication of overfishing. It is interesting to note that a similar picture can emerge even in fishery that is managed in a socially optimal way. The discounted supply curve is also backward bending for positive discount rates. As a result, there are demand conditions under which multiple equilibria and instabilities are present even in optimally controlled fisheries.

The problems caused by the emergence of instabilities and overfishing in fisheries are further intensified by uncertainty, which is an important aspect of resource economics. Uncertainty in this context can be associated with the evolution of the resource stock[2] or with demand conditions. Thus both supply and demand shocks could disturb a locally stable fishery and lead to instabilities and overfishing. As Clark (1990) points out, many stock-recruitment relationships are poorly understood and difficult to estimate given the existing data, which in most cases is of low quality. As a result regulation based on mispecified biological dynamics might be ineffective in achieving the desired targets. This brings into the picture the issue of scientific uncertainty and its effects on fishery management.

Our use of the term uncertainty refers to cases where the possible outcomes are known but the decision maker is unable to assign unique probabilities. The possibility of multiple prior distributions has largely been absent from recent economic literature, although it is often a more appropriate setting (see Woodward and Bishop (1997)). Introducing an axiom of uncertainty aversion, as in Gilboa and Schmeidler (1989), a maximin model is obtained where the optimal choice maximizes utility for the worst probability distribution in a given set.[3] In our analysis of fisheries, scientific uncertainty relates

[1] See for example Clark (1990).

[2] See for example Conrad and Clark (1988, Ch. 5), McDonald and Hanf (1992), Clark (1990, Ch. 11), Danielson (2002), Tu and Wilman (1992), Conrad (2000, Ch. 7), Weitzman (2002), Androkovich and K.R.Stollery (1989).

[3] See also Roseta-Palma and Xepapadeas (forthcoming) for an application of robust

to the stock-recruitment equation. It reflects the possibility that although the estimated model, often referred to as the approximating or benchmark model, is consistent with the data, there is a set of alternative models describing the evolution of the resource stock which are also consistent with the data, and thus could be regarded as possibly true. It is important to stress that if the benchmark model is mispecified, and resource stock evolution corresponds to a worse than expected scenario, then the optimal control solution for the benchmark model could result in a fishery with instabilities and overfishing. This observation provides support for adopting a "precautionary principle" in fishery management when there is scientific uncertainty. When the extensive collapse of fisheries over the last century is considered, precaution in designing management rules for regulating fisheries seems to be desirable.

Managing a fishery in this context suggests formulating the management problem as a *robust control* problem along the lines developed in Hansen and Sargent (2001), Hansen and Sargent (2003). The objective is to choose a harvesting rule that will work, in the sense of preventing instabilities and overfishing, under a range of different model specifications of the stock-recruitment equation. Robust control can be directly related to uncertainty aversion and precaution, and as Hansen and Sargent (2001) explicitly state "a preference for robustness induces context-specific precaution".

The purpose of this paper is to address the issue of scientific uncertainty and the potentially induced instabilities and overexploitation in fisheries by introducing robust control methodologies in fishery management. Our main finding is that by an appropriate choice of the robustness parameter, which is a parameter indicating preference for robustness, a regulator that manages a fishery for the social optimum could eliminate multiple equilibria instabilities and potential overfishing. The robust harvesting rules that lead to a unique equilibrium can be used to design decentralized regulation with policy instruments such as transferable quota or landing fees.

control to water management, and Chevé and Congar (2000), Chevé and Congar (forthcoming) for alternative set definitions.

2 Bionomic Instabilities in Fishery Management[4]

We begin by considering a standard fishery model where biomass evolves deterministically according to

$$\dot{x}(t) = F(x(t)) - h(t) \tag{1}$$

where $x(t)$ is fish biomass, $h(t)$ denotes the harvest rate and $F(x(t))$ is the growth function for stock-recruitment. One common example is the logistic growth function, where $F(x) = rx(1 - x/k)$. Biomass stock for the maximum sustainable yield is defined as $x_{msy} = \arg\max_x F(x)$, while $x_k : F(x_k) = 0, x_k > 0$ denotes the carrying capacity biomass. Let unit harvest cost, $c(x(t))$ be a nonincreasing function of the fish stock x. Then for any price p, the profit flow is determined as[5]

$$\pi = (p - c(x))\, h \tag{2}$$

The open access supply in equilibrium is determined by the conditions

$$h = F(x) \tag{3}$$
$$p = c(x) \tag{4}$$

Solving (4) for x and substituting into (3) we obtain equilibrium supply as $h = F(x(p))$. If demand is given by $h = D(p)$, market equilibrium under open access is determined as:

$$\left(p^0, h^0\right) : D\left(p^0\right) = F\left(x\left(p^0\right)\right)\ ,\ p^0 = P\left(h^0\right)$$

where $p = P(h)$ is the inverse demand curve. Typical bell-shaped growth functions together with stock effects on harvest cost and a positive discount rate may give rise to a backward bending supply curve. Combined with a downward sloping demand curve, this could induce multiple equilibria. With three equilibria, the middle one indicates bionomic instability while one of the locally stable equilibria indicates overfishing with low equilibrium harvesting at a relatively high price.[6] Multiple equilibria could be the result of prevailing demand conditions, or could arise from demand shocks.

[4] This section follows Clark (1990, section 5.2), and will serve as background for the development of robust control methodology in the following section.

[5] t is dropped to simplify notation.

[6] See for example figure 5.11 in Clark (1990), for logistic growth and unit cost equal to $\frac{c}{x}$.

To analyze socially optimal fishery management we introduce a social planner or a regulator maximizing net surplus defined as $U(h) - c(x)h$, where $U(h) = \int_0^h P(u)\,du$ so that $U'(h) = P(h)$. The welfare maximization problem is defined as:

$$\max_{\{h(t)\}} \int_0^{\infty} e^{-\rho t} \left[U(h(t)) - c(x(t))\,h(t)\right] dt \tag{5}$$

$$\text{s.t. } \dot{x}(t) = F(x(t)) - h(t)\ , x(0) = x_0 > 0 \tag{6}$$

The current value Hamiltonian for the problem is:

$$H = U(h) - c(x)\,h + \lambda\left[F(x) - h\right] \tag{7}$$

with optimality conditions

$$U'(h) = \lambda + c(x)\ ,\ U'(h) = P(h) \tag{8}$$

$$\dot{\lambda} = \left[\rho - F'(x)\right]\lambda + c'(x)\,h \tag{9}$$

along with biomass evolution (6) and the transversality condition at infinity. Differentiating (8) with respect to time and substituting into (9) we obtain the dynamic system that characterizes the optimal paths of harvest and fish stock. The behavior of harvest is given by

$$\dot{h} = \frac{1}{U''(h)}\left[(\rho - F'(x))\,(U'(h) - c(x)) + c'(x)\,F(x)\ ,\ U'(h) = P(h)\right] \tag{10}$$

whereas stock behaves according to (6). The deterministic steady state equilibrium is defined as $\dot{h} = \dot{x} = 0$. At the steady state, market equilibrium is characterized by

$$P(h) = p, \quad p = c(x) - \frac{c'(x)\,F(x)}{\rho - F'(x)} = S_\rho(x)\,, \quad h = F(x) \tag{11}$$

which describe demand, supply, and biological equilibrium respectively. Solving the stock equilibrium equation of (11), market equilibrium when the fishery is optimally managed is defined as

$$(p^*, h^*) : P(h^*) = S_\rho(x(h^*))\ ,\ p^* = P(h^*) \tag{12}$$

The discounted supply curve determined by (11) is backward bending as in the case of open access fishery and could induce multiple equilibria, as presented in the phase diagram of figure 1.[7]

[7] See also Clark (1990) figures 5.17 and 6.12.

[Figure 1]

For the $\dot{h}_1 = 0$ isocline there is a unique steady state which is saddle point stable at M. However, a demand shock could shift this isocline to $\dot{h}_s = 0$ and induce multiple equilibria, at M_1, M_2, and M_3, with the middle one being unstable and M_3 indicating overfishing. Furthermore, if the benchmark model for stock evolution is misspecified, it is possible for a worse than estimated model for the stock-recruitment relationship $F(x)$ to be realized. Under demand shocks and misspecification of the stock-recruitment relationship both the $\dot{x} = 0$ isocline and the $\dot{h} = 0$ isocline shift and multiple equilibria could also be induced. If these shifts yield a system such as $\dot{x}_2 = 0$, $\dot{h}_2 = 0$ then multiple equilibria emerge at E_1^D, E_2^D, E_3^D. It is also possible for the true model to correspond to an $\dot{x} = 0$ isocline even further below $\dot{x}_2 = 0$, so that an equilibrium with harvesting rule $\dot{h}_2 = 0$ does not exist. This harvesting rule would lead to resource collapse under such circumstances.

The possibility of multiple equilibria at the social optimum presents problems for regulation. For example, the regulatory instruments could have been designed to steer the system towards M_1 but due to demand shocks and/or misspecification, as described above, the systems could converge, for appropriate initial conditions, to a state like E_3^D which is an overfishing steady state. To prevent regulatory complications arising from such cases a different type of regulation is required. The idea behind the robust control methodology, as applied in this paper to fishery management, is to help design rules which under the worst possible scenario for the stock-recruitment relationship will prevent instabilities, steady state multiple equilibria and biological overfishing.

3 Robust Control and Fishery Management

To develop the robust control methodology we introduce uncertainty in the stock-recruitment equation. Let $(\Omega, \mathcal{F}, \mathcal{G})$ be a complete probability space, and let $x_t = x(\omega, t)$, $h_t = h(\omega, t)$ be the stochastic processes for the fish biomass and harvesting, respectively. Moreover, let $B_t = B(\omega, t)$ be a Wiener process, $\mathcal{E}(dB_t) = 0$, $var(dB_t) = dt$.

The stochastic social optimization problem for the fishery can be defined as the choice of a nonanticipating harvesting process $h(\omega, t)$ that maximizes the expected value of net surplus, subject to the constraints imposed by species growth rate:[8]

$$\max_{\{h(t)\}} \mathcal{E}_0 \int_0^{\infty} e^{-\rho t} \left[U(h_t) - c(x_t) h_t \right] dt \tag{13}$$

$$\text{s.t. } dx(t) = \left[F(x_t) - h_t \right] dt + \sigma dB_t \tag{14}$$

$$\sigma > 0, \quad x(0) = x_0 > 0 \text{ nonrandom} \tag{15}$$

$$x_t \geq 0, h_t \geq 0 \tag{16}$$

where x_t is the state variable and h_t is the control variable of the stochastic control problem.

In equation (14) the term $F(x_t) - h_t$ represents the expected change in the fish biomass at any given point in time, while the term σdB_t is the random amount of biomass change, with zero mean and variance σ^2. In this setup, which is a typical stochastic control problem, the manager is assumed to know the behavior of stochastic shocks well enough to fully trust the characterization of the probability distribution implied by (14). This basic assumption leads to a decision on optimal harvest paths. However, it is quite possible (indeed likely, given natural system characteristics and information gaps) that the distribution is only an estimate, so that there is a degree of uncertainty attached not just to the specific realization of the random shock but also to the distribution itself. In other words, the planner might want to consider his own doubts about the model he is using to represent randomness. [9]

Following Hansen et al. (2002), we regard (14) as a benchmark model. If we assume that the social planner knows the benchmark model then there are no concerns about robustness to model misspesification. Otherwise, these concerns for robustness to model misspecification are reflected by a family of stochastic perturbations to the Brownian motion $\{B_t : t \geq 0\}$. The perturbation distorts the probabilities $\mathcal{G}$ implied by (14) and replaces $\mathcal{G}$ by another probability measure $\mathcal{Q}$. The main idea is that stochastic processes under $\mathcal{Q}$ will be difficult to distinguish from $\mathcal{G}$ using a finite amount of data. The perturbed model is constructed by replacing B_t in (14) with

$$B_t = z_t + \int_0^t R_s ds, \text{ or } dB_t = dz_t + R_t dt \tag{17}$$

where $\{z_t : t \geq 0\}$ is a Brownian motion and $\{R_t : t \geq 0\}$ is a measurable drift distortion. Changes in the distribution of B_t will be parametrized as

[8] The basic assumption is that species biomass fluctuates continuously and that these stochastic influences are adequately represented by Wiener processes.

[9] There are two essentially different types of uncertainty involved. Chevé and Congar (2000) refer to these as risk (not knowing the precise value the shock will take) and imprecision (not being sure of the model).

drift distortions to a fixed Brownian motion $\{z_t : t \geq 0\}$. The distortions will be zero under the measure $\mathcal{G}$, in which case B_t and z_t coincide.

Now the social planner's concerns about misspecification of the model describing the evolution of fish biomass can be expressed using (17) to write the distorted model

$$dx_t = [F(x_t) - h_t + \sigma R_t]dt + \sigma dz_t \tag{18}$$

Thus, in the fishery management problem under model misspecification, equation (14) is replaced by (18). Now, following Hansen et al. (2002), the corresponding multiplier robust control model for the fishery can be written as:

$$\max_{h_t} \min_{R_t} \mathcal{E} \int_0^\infty e^{-\rho t} \left[U(h_t) - c(x_t) h_t + \theta \frac{R_t^2}{2} \right] dt \tag{19}$$
$$\text{s.t. } (18),(15) \text{ and } (16)$$

In problem (19) the social planner is the maximizing agent that chooses harvesting h_t to maximize surplus, while "Nature" is the minimizing agent that chooses the "worst case distortion" to the stock-recruitment relationship. The robustness parameter θ can be interpreted as the Lagrangian multiplier associated with an entropy constraint, which defines the maximum specification error in the stock-recruitment relationship that the social planner is willing to accept.[10] . A value $\theta = +\infty$ signifies no preference for robustness in the sense that the decision-maker has no doubts on the model, while lower values for θ indicate such a preference and such doubts.

Note that a specific choice of a maximum specification error that the regulator is willing to consider implies a specific choice of θ. Conversely, a specific choice of the robustness parameter θ implies a specific maximum specification error. Thus a desire to be robust, as reflected in θ, can be translated to a maximum acceptable specification error and vice-versa. Infinite θ implies that the regulator is not willing to consider any specification error and regards the benchmark model as a good model, or rather, as *the* model.

[10] Relative entropy is a measure of the distance between the distributions $\mathcal{G}$ and $\mathcal{Q}$. It must be limited, otherwise they would be distinguishable. More rigorously, the entropy constraint is $\int_0^\infty e^{-\delta u} \mathcal{E}_Q \left(\frac{|R_u|^2}{2} \right) du \leq \eta$ (see Hansen et al. (2002)). Then θ can be interpreted as the Lagrangian multiplier associated with the constraint robust problem $\max_{h_t} \min_{R_t} \mathcal{E} \int_0^\infty e^{-\rho t} [U(h_t) - c(x_t) h_t] dt$, subject to (18),(15), (16) and the above entropy constraint, with η being the maximum specification error that the regulator is willing to consider. As Hansen et al. (2002) show, the constraint problem and the multiplier problem are equivalent.

Using the Fleming and Souganidis (1989) result on the existence of a recursive solution to the multiplier problem, Hansen et al. (2002) show that problem (19) can be transformed into a stochastic infinite horizon two-player game where the Bellman-Isaacs conditions imply that the value function $J(x_t, \theta)$ satisfies[11]

$$\begin{aligned}\rho J(x,\theta) &= \max_h \min_R \left\{ \begin{array}{c} \left[U(h) - c(x)h + \theta \frac{R^2}{2}\right] + \\ J_x\left[F(x) - h + \sigma R\right] + \frac{1}{2}\sigma^2 J_{xx} \end{array} \right\} \\ &= \min_R \max_h \left\{ \begin{array}{c} \left[U(h) - c(x)h + \theta \frac{R^2}{2}\right] + \\ J_x\left[F(x) - h + \sigma R\right] + \frac{1}{2}\sigma^2 J_{xx} \end{array} \right\}\end{aligned} \tag{20}$$

A solution for game (20) for any given value of the robustness parameter θ will determine the socially optimal robust harvesting policy.

3.1 Robust harvesting rules

The optimality conditions associated with the optimization in the right hand side of (20) imply

$$U'(h) - c(x) = J_x \tag{21}$$

$$R = -\frac{\sigma}{\theta} J_x \tag{22}$$

Equation (21) is the usual result that at the optimal harvest the net marginal benefit of an additional unit of catch must be equal to the resource cost, whereas equation (22) is the worst possible distortion that is admissible, which is negative as expected and depends on θ. When θ is large, R is small and the benchmark model is a good approximation. More specifically, when $\theta \to \infty$ there is no distortion at all and the model yields the same solution as the typical optimal stochastic control model.

Going through the required derivations (see Appendix A), we obtain the solution for the evolution of harvesting (in expected terms), which depends on the distortion R :

$$(1/dt)\,\mathcal{E}dh = \frac{1}{U''(h)} \left\{ \begin{array}{c} \left[\rho - F'(x)\right]\left(U'(h) - c(x)\right) + c'(x)\left[F(x) + \sigma R\right] \\ +\frac{1}{2}\sigma^2 c''(x) - \frac{1}{2}U'''(h)\,\sigma^2 h_x^2 \end{array} \right\} \tag{23}$$

substituting the worst case distortion R from first order condition (22), we have the differential equation governing the change of the expected value of

[11] t is dropped again to simplify notation.

robust harvesting along the optimal path.

$$(1/dt)\,\mathcal{E}dh = \frac{1}{U''(h)}\left[\begin{array}{c}\left[\rho - F'(x) - \frac{\sigma^2}{\theta}c'(x)\right](U'(h) - c(x)) + c'(x)\,F(x) \\ +\frac{1}{2}\sigma^2\left(c''(x) - U'''(h)\,h_x^2\right)\end{array}\right] \tag{24}$$

Likewise, the evolution of the expected value of biomass, after substituting R from equation (22) into equation (18) and taking expected values, becomes

$$(1/dt)\,\mathcal{E}dx = F(x) - h - \frac{\sigma^2}{\theta}(U'(h) - c(x)) \tag{25}$$

Equations (24) and (25) summarize the evolution of the expected values of harvesting and biomass under socially optimal management with robust control.

4 Robust Equilibrium: Uniqueness and Regulation

In equilibrium $(1/dt)\,\mathcal{E}dh = (1/dt)\,\mathcal{E}dx = 0$. Using (24) and recalling that $U'(h) = P(h)$, the socially optimal expected steady state harvest under robust control will be determined by:

$$\rho = F'(x) + \frac{\sigma^2}{\theta}c'(x) - \frac{c'(x)\,F(x) + \frac{1}{2}\sigma^2\left(c''(x) - U'''(h)\,h_x^2\right)}{P(h) - c(x)} \tag{26}$$

Under certainty $\sigma = 0$, in which case (26) is reduced to the well known rule for optimal fishery management, equation (11). Similarly, the management rule under "typical", risk-type uncertainty in stock-recruitment, without a preference for robustness, is obtained by setting $\sigma \neq 0$ and $\theta \to \infty$.

Solving (26) for $P(h)$ the robust equilibrium market clearing conditions become:

$$\begin{aligned} p &= P(h) = \\ & c(x) - \left[\frac{c'(x)\,F(x) + \frac{1}{2}\sigma^2\left(c''(x) - U'''(h)\,h_x^2\right)}{\rho - F'(x) - \frac{\sigma^2}{\theta}c'(x)}\right] \end{aligned} \tag{27}$$

$$h + \frac{\sigma^2}{\theta}U'(h) = F(x) + \frac{\sigma^2}{\theta}c(x) \tag{28}$$

where condition (28) indicates stationary biomass, $x_\theta(h, \theta)$. Substituting into (27) we obtain the robust supply curve $p = S_\theta(h, \theta)$. Then market equilibrium is obtained as:

$$(p_\theta^*, h_\theta^*) : P(h_\theta^*) = S_\theta(h_\theta^*, \theta) \text{ and } p_\theta^* = P(h_\theta^*) \tag{29}$$

Setting $\theta \to \infty$ we obtain the corresponding equilibrium condition under risk. It is interesting to note that the simpler type of randomness (assuming a known distribution) affects only the supply curve (**??**), but not the stock equilibrium condition (**??**). However, once we allow for model uncertainty the stock equilibrium condition is also affected by the robustness parameter, so that both harvest and stock expected paths are modified. The chosen equilibrium will depend on σ (which is assumed to be exogenous) as well as θ. Now the interesting question is how to choose an appropriate value for this parameter. One possibility is to use the detection error probabilities associated with a given sample of observations for biomass evolution, calculating likelihood ratios between different worst case distributions and the benchmark (see Hansen and Sargent (2003)).

Alternatively, the discussion in section 2 suggests that the dynamic fishery model could be associated with problems of multiple equilibria and bionomic instabilities, which suggests that θ could also be used to eliminate such problems. To make the point clear, assume that the fishery is controlled using only the benchmark model (14), which implies that $\theta \to \infty$. The dynamic system for expected harvesting and biomass is defined, using (23) and (25) for $\theta \to \infty$, by:

$$(1/dt)\,\mathcal{E}dh = \frac{1}{U''(h)}\left\{\begin{array}{c}[\rho - F'(x)](U'(h) - c(x)) + c'(x)F(x) \\ +\frac{1}{2}\sigma^2 c''(x) - \frac{1}{2}U'''(h)\sigma^2 h_x^2\end{array}\right\} \tag{30}$$

$$(1/dt)\,\mathcal{E}dx = F(x) - h \tag{31}$$

Suppose that this system has a unique equilibrium with the usual saddle point property, shown, in Figure 1, as the intersection of $\dot{x}_1 = 0$ and $\dot{h}_2 = 0$ at point E. Assume now that the benchmark model is not the true one, but that the true one is a distorted model for some $R^D < 0$. Since there are no robust control considerations by the manager, the corresponding dynamic system in expected values is given by (30) and

$$(1/dt)\,\mathcal{E}dx = F(x) - h + R^D$$

In this case while the $(1/dt)\,\mathcal{E}dh = 0$ isocline remains the same, the $(1/dt)\,\mathcal{E}dx = 0$ isocline shrinks inward, possibly as far as the $\dot{x}_2 = 0$ isocline in Figure 1, inducing multiple equilibria at E_1^D, E_2^D, and E_3^D. If R^D is sufficiently large in absolute value, then there could be no steady state equilibrium at all and the resource might collapse. Thus controlling with the benchmark model when the distorted model is true could lead to instabilities or even resource collapse.[12]

[12] These effects will be more profound and detrimental the faster the biomass and harvest dynamics.

The idea behind stabilization through robust control is to choose a harvesting rule such that the system has a unique equilibrium not only for the benchmark model but for the worst possible distortion R that Nature could choose. If a unique equilibrium exists under the worst possible distortion, we want to show that uniqueness will also hold for milder distortions of the benchmark model.

Under robust control the equilibrium harvesting and biomass are determined by (24), (25). In this system θ can be used as a free parameter. Therefore, it could be chosen in principle so that the system has a unique equilibrium. This idea can be explained with the help of Figure 2, which depicts again the three equilibria that emerge from the distorted model without robust control, E_1^D, E_2^D, and E_3^D of Figure 1. Choosing a specific θ implies that the $(1/dt)\, Edh = 0$ and the $(1/dt)\, Edx = 0$ isoclines of the system (24), (25) will shift. The idea is to choose θ so that the isoclines shift to positions such as $H_R H_R$ and Ax_k^R, intersecting only once at point E^R.

[Figure 2]

Choosing θ this way implies that the preference for robustness is combined with a preference for uniqueness. A specific value of θ that guarantees a unique, stable equilibrium can be translated to a maximum specification error that the manager or regulator is willing to accept, by recalling θ's role as multiplier of the entropy constraint in the constraint problem formulation.[13] Provided that uniqueness is preserved under milder distortions, the use of robust control ensures that a unique equilibrium exists for all distortions from the benchmark case to the worst one. Thus, if a milder distortion shifts the $(1/dt)\, Edx = 0$ isocline to Bx_k^M in Figure 2, since the robust control solution fixes the $(1/dt)\, Edh = 0$ isocline at $H_R H_R$, uniqueness is preserved at E^M.

An approach for choosing such a θ can be described as follows. Let $(x, h) \in \mathcal{A} \subset \mathcal{R}_+^2$, where $\mathcal{A} = (0, h^{\max}) \otimes (0, x^{\max})$, $x^{\max} > x_k$. and $h^{\max}$ sufficiently large but without violating any technical constraints. Let $\theta \in \Theta = (\underline{\theta}, \infty)$, where $\underline{\theta}$ defines the lower bound of admissible values of θ, ie. the nonnegative values of θ for which the objective function can be larger than $-\infty$. The $(1/dt)\, \mathcal{E}dx = 0$ isocline defines, using (25), the curve $G(x, h; \theta) = 0$, while the $(1/dt)\, \mathcal{E}dh = 0$ isocline defines, using (24), the curve $K(x, h; \theta) =$

[13] The uniqueness - stabilization argument used in this paper can be complementary to the detection error probability approach. For instance, it is possible that more than one value of θ achieve uniqueness, in which case detection error probabilities can provide additional input into the final choice.

0. If a θ^* exists such that $G(x,h;\theta^*) = 0$ and $K(x,h;\theta^*) = 0$ have a unique solution (x^*, h^*), then robust control leads to a unique equilibrium. Sufficient conditions for the existence of such a θ can be derived.

Consider the Jacobian determinant of the system (24), (25):

$$D(x,h;\theta) = \begin{vmatrix} G_x(x,h;\theta) & G_h(x,h;\theta) \\ K_x(x,h;\theta) & K_h(x,h;\theta) \end{vmatrix} \text{ for } (x,h) \in \mathcal{A},\ \theta \in \tilde{\Theta} \subseteq \Theta \quad (32)$$

where $\tilde{\Theta}$ is the subset of values of θ for which the a solution for the system exists.

Proposition 1 *If $D(x,h;\theta)$ does not change sign in $\mathcal{A} \otimes \tilde{\Theta} \subseteq \Theta$ then a unique robust equilibrium exists for the expected values of harvest and biomass.*

For proof see Appendix B.

A possible illustration of this result can be presented with reference to Figure 2. The uniqueness condition means that a θ^* is selected such that the $H_R H_R$ curve cuts the horizontal axis between A and x_k^R, that it is monotonic increasing at least up to x_k^R, and that the intersection takes place at the non increasing part of the Ax_k^R curve.[14] At the equilibrium point the slope condition for the $H_R H_R$ and Ax_k^R curves implies, using (32), that

$$-\frac{K_x(x,h;\theta^*)}{K_h(x,h;\theta^*)} > -\frac{G_x(x,h;\theta^*)}{G_h(x,h;\theta^*)} \text{ or } \left.\frac{dh(\theta^*)}{dx}\right|_{(1/dt)\mathcal{E}dh=0} > \left.\frac{dh(\theta^*)}{dx}\right|_{(1/dt)\mathcal{E}dx=0}$$

For $D(x,h;\theta^*) = G_x K_h - K_x G_h < 0$ the robust equilibrium has the saddle point property.

To locate sufficient conditions for uniqueness to be preserved under milder distortions, consider a θ^* that provides a unique equilibrium satisfying Proposition 1. For this value of θ the triplet (x^*, h^*, θ^*) will determine a corresponding R^* which is the worse possible distortion. Consider now arbitrary distortions $\tilde{R} \in [R^*, 0]$, with $\tilde{R} = 0$ corresponding to the benchmark model and $\tilde{R} = R^*$ corresponding to the robust model. Thus as R increases toward zero we have milder distortions and the $(1/dt)\,\mathcal{E}dx = 0$ isocline shifts. In terms of Figure 2 this means that the Ax_k^R curve shifts outwards uniformly. Keep the $H_R H_R$ to the robust equilibrium position determined by the triplet (x^*, h^*, θ^*), and consider the sequence of determinants:

$$\tilde{D}\left(x,h;\tilde{R}\right) = \begin{vmatrix} G_x\left(x,h;\tilde{R}\right) & G_h\left(x,h;\tilde{R}\right) \\ K_x(x^*,h^*;R^*) & K_h(x^*,h^*;R^*) \end{vmatrix} \quad (33)$$

[14] *An intersection could take place at the increasing part of the Ax_k^R curve, but additional conditions would be required to ensure uniqueness in that case.*

It is clear that $\tilde{D}(x,h;R^*) = D(x,h;\theta^*)$.

Proposition 2 *If $\tilde{D}\left(x,h;\tilde{R}=0\right)$ has the same sign as $D(x,h;\theta^*)$ and it is monotonic in $\tilde{R}$ then the uniqueness of the robust equilibrium is preserved under milder distortions in $(R^*,0]$*

For proof see Appendix **??**.

In terms of Figure 2, uniqueness is obtained if the $(1/dt)\,Edh = 0$ isocline is increasing at least up to the carrying capacity of the benchmark model. Furthermore, since Ax_k^R shifts uniformly outwards, say to Bx_k^m in figure 2, while $H_R H_R$ remains fixed, uniqueness with the saddle point property is preserved up to the benchmark model.[15]

Of course it is possible that several θ satisfy the sufficient conditions described above. In such a case, the value for θ can be chosen to ensure the highest expected value for the robust control problem.[16] More formally, among the set of θ that satisfy conditions for uniqueness and preservation of uniqueness under milder distortions, a θ^{**} is chosen such that:

$$\theta^{**} = \arg\max_{\theta} \mathcal{E} \int_0^{\infty} e^{-\rho t} \left[U\left(h_t^*(\theta)\right) - c\left(x_t^*(\theta)\right) h_t^*(\theta) + \theta \frac{R^*(\theta)^2}{2} \right] dt$$

where $h_t^*(\theta)$, $x_t^*(\theta)$, $R^*(\theta)$ are solutions of the robust control problem evaluated at each θ.

If a unique robust equilibrium is defined, the value obtained for harvesting in these conditions can be used as a robust quantity limit for designing tradable quota systems. In this case a robust quota is determined by a policy function $h_t^R = \phi(x_t)$ which is the function characterizing an approach path to the unique robust equilibrium. This is the path RR corresponding to the one dimensional stable manifold of the saddle point robust equilibrium, converging to E^R in Figure 2.[17] This result can be related to the safe quota

[15] If milder distortions are realized, updates of the policy might be possible. The analysis of the updating process for a robust rule is left for future research.

[16] Given empirical data, the set of allowable θ can be narrowed down to those that generate reasonable detection error probabilities. See footnote 13.

[17] The stable manifold or equivalently the policy function $h_t^R = \phi(x_t)$ can be recovered by numerical methods. Using the time elimination method, the stable manifold is determined by the solution of the differential equation

$$\frac{dh}{dx} = \frac{(1/dt)\mathcal{E}dh}{(1/dt)\mathcal{E}dx}$$

with initial conditions (x^*, h^*), which is the robust steady state corresponding to E^R in Figure 2.

concept discussed in Homans and E.Wilen (1997). They assume a quota that is a linear function of the biomass, so that the safe quota is determined as $h^S = \max\{0, c + dx\}$, with $c < 0$, $d > 0$. Thus if the stock is below some minimum value then $h^S = 0$ (as negative harvesting is obviously ruled out), while the quota is below, equal or above biological growth if $x \gtreqless x_{safe}$, respectively. In our case for each stock level the quota is "safe" in the sense that it ensures that the robust equilibrium biomass is attained in the long run even under the worst possible scenario for stock-recruitment.

It should be noted that the robust quota rule which attains a steady state biomass equilibrium for the worst possible case of the stock-recruitment equation implies smaller harvesting relative to the benchmark model. If the benchmark model was actually the true model, then with initial condition x_0^R in Figure 2, the benchmark quota would be determined by the stable manifold NN converging to E_1^D, which defines the policy function $h_t^N = \psi(x_t)$. The difference $h_t^N - h_t^R$ can be interpreted as the reduction in harvesting induced by the decision to follow robust rules. Moreover, the difference

$$\frac{1}{\rho}\mathcal{E}\left\{\left[U(h^*) - c(h^*, x^*)\right] - \left[U\left(h^+\right) - c\left(h^+, x^+\right)\right]\right\}$$

will indicate the change in expected steady state welfare between robust and benchmark rules. Since this difference is negative, it can be interpreted as the steady state cost of wanting to be robust, or to put it in a different way, as the cost of precaution.

5 Concluding Remarks

Bionomic instability is an inherent characteristic of fishery models induced by a backward bending supply curve. This instability emerges both in open access and in optimally controlled fisheries. Given the uncertainties associated with fisheries, these instabilities could be intensified by demand shocks or uncertainties associated with the stock-recruitment relationship.

In the present paper we consider the case of scientific uncertainty in the stock-recruitment relationship and we introduce robust control methods in fishery management. We show that robust control could act as a tool to prevent instabilities, by an appropriate choice of the robustness parameter. This is obtained by designing a rule so that the optimally managed fishery is stable under a worst possible scenario for the stock-recruitment relationship.The robust management rule can be used to design a robust quota rule that work better than typical prescriptions under uncertainty, both in the sense of maintaining stable harvests and in avoiding biomass collapse. This management rule will, however, have a cost in terms of foregone expected harvesting benefits.

The robust harvesting solution can be used as a basis for setting "safe" quotas to be applied in a fishery. The question of whether and when it makes sense to update the robustness parameter as more information becomes available on stock-recruitment, and thus to update the harvesting rule accordingly, is one potencially important question which should be addressed in future research.

Finally, the basic model developed here can also be extended along different lines, such as depensation or non-linear cost effects, or by considering the fishery as a dynamic game between the planner/regulator and the fishermen, and seeking robust solutions with possible heterogenous preferences for robustness.

A Derivation of optimal solution

This appendix shows how to derive equation (23).

Differentiating the value function with respect to x and using (21) and (22) we obtain [18]

$$\rho J_x = [F(x) - h + \sigma R] J_{xx} - c'(x) h + F'(x) J_x + \frac{1}{2}\sigma^2 J_{xxx} \tag{34}$$

since $J(x)$ is a function of the stochastic variable x we have by Ito's lemma for $J_x(x)$

$$dJ_x(x) = J_{xx} dx + \frac{1}{2} J_{xxx} (dx)^2 \tag{35}$$

Using equation (18), taking expected values, and dividing by dt we obtain

$$(1/dt)\,\mathcal{E} dJ_x(x) = J_{xx}[F(x) - h + \sigma R] + \frac{1}{2}\sigma^2 J_{xxx} \tag{36}$$

Substituting in (34) and rearranging with (21), the expected evolution of the resource cost is

$$(1/dt)\,\mathcal{E} dJ_x = [\rho - F'(x)](U'(h) - c(x)) + c'(x) h \tag{37}$$

To express the solution in terms of the expected evolution of harvesting, apply the differential operator $(1/dt)\,\mathcal{E} d(\cdot)$ to (21)

$$(1/dt)\,\mathcal{E} d(U'(h) - c(x)) = (1/dt)\,\mathcal{E} dJ_x \tag{38}$$

We need to expand the left hand side of (38), by applying Ito's lemma to $c(x)$ and $U'(h)$, which yields the following second order expansions:

$$\mathcal{E} dc(x) = \left[c'(x)[F(x) - h + \sigma R] + \frac{1}{2}\sigma^2 c''(x)\right] dt \tag{39}$$

$$dU'(h) = U''(h)\, dh + \frac{1}{2} U'''(h)(dh)^2 \tag{40}$$

Since along the optimal path $h = h(x)$, where x is a stochastic variable, using Ito's lemma once again yields

$$dh = \left[h_x [F(x) - h + \sigma R] + \frac{1}{2}\sigma^2 h_{xx}\right] dt + \sigma h_x dz \tag{41}$$

[18] For a basic explanation of the methods used in this section see for example Dixit and Pindyck (1994, Ch.4).

When taking the expected value, terms of order higher than t go to zero, so that $\mathcal{E}\left(dh\right)^2 = \sigma^2 h_x^2 dt$, and (40) becomes

$$\mathcal{E}dU'\left(h\right) = U''\left(h\right)\mathcal{E}dh + \frac{1}{2}U'''\left(h\right)\sigma^2 h_x^2 dt \tag{42}$$

Using equations (39) and (42) to plug into (38), and recalling (37) we finally obtain

$$\left(1/dt\right)\mathcal{E}dh = \frac{1}{U''\left(h\right)}\left\{\begin{array}{c} \left[\rho - F'\left(x\right)\right]\left(U'\left(h\right) - c\left(x\right)\right) + c'\left(x\right)\left[F\left(x\right) + \sigma R\right] \\ +\frac{1}{2}\sigma^2 c''\left(x\right) - \frac{1}{2}U'''\left(h\right)\sigma^2 h_x^2 \end{array}\right\}. \tag{43}$$

B Proof of Proposition 1

We locate sufficient conditions for the existence of a

The proof follows from the proof of proposition 1. All the $\tilde{D}\left(x, h; \tilde{R}\right)$ determinants are different than zero and do not change sign. Then the implicit value theorem and the index theorem provide existence and uniqueness under milder distortions. ■

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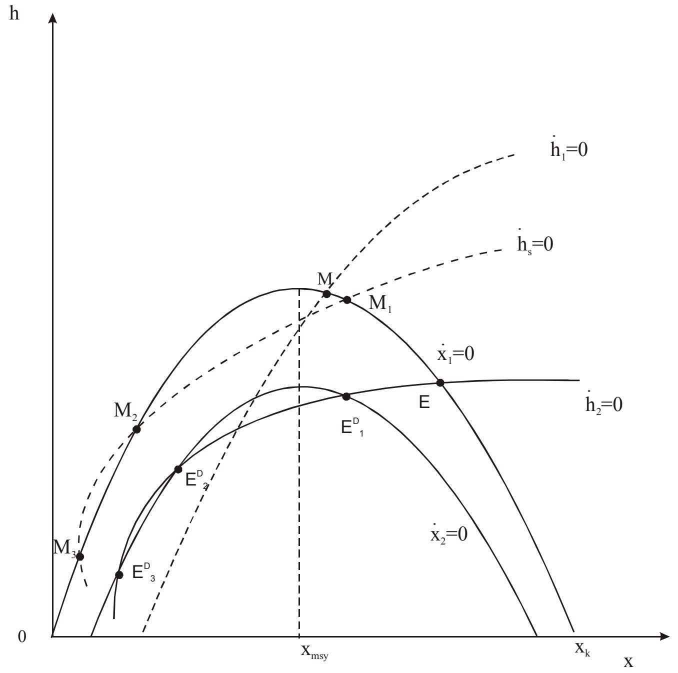

Figure 1:

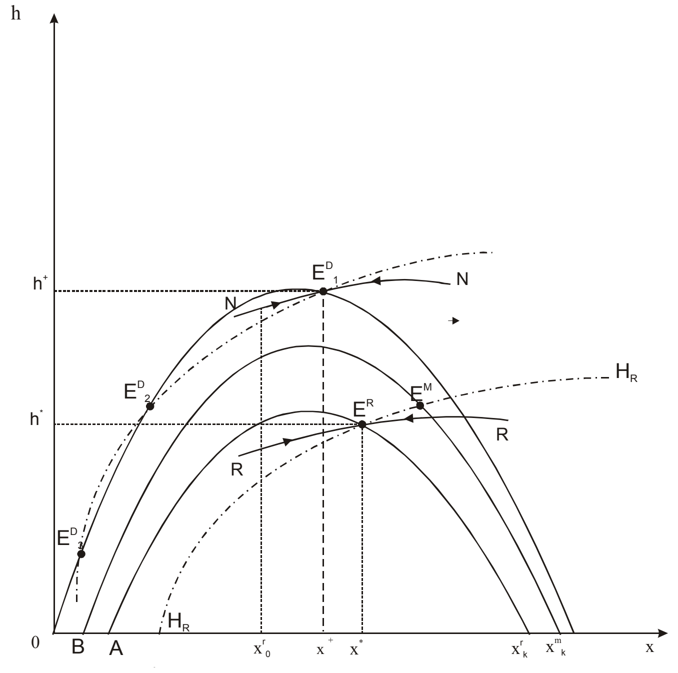

Figure 2: