Towards the Measurement of Spatial Welfare

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Abstract

This paper develops a framework for assessing the welfare effects of the capitalisation of the value of environmental quality in the price for land, combining elements from environmental and spatial economics on the basis of equilibrium considerations for population games. Its main contribution to environmental economics consists of introducing a consistent concept for *spatial welfare*. Following the introduction of estimable locational sorting models in valuation methodology, the relationship between the theoretical underpinnings of the hedonic pricing model and the bid rent concept in urban economics is re-examined. This is done along the definition of a general equilibrium willingness to pay (GE-WTP) that is at the heart of most applications of locational sorting models in environmental economics. A GE-WTP should be able to value non-marginal changes in a spatially explicit distribution of local public goods. Commonly, such a GE-WTP is derived as a Hicksian WTP adjusted for endogenous land prices. This paper offers an alternative interpretation. For a GE-WTP, endogenous prices are typically enforced by a market clearing condition, often an exogenous supply. It is demonstrated how for a discrete choice formulation, a fixed supply per location results in a Nash equilibrium in bid rents. Furthermore, it is shown that this Nash equilibrium corresponds exactly to a spatial equilibrium in urban economics. In this stylised context a GE-WTP could be thought of as a measure for the difference in social welfare between two spatial equilibria. As a money-metric measure, the GE-WTP could be translated to the difference in expenditures in which capitalisation is accounted for in quality-adjusted prices for land. These observations allow for a novel spatially explicit approach to the evaluation of land policy options, combining current cost-benefit practice with the optimisation of land use. Suggestions for operationalisation are put forward.

Keywords: Locational Sorting, Willingness to Pay, Capitalisation, Welfare Economics, Hedonic Price Index

JEL Classification: Q51, Q58, R52, D61

1 Introduction

When accounting for the relation between land use and social welfare, a formalisation of the land or housing market is needed. Traditionally the land market receives attention from various economic subdisciplines. In all these subdisciplines the distinction between the land and the housing market is often fuzzy. Land in urban economics is often considered a consumption good, with a corresponding competitive market. This market is sometimes identified with the housing market for simplicity, in which case it is typically referred to as the rental market for location space. In environmental economics land plays a role in property valuation, where the value of land is assessed in revealed preference methods, especially in *hedonic pricing*; again often projected on housing. And in finance land is considered an asset—with extensions to real estate in general—, competing with other investments in diversified risk portfolios or as a basis for a credit loan (mortgage). These different interpretations highlight various aspects of land in economics. Leaving aside for the moment the asset, or *capital*, quality of land—primarily because of the complication of introducing time as yet another dimension besides space—the question arises which combination of elements from spatial and environmental economics would serve policy interests concerning land markets from a social welfare perspective. This paper proposes a preliminary answer to that question, that might serve as a basis for a novel approach to land use planning, defining the role of a *spatial social planner*.

One binding element for land prices in both environmental and spatial economics is the concept of the *bid rent*. In spatial economics this concept plays a central role in land use models. The best-documented spatial economics tradition in land use models employing a bid rent dates back to von Thünen (1826) for agricultural land use. Von Thünen's method was extended by Alonso (1964), Muth (1968) and Mills (1972) for location choices of consumers and producers. These prototype models of spatial economics have always been interpreted as being part of the neoclassical economics tradition, because they conform to the conditions of competitive markets. They have also often been criticised, because of their unrealistic assumptions. However, it seems fair to say that the assumptions are the price of maintaining a reference to the neoclassical framework, identifying market allocation with welfare maximisation. In the land use models of spatial economics—or more particularly *urban economics*—, the market equilibrium price for land is assumed to be identical to the maximum bid rent, that represents the price a consumer is willing to pay as the rental price for space, after travel or transport costs are subtracted from her income. Travel or transport costs are the only connotation with geography, as they are calculated based on the distance to an single, exogenously given Central Business District (CBD), or market place. Fujita and Thisse (2002, p.79) derive the Alonso model starting by referring first to the following regular maximisation problem:

$$\max_{z,s} u(z,s) \qquad s.t. \qquad z + ps = y - \tau(r). \tag{1.1}$$

Here, y is income and p is the rental price per quantity s of space. The transportation costs τ are a function of the distance r to the CBD. The bid rent is defined as:

$$p(r,u^*) = \max_{z,s} \left\{ \frac{y - \tau(r) - z}{s} \qquad s.t \qquad u(z,s) = u^* \right\}. \tag{1.2}$$

From a public policy point of view it is relevant that in these models market competition—i.e., assigning the land to the highest bid—implies an efficient allocation of land[1]. Therefore the traditional policy directive concerning liberalisation and free markets based on neoclassical Welfare Economics could be extended to land markets on the basis of these models.

In environmental economics, where land use at first sight would be an obvious research topic, the literature until recently showed little coverage of the spatial dimension (Bockstael, 1996; van der Veen and Otter, 2001). Irwin and Bockstael (2002) present an important exception. Their framework, though, is somewhat different from the land use models in the urban economics tradition, as it focuses on the timing of decisions by land developers, rather than on location choices of consumers and producers. Housing prices, on the other hand, have been used extensively for a long time already in environmental economics. In that respect, the perspective on land use in environmental economics seems to have been dominated by the theoretic underpinning of hedonic pricing by Rosen (1974). Rosen proposed a perfectly competitive market for the *characteristics* of consumer goods, also making use of a *bid rent* concept, thereby referring to Alonso (Rosen, 1974, p. 38). He defines a bid rent times the market equilibrium amount of

[1] both, locally at r and in terms of city size (r_{max} measured from the CBD), see Fujita and Thisse (2002, p. 84–85).

space, p^*s^*, as a 'willingness to pay' for achieving a specified utility level, u^*, according to:

$$u\left(y - p^*s^*; q\right) = u^*. \tag{1.3}$$

The parameter q could be interpreted as a quality index[2]. Quality itself can be interpreted as either a characteristic of a good or as public good itself (amenity). Since the quality level is beyond the consumer's choice set, it is exogenous to her and there is no formal basis for a distinction from a public good. The two can be regarded synonyms. Hedonic pricing enables an interpretation of a land (or housing) price as a function of quality, $p(q)$, but at first sight it thereby seems to differ with the interpretation of markets where space is traded as a good itself, which would result in a regular inverted demand, $p(s)$. Furthermore, the focus on markets for characteristics makes a spatial interpretation less obvious[3].

Hedonic pricing is probably one of the most popular revealed preference methods for valuing non-market goods. The impact of changes in quality on welfare is usually measured in terms of marginal price effects. In that case the changes need to be assumed marginal too, effectively excluding the relocation of the households which would be reflected in changes of demand and supply. In order to circumvent this limitation, recently, frameworks based on a locational equilibrium have been introduced. A locational equilibrium can be thought of as a mapping of a spatial equilibrium on the price equilibrium in the land (or housing) market, where demand is derived from individual location choices (Epple and Sieg, 1999; Bayer et al., 2002). Locational sorting models contribute to the valuation literature by defining a *general equilibrium willingness to pay* (GE-WTP) as (Smith et al. (2004)):

$$v\left[p^*\left(\hat{q}\right), y - WTP_{GE}; \hat{q}\right] = v\left[p^*\left(q\right), y; q\right]. \tag{1.4}$$

Here, $p^*\left(\hat{q}\right)$, denotes the equilibrium price that corresponds to the new quality level $\hat{q}$, after increasing q. This GE-WTP is to be contrasted with the general definition of a Hicksian (partial equilibrium, short-run) WTP—or *compensating variation*—for changes in the quality level only (keeping prices, p^*, fixed):

[2]Throughout this paper the quality index, q, is assumed to be a scalar. This done mainly to stress the similarity with the distance, r, in spatial economic models.

[3]Rosen (1974, p. 34) refers to 'locational decisions in characteristics space'.

$$v\left[p^*\left(q\right), y - WTP_{PE}; \hat{q}\right] = v\left[p^*\left(q\right), y; q\right]. \tag{1.5}$$

The supply of housing in locational sorting models is often taken to be locally fixed, assuming that the population will resort itself over the existing stock of houses. The specification of both demand and supply introduces the endogenous prices, $p^*\left(\hat{q}\right)$, and thereby the definition of the 'general equilibrium'—which is actually a spatial equilibrium based on location choices—in the model. Palmquist (2004) suggests continuing further research on applications in environmental economics of these frameworks. Smith et al. (2004) present such an application. Palmquist (2004, p. 59) also notes that:

> *'The theoretical hedonic model describes an equilibrium, but there has been little formal work on modelling how that equilibrium would change if there were changes in exogenous factors.'*

This paper aims at contributing to the understanding of the market equilibrium in hedonic pricing in terms of a spatial equilibrium. For the recently introduced frameworks in environmental economics, spatially explicit assessments are not the first goal. The spatial dimension forms a necessity, rather, because of the definition of general equilibrium adopted. Although similar in spirit, models of spatial economics—and more specifically agglomeration or urban economics (Fujita and Thisse, 2002)—do focus on spatially explicit assessments in the first place. Besides, while welfare considerations in agglomeration economics are mainly normative and applied to a market good (optimal land consumption), sorting models in environmental economics focus on the valuation of non-market goods. Since efficiency considerations in agglomeration economics are based on the maximisation of social welfare, as in a regular neoclassical market, this paper seeks to connect this notion of welfare directly to the welfare measures valuing public goods in locational equilibrium models applied in environmental economics.

The structure of the paper is as follows. First, a suggestion is put forward for the general formulation of a policy goal, based on the concept of a 'benevolent spatial social planner' dealing with *capitalisation*. Then, in section 3, the perspectives in environmental and spatial economics on land prices and the value of public goods are compared under the heading of *expenditure*. In section 4 land prices are related to the concept of spatial equilibrium in urban economics, focusing on welfare or *utility* level and efficiency. Here, the spatial equilibrium is identified as a Nash equilibrium in both population densities and prices. Section 5 translates the concepts to a framework that is closer to an econometric implementation, together with possible definitions for indicators, especially an aggregate *quality adjusted expenditure*. Interpretation and discussion follow in section 6. Finally, in section 7 conclusions are summarised.

2 Land Policy and Social Welfare

In the introduction the concept of the bid rent was reviewed from the perspective of a spatial equilibrium and valuation methodology, respectively. Both perspectives seem to accommodate a different definition of efficiency. On one side there is the spatial economics literature stressing the optimal allocation of land through markets, while on the other side the environmental economics and valuation literature put forward the public good character of quality and its contribution to welfare in terms of the consumption of non-market goods. Efficiency in the latter context concerns the optimal allocation of public goods, in the spirit of cost-benefit analysis (CBA). From a public policy point of view, a GE-WTP—as a welfare measure—seems to address both aspects of land simultaneously. It values changes in the level of public goods, but it also values the difference in welfare between two situations while land is allocated by competitive markets.

As mentioned in the introduction, the land market in environmental economics is usually identified with hedonic pricing. In the context of methods applied in environmental economics in general (not restricted to land use), hedonic pricing takes a special position. It is one of the few valuation methods with an explicit reference to market prices. Other methods rather deal with the valuation of public goods exclusively. Starting with Mäler (1974), environmental economics has developed a theoretical basis for incorporating public goods in an essentially neoclassical framework. The most rigorous implementation concerns the *contingent valuation method* for valuing pure public goods. Because public goods lack a market for achieving an efficient allocation, different criteria had to be developed. Mäler proposed a concept of shadow prices, or *virtual prices*, for public goods that is consistent with regular definitions of expenditure minimisation, compensated (Hicksian) demand and compensating or equivalent variation. For market goods a change in price results in a change of the Hicksian consumer's surplus that equals the difference in the expenditure needed to maintain the same level of utility. With a similar definition for changes in the supply of non-market goods, a willingness to pay (WTP) can be derived, that serves as a monetary measure for the value of the change in (environmental) quality. The willingness to pay corresponds to the shadow price mentioned above—as the *marginal* WTP—integrated over the amenity improvement, $\Delta q = \hat{q} - q$. This yields something similar to a Hicksian consumer's surplus for price changes, with the public good, q, treated as a *quantity* (Hanemann, 1999, p. 52). The general result maintains that the allocation of public goods which yields the highest WTP, would also yield the highest increase in utility, and is therefore optimal.

If, in a first assessment, the quality of a location would indeed be considered a *local* pure public good, exogenous to both consumer and 'producer' (i.e., developer or land owner), it appears as if two goods would be traded simultaneously on a land market:

1. *land*, as a consumption good, following the spatial economics land use tradition of Von Thünen,
2. *quality*, as a local public good (amenity, environmental quality) in the Mäler tradition.

In this stylised case, public policy is confronted with two aspects of a socially optimal allocation of land:

1. securing optimal allocation of land by markets,
2. supplying (an optimal distribution of) local public goods.

Unlike in standard neoclassical markets for consumption goods, the role of a government cannot be restricted to the elimination of external effects as market distortions. Instead, optimal allocation of land requires insight in the reaction of the land market in response to changes in amenities. The GE-WTP as a new measure, embodying a Marshallian demand as the market response, might not only be suitable for solving the problem that arises in the need of deriving a Hicksian measure for non-marginal quality changes adjusted for new equilibrium prices, it could also address directly the 'spatial social planning aspects' of local amenity improvements in terms of the welfare level of a spatial equilibrium.

In the remaining sections the elements introduced above, especially the bid rent and the willingness to pay, will be combined in one consistent framework. With this framework it is demonstrated how the concept of spatial equilibrium facilitates an alternative interpretation of a change in welfare reflected in the GE-WTP. It is suggested that locational sorting models—similar to urban economic models—in general rely on a generic principle that defines market equilibrium prices as a function of utility, while utility in turn is equalised over space for identical agents in spatial equilibrium. In contrast with urban economic models, however, equalising the utility level can be interpreted as an endogenous property of a locational sorting model. If consumers are assumed identical, the level of utility in spatial equilibrium is also a measure for social welfare (multiplying the individual utility level by the population size). Therefore, a GE-WTP could also be read directly as a monetary measure for a change in *spatial welfare*, i.e.:

$$WTP_{GE} \sim \left\{ v\left[p^*\left(\hat{q}\right), y; \hat{q}\right] - v\left[p^*\left(q\right), y; q\right] \right\}. \tag{2.1}$$

This interpretation will be elaborated upon in section 4, addressing in more detail the level of utility in spatial equilibrium, followed by an interpretation of the money metrics of (2.1) in sections 5 and 6. First, the notion of compensation in expenditure and the relation with hedonic prices will be explored further in section 3.

3 Expenditure

Starting with the dual problem for utility maximisation under a budget constraint (1.1), i.e., the minimisation of expenditure for achieving a given utility level, a willingness to pay—as a monetary measure—can be thought of as a virtual correction of expenditure. If quality is treated analogously to a consumption good, the virtual correction accounts for changes in the supplied level of a public good. If, however, market prices are correlated with the level of the public good, or quality,—as is assumed in hedonic pricing—a change in the quality level would also affect 'real' expenditure of the individual through the price of land. In this section the relation between market prices for land and the effects in real and virtual expenditure will be explored.

3.1 Public goods and capitalisation

Most analyses of the relation between the supply of local public goods and the impact on social welfare can be found in the public finance literature that takes Tiebout (1956) as a starting point. Tiebout proposed to interpret a special kind of spatial equilibrium—where a population is distributed over a given number of municipalities—as equivalent to a market equilibrium. The size of the population in a municipality would represent the demand for the locally supplied public good, determining in equilibrium its aggregate price as the municipal expenditure. Tiebout does not refer to capitalisation of the value of the public good in property prices directly, but Oates (1969) suggests testing Tiebout's hypothesis by examining differences in property tax, where this tax could be considered as the entry price of a municipality. According to Oates, property values must be higher in municipalities with a high level of public goods supplied. If people accept higher taxes for financing this supply, the net benefit for property owners has to be positive and Tiebout's conceptualisation would be correct. A part of the following research in this tradition focuses on measuring net benefit by measuring changes in rent, i.e., *capitalisation* (Lind, 1973). Starrett (1981) extends Lind's analysis, noticing that capitalised rent counts as a benefit to the owner, but as a cost for the tenant. Although Starrett does not refer to Alonso (1964), his approach resembles the urban economic tradition of linking location choices to a trade-off between rent and travel costs, since the price a consumer pays for enjoying the public good is related to a trip to the location where the public good is supplied. In this way—as in Arnott and Stiglitz (1979)—capitalisation could be linked to the Alonso model if it is assumed that the public good is supplied exclusively at the Central Business District (CBD). In the Tiebout model, the number of inhabitants of each municipality is endogenous. The spatial equilibrium in the sense of Tiebout, applied to several municipalities, would in that case imply the same level of welfare in every municipality. In urban economics the size of one city—in terms of total land use—is often assumed to be endogenous, while keeping the number of inhabitants fixed[4]. In both cases, the value of public goods supplied in a municipality or city is only reflected in the land price (rent). Theoretically, the supplied public goods could be financed completely by a property tax that equals the net rent; no further tax would be necessary (see e.g. Fujita and Thisse (2002, p. 138))

3.2 Bid rent and valuation

Scotchmer (1986) derives an expression for the bid rent in *hedonic pricing* by adopting a reduced expenditure function, consisting of all expenditures except those for housing. The use of an expenditure function, with its implicit reference to compensation, makes her analysis conceptually related to the environmental economics tradition of Mäler (1974) for the valuation of non-market goods. Because Scotchmer also relates the hedonic marginal willingness to pay for amenity changes directly to an Alonso type of bid rent, her derivation can be linked directly to Fujita and Thisse (2002) as referred to in the introduction (eq. (1.1) and (2.1)). The maximisation problem then would read

$$\max_{z,s} u\left(z, s; q\right) \qquad s.t. \qquad z + ps = y. \tag{3.1}$$

And the bid rent is defined as

$$p\left(q, u^*\right) = \max_{z,s} \left\{ \frac{y - z}{s} \qquad s.t \qquad u\left(z, s; q\right) = u^* \right\}. \tag{3.2}$$

Because u is strictly increasing in z, $z(s; u^*, y)$ is defined (as the inversion) and the bid rent is only a function of s:

[4] A model of one city with fixed land use and an endogenous population size is referred to as an *open city* in urban economics literature.

$$p\left(q,u^{*}\right)=\max_{s}\frac{y-z\left(s;q,u^{*}\right)}{s}. \tag{3.3}$$

In this case, the bid rent values the *local* quality level q, instead of the level of a public good supplied at a central place. Similar to the transportation costs, $\tau(r)$ in (1.2), q could be thought of as a mapping, $q(r)$, of location r to an impact on welfare. This translates the 'characteristics space' of Rosen (1974) directly to the geographical space of Alonso (1964).

If it is assumed that the level of utility, u^{*}, corresponds to the maximised level in a market equilibrium, the bid rent remains essentially the price, or the *marginal rate of substitution*:

$$p\left(q,u^{*}\right)=-\frac{\partial u\left(z,s;q\right)/\partial s}{\partial u\left(z,s;q\right)/\partial z}=-\frac{\partial z\left(s;q,u^{*}\right)}{\partial s}. \tag{3.4}$$

This observation leads to the following justification of using hedonic prices for deriving a marginal willingness to pay for amenity q, with s^{*} (utility maximising amount of space) kept fixed:

$$\frac{\partial p\left(q,u^{*}\right)}{\partial q}=-\frac{1}{s^{*}}\frac{\partial z\left(s;q,u^{*}\right)}{\partial q}. \tag{3.5}$$

The conclusion especially important for the analysis in section 4 of this paper is that if only a hedonic price function is derived, the level of utility is implicitly assumed as given (see also Scotchmer (1986, p. 64)). With s^{*}—as before—as the fixed, maximised space and $p^{*}\equiv p\left(q,u^{*}\right)$, the willingness to pay (WTP) for an amenity improvement $\Delta q=\hat{q}-q$ for tenants is defined by

$$u\left(y-p^{*}s^{*}-WTP;\hat{q}\right)=u\left(y-p^{*}s^{*};q\right)=u^{*}. \tag{3.6}$$

A comparison with (1.3) reveals that the notion of 'willingness to pay' in terms of a bid rent—or expenditure on space $p^{*}s^{*}$—as adopted by Rosen (1974) concerns real money. Because in the derivation of Fujita and Thisse (2002), the expenditure on all other goods than space are condensed in the numéraire, the reduced expenditure function of

Scotchmer (1986) consists only of z^*. In terms of the marginal analysis in (3.5), the WTP in (3.6) conforms to (keeping u^* fixed)

$$WTP \approx \partial z\left(s; q, u^*\right) = -s^* \partial p\left(q, u^*\right). \tag{3.7}$$

This assumes that $\Delta q \approx \partial q$. Stated otherwise, only a perturbation of the equilibrium rent allows for an interpretation of the WTP for a quality change in terms of real money in the expenditure.

A marginal change in the amenity level while keeping land prices essentially fixed, implies that the population will not move in the Tiebout sense. Because the local *population density* in the Alonso model is usually related to the inverse of the locally demanded amount of space per person, the analysis of Scotchmer (1986) is also helpful in interpreting locational sorting models in more detail. Scotchmer makes a distinction between short-run and long-run benefits of an environmental improvement. The difference between the two is that the long-run benefits take into account changes in the distribution of the population, i.e., the local population densities. This can be also interpreted as a difference between partial and general equilibrium analysis in terms of locational equilibrium models (Smith et al., 2004).

3.3 Price and demand

The argument developed in this paper concerning the alternative interpretation of the GE-WTP, is based on the suggestion that a GE-WTP could be thought of as being defined purposefully on the basis of an Marshallian (uncompensated) demand, because of the reference to *market prices* for the private good. In locational sorting models, the change in the market price for land follows a change in Marshallian demand, reflecting the process of relocation. By definition of the bid rent the following equality holds (Fujita and Thisse, 2002, p. 80):

$$u^* \equiv v\left[p\left(q, u^*\right), y; q\right]. \tag{3.8}$$

This is due to the maximisation of space, s, and the definition of the indirect utility. Also, because of this, Marshallian demand will equal Hicksian demand:

$$s\left(q,u^*\right)\equiv s^M\left[p\left(q,u^*\right),y\right]\equiv s^H\left[p\left(q,u^*\right),u^*\right]. \tag{3.9}$$

It allows eq. (3.6) to be written in terms of an indirect utility, thereby already conforming to the definition of a regular, Hicksian WTP:

$$v\left(y-p^*s^*-WTP;\hat{q}\right)=v\left(y-p^*s^*;q\right)=u^*. \tag{3.10}$$

The WTP in (3.10) could in principle be interpreted as the virtual correction of expenditure mentioned in section 2 and quality q would be a pure public good unrelated to any market as in contingent valuation. For a *marginal* change Δq, (3.7) therefore defines the *virtual price*. For a *non-marginal change*, while keeping p^* and s^* fixed, the WTP would be accounted for by an integration over this virtual price, leaving the real money part of the expenditure unaffected. In that case, the relation with the real, reduced expenditure z—as in(3.7)—can not be sustained.

Eq. (3.10) can be used as the basis for calculating a GE-WTP directly an 'adjusted Hicksian' WTP that is implicitly compensated for changes in rent as illustrated by (1.4). But a new rent, $\hat{p}^*$ that corresponds to the market—or 'general' in locational sorting terminology—equilibrium price for land at the new quality level $\hat{q}$, also corresponds to a new demand $\hat{s}^*$. Market demand can be derived from the indirect utility directly through application of Roy's Identity:

$$s^*=-\frac{\partial v/\partial p}{\partial v/\partial y} \tag{3.11}$$

However, in the new equilibrium, both rent and demand depend on the maximised utility level $\hat{u}^*$: $\hat{p}^*\equiv p(\hat{q},\hat{u}^*)$ and $\hat{s}^*\equiv s^M\left[p\left(\hat{q},\hat{u}^*\right),y\right]$. In general, it is to be expected that (otherwise $WTP_{GE}=0$)

$$\hat{u}^*\neq u^*. \tag{3.12}$$

Critical is therefore the characterisation of the market equilibrium prices, that constitute the general equilibrium on which the GE-WTP—as adjusted WTP—is based. Market, or Marshallian, demand in the context applied thus far depends only on the price, $p(q, u^*)$. With quality, q, exogenously given and the utility level u^* assumed to correspond to a market equilibrium. The only variable determining the endogeneity of the price—and thereby the value of the GE-WTP—is the utility level u^*.

The logic of the Tiebout model can still be applied, if instead of the scale of municipalities, equilibrium considerations would be applied at a lower scale level, e.g. to locations within one city. Capitalisation of a change in the public good level at a location in the rent (real money) depends on the endogenous welfare level, that must be consistent with a relocation process. This observation establishes the possibility of an interpretation of the market equilibrium as a spatial equilibrium analogous to agglomeration economics, as will be shown in section 4.

4 Utility

As was argued in the introduction, the logic of a GE-WTP underlying its initial purpose, is most clearly illustrated against the background of the limitations of a standard hedonic welfare measure. Hedonic pricing is generally restricted to the valuation of marginal changes. This is because prices are assumed to be fixed and, as a result, only a marginal willingness to pay can be derived from these prices. Fixed prices imply that changes in amenities, or—more broadly—environmental quality, do not alter location choices. This leaves the demanded quantities for space unaffected in accordance with the analysis in section 3.2.

The derivation of a marginal WTP on the basis of the current price also implicitly assumes that the utility remains unchanged at the maximised level. If it is suggested that non-marginal changes in the local quality level might induce households to move to other locations, a demand function would needed to be specified, so that—together with a specification of supply—new equilibrium prices could be derived. Because hedonic prices are considered market equilibrium prices where consumers maximise their utility level u, equating demand and supply should always result in prices that remain consistent with utility maximisation. Stated otherwise, the calculation of endogenous equilibrium (market clearing, or hedonic) prices depends on u^* as an endogenous variable. The endogeneity of the utility level and, more specifically, its dependency of the specification of supply in locational sorting models is the main topic of this section.

4.1 Land prices in spatial equilibrium

Usually, in spatial economics a public good is seldom identified as such. Occasionally the existence of a CBD is justified by the assumption that a composite public good is supplied there (see section 3.1). Supporting the analogy between amenity and distance to the CBD in this paper, the distance to the CBD could considered an amenity *in itself*[5] (see also Scotchmer (1986, p. 68, footnote 8)). Furthermore, spatial economics often assumes a continuous featureless plain, apart from the presence of a CBD. Here a discrete choice framework will be used, dividing the plain in discrete locations, each with

[5]The shorter the distance to the CBD, the higher the local quality.

a distinct quality level. This conforms to the econometric implementation of locational sorting models. Particularly it corresponds to the framework Bayer et al. (2002) and especially Timmins (2003). Maintaining the analogy between distance and amenity level, this framework would amount to discrete distances in the one dimensional case[6].

After changing the local quality levels in a locational sorting model, the new—and hypothetical, or counterfactual—market prices are usually calculated by means of simulation. The two main current approaches, (Epple and Sieg, 1999; Bayer et al., 2002), estimate the parameters of a specified indirect utility function, from which a consistent demand function can be derived by applying Roy's Identity, as sketched in section 3.3. Supply is in both approaches characterised by the currently existing housing stock, so that the population is assumed to relocate over a fixed supply per location[7]. The GE-WTP for non-marginal changes in local quality could be thought of as a correction of an 'expenditure' function with both, new equilibrium quantities and new equilibrium prices, restoring the previous utility level, as illustrated in eq. (1.4).

In the urban economics land use models, bid rents reflect a *spatial* equilibrium. The spatial equilibrium itself is in urban economics defined as a distribution of agents. It is Pareto-optimal in the sense that no agent is able to improve her utility by moving to another location, without reducing the utility of other agents. For a population of identical agents, this definition of a spatial equilibrium implies that every agent—regardless her location—enjoys the same level of utility. Behind the use of bid rents there is often the implicit assumption of a mechanism in which agents establish their bids in strategic interaction on the basis of equalising differences in utility, this mechanism suggests an analogy with interpersonal comparisons of utility similar to *game theory* and strategic price setting producers in oligopoly. In urban economics in the Alonso tradition then consumers, instead of producers, could be thought of as being involved in strategic price setting by means of their bid rents. This analogy will be elaborated upon in section 4.2.

The market price for land at location j in a discrete spatial equilibrium is defined by

[6]If local quality is replacing the distance to the CBD as the characteristic of a location, there is no need to resort to the simplifying assumptions of urban economics that are often criticised. A city does not need to be represented by a one-dimensional line or a two-dimensional disc. Any specification of $q(r)$ or $q(x, y)$ is allowed.

[7]At the highest level of resolution, when using individual-based data, a location would correspond to one house.

(continuing the analysis based on Fujita and Thisse (2002, p. 79–81), started in section 3.2)

$$p_j^* = p\left(q_j, u_j^*\right). \tag{4.1}$$

This is consistent with the previous analysis, where the former degree in freedom, u_j^*, now denotes the utility level in a *local* market equilibrium. The indirect utility at j can therefore be written as

$$u_j^* = v\left(p_j^*, y; q_j\right). \tag{4.2}$$

As stated above, in urban economics it is assumed that in a spatial equilibrium the utility level is independent of location, or in this case independent of local utility level q_j: $u_j = u^*$ for all j. This independence will be justified in the next subsection. The indirect utility in the new spatial equilibrium—after changing from q_j to $\hat{q}_j$—is given by

$$v\left(\hat{p}_j^*, y; \hat{q}_j\right) = \hat{u}^*. \tag{4.3}$$

Repeating the expectation (3.12) here:

$$\hat{u}^* \neq u^*. \tag{4.4}$$

Stated otherwise, the counterfactual equilibrium is probably characterised by a different level of equilibrium utility. Against this background, the GE-WTP is mainly restoring the old utility level:

$$v\left(\hat{p}_j^*, y - WTP_{GE}; \hat{q}_j\right) = u^*. \tag{4.5}$$

It is therefore conjectured that a general equilibrium willingness to pay is likely to value the difference in utility of two *spatial* equilibria. This welfare measure then would critically depend on the definition of this spatial equilibrium, or—more precisely—on the relation between the competitive (market) equilibrium and the spatial equilibrium. Both connotations of the market equilibrium shed different lights on a GE-WTP. The relation with a hedonic bid rent facilitates an interpretation in terms of adjusting, or correcting, a pure Hicksian willingness to pay for quality changes. But from the perspective of a spatial equilibrium, the GE-WTP would be monetary measure for comparing the welfare level of two different simultaneous distributions of agents and amenities.

The analysis can now be completed with a specification of supply that is in accordance with the independence of the spatial equilibrium u^* on the local quality q_j.

4.2 Spatial equilibrium as a Nash equilibrium

In the previous subsection the degree of freedom for equilibrium utility in spatial economic models was stressed and a relation with game theory—in terms of population densities—was already indicated. In this subsection it is shown how the value of the equilibrium utility can be established in locational sorting models, drawing a parallel with *population games*.

Assume a population of N agents, sorting over M locations. With j as the index for a location, the equilibrium price at location j is defined in (4.1). On the other hand, individual demand at equilibrium prices in a spatial equilibrium is given by

$$s_j^* = s\left(q_j, u_j^*\right). \tag{4.6}$$

The number of agents at location j is given by n_j. Total demand for location j in spatial equilibrium therefore equals $D_j^* = n_j^* s_j^*$. Continuing the establishment of a relation with the urban economics model in the Alonso tradition, a *fixed supply* would close the model. Fixed supply would in this case correspond to the notion of *scarcity* in the sense of a limited amount of space per location, plus the condition that all space is used[8] (no vacancy). For simplicity is assumed that the supply of land at every location j equals the same amount[9]:

$$n_j^* s_j^* = A. \tag{4.7}$$

[8] Fujita and Thisse (2002, p. 82) define a similar condition as $ns = 1$, or $ns = 2\pi r$, depending on whether the city is depicted on a line or a circular plain.

[9] This is not necessary, as A_j could also be defined as location-dependent, thereby allowing for a parcel of land of any shape.

Given the population size N, equation (4.7) can also be written in terms of fractions ($x_j \equiv n_j/N$):

$$N x_j^* s_j^* = A. \tag{4.8}$$

Referring to a population game, these fractions correspond to probabilities in a mixed strategy equilibrium. The supply constraint effectively relates demand s_j^*, through inversion of the demand function, to price p_j^*. It also relates the price to relative scarcity[10], because the (Marshallian) demand s_j^* could act as for a normal good, in the sense that

$$s_j \sim \frac{1}{p_j}, \tag{4.9}$$

but for a 'total local demand' in terms of the number of people normalised to a fraction, (4.8) implies $x_j^* \sim p_j^*$. Finally, in terms of evolutionary game theory, the equilibrium distribution could be derived from ad hoc use of the *replicator dynamics* (Weibull, 1995), similar to its use in the *New Economic Geography* (Fujita et al., 1999):

$$\frac{dx_j}{dt} = x_j\left(v_j - \overline{v}\right) = 0. \tag{4.10}$$

Here, $\overline{v}$, would be the average (expected) fitness, payoff or utility. If $x_j^* > 0$, it follows that $v_j^* = \overline{v}^* = u^*$ at every location j, which is consistent with the assumption that the level of utility is equal across all locations. The level of utility in a Nash equilibrium in a discrete locational sorting model is therefore shown to be consistent with the equilibrium utility level in agglomeration economics.

From (4.10) it follows that $u^* = u(\mathbf{x}^*)$, expressing that the equilibrium utility level depends on the entire *vector* of population densities. Combined with (4.1), it leads to $p_j^* = p_j[u(\mathbf{x}^*)] = p_j(\mathbf{x}^*)$. However, the following expression is preferred:

$$p_j^* = p_j(x_j, \mathbf{x}^*). \tag{4.11}$$

Expression (4.11) stresses the fact that land price at location j is a function of the population density at that location, *relative* to the population densities at all locations.

[10] In effect, this property reflects the *uniqueness* of every location.

The inversion of (4.12) with respect to x_j, followed by a substitution in (4.10), leads to the identification of a spatial equilibrium with a Nash equilibrium in *bid rents*. This will become more apparent in section 5, where a functional form for $u(s, z)$—and for the corresponding expression (4.11)—will be specified.

4.3 Efficiency

If, as is common in urban economics, it is assumed that the 'production costs' for the owner of land equal the opportunity costs for alternative use (e.g. rent from agricultural use), with a rent p_a, profit could be defined as

$$\pi_j = A_j \left(p_j^* - p_a\right). \tag{4.12}$$

Assuming further that the M locations are given exogenously, with fixed size A_j, the profit for the landowners would be based entirely based on the *pure economic rent*. Nevertheless, the price still is the marginal rate of substitution, as shown by eq. (3.4). This implies that the land owner can apply a considerable mark-up above the marginal costs of production or—in this case—opportunity costs. Since in this stylised case the Nash equilibrium is unique, the individual level of welfare, $u^* = \overline{v}^*$, is unique too. Social welfare could easily be identified as $U^* \equiv Nu^*$. This variant of a competitive market results in a Pareto-efficient distribution of agents. Complementary to (4.11) the level of utility at location j could be written as

$$v_j^* = v_j(x_j^*, \mathbf{x}^*). \tag{4.13}$$

With utility in (4.13) decreasing in x_j (a preference for space translates to a negative impact of local population density), bid rents are a means in optimising the distribution of agents, thereby still maximising social welfare U^*. This mechanism is expected to be generic for locational sorting models.

It is to be noted that a full correspondence with urban economics models would also imply endogeneity for the set of locations M, with (4.12) as a 'boundary' condition[11].

[11] For a complete analysis of efficiency for the continuous model of a city of endogenous size, including the comparison with pure rent, see Fujita and Thisse (2002, ch. 3).

The impact on social welfare, U^*, of the optimisation of M in this model is addressed in Grevers and van der Veen (2005).

5 Measuring Spatial Social Welfare

In this section, the discrete choice model, outlined in section 4 will be specified and brought closer to an empirical implementation. This is done by using a multinomial logit behavioural model (McFadden, 1984). The logit model also forms the basis of the locational sorting framework developed by Bayer et al. (2002) (see also Bayer and Timmins (2002) and Timmins (2003)). As was argued in section 4, in a simplified locational sorting model, the utility in spatial equilibrium must equal the average of the utility level at all locations and the spatial equilibrium itself was enforced an exogenous supply. The use of a discrete choice framework in a setting where both demand and supply are specified has been pioneered by Anderson et al. (1992) in the theoretical and by Berry et al. (1995) in the empirical literature on markets for differentiated products.

5.1 Model

The indirect utility for individual i from choice option j is specified as

$$\tilde{v}_{ij} = B \frac{y}{p_j^{\beta}} q_j^{\gamma} e^{\varepsilon_{ij}}, \tag{5.1}$$

where $B \equiv \beta^{\beta}(1-\beta)^{(1-\beta)}$. Individuals are assumed to be identical (i.e., $y_i = y$), except for the idiosyncratic preference for location j reflected in $e^{\varepsilon_{ij}}$. Locations are available at prices p_j and differentiated by quality q_j. The choice framework of (5.1) can be considered as the result of the first stage in a two-stage optimisation procedure, where individuals first solve

$$\max_{z,s_j} u_j = s_j^{\beta} z^{1-\beta} q_j^{\gamma} \qquad \text{s.t.} \qquad y = z + R_j s_j, \tag{5.2}$$

for each location j separately. This stage is a regular maximisation of a Cobb-Douglas utility, $s^{\beta} z^{1-\beta}$, with *space* s and a consumption bundle z as numéraire, adjusted for quality q—which is beyond the consumer's control, or choice set. At this level the problem is identical to e.g. the specification (3.1), which was shown to be consistent with the bid rent concept in section 3. Taking logarithms of (5.1) results in

$$\ln \tilde{v}_{ij} = \ln v_j + \varepsilon_{ij} = \ln y - \beta \ln p_j + \gamma \ln q_j + \ln B + \varepsilon_{ij}. \tag{5.3}$$

Specification (5.3) can be interpreted as a simplified theoretic equivalent to the full empirical specification in Timmins (2003). The second stage—i.e., solving market equilibrium prices p_j^*—consists of choosing the location with the maximum utility. With a i.i.d. for ε_{ij} according to the double exponential distribution, the probability is specified according to a standard multinomial choice model (with a standard deviation $\sim \mu$):

$$\Pr_j = \frac{\exp\left[(\ln y - \beta \ln p_j + \gamma \ln q_j + \ln B)/\mu\right]}{\sum_k \exp\left[(\ln y - \beta \ln p_k + \gamma \ln q_k + \ln B)\mu\right]} = \frac{\left(\frac{q_j^\gamma}{p_j^\beta}\right)^{\frac{1}{\mu}}}{\sum_k \left(\frac{q_k^\gamma}{p_k^\beta}\right)^{\frac{1}{\mu}}}. \tag{5.4}$$

As in section 4 with mixed strategies, if the population is large enough (formally $N \to \infty$), the choice probability $\Pr_j$ can be interpreted as the *fraction*, x_j, of the population choosing location j. In terms of the absolute number of individuals: $n_j = N x_j$.

5.1.1 Average utility

The expected value of the logarithm of the individual indirect utility, maximised over all possible options j, is given by Anderson et al. (1992, p. 61)

$$\mathcal{E}(\max_{j=1\ldots M} \ln \tilde{v}_{ij}) = \mu \ln \left[\sum_{j=1}^{M} \exp\left(\frac{\ln v_j}{\mu}\right)\right]. \tag{5.5}$$

This is the equivalent of the average utility $\overline{v}^*$ in (4.1). Since this expression is based on the distribution $F(x) = \Pr(\ln v_j + \varepsilon_{ij} \leq x)$, a monotonic transformation leaves the result unaffected, because the expected value essentially depends on the probability (5.4) with which each of the options are chosen:

$$\Pr(\ln v_j + \varepsilon_{ij} \leq x) = \Pr(v_j e^{\varepsilon_{ij}} \leq e^x). \tag{5.6}$$

As a result:

$$\mathcal{E}(\max_{j=1...M} v_j e^{\varepsilon_{ij}}) = \left(\sum_{j=1}^{M} v_j^{\frac{1}{\mu}}\right)^{\mu}. \tag{5.7}$$

From (5.1) it follows that

$$v_j = B\frac{yq^{\gamma}}{R_j^{\beta}}. \tag{5.8}$$

Finally, substitution of (5.8) in (5.7) yields

$$\mathcal{E}(\max_{j=1...M} v_j e^{\varepsilon_{ij}}) = ByG^{-\beta}, \tag{5.9}$$

where

$$G \equiv \left[\sum_{j=1}^{M}\left(\frac{q_j^{\gamma}}{p_j^{\beta}}\right)^{frac1\mu}\right]^{-\frac{\mu}{\beta}} = \left[\sum_{j=1}^{M}\left(\frac{p_j}{q_j^{\frac{\gamma}{\beta}}}\right)^{-\frac{\beta}{\mu}}\right]^{-\frac{\mu}{\beta}}. \tag{5.10}$$

Using (5.9), the utilitarian social welfare function can be written as

$$V = N\mathcal{E}(\max_{j=1...M} v_j e^{\varepsilon_{ij}}) = \beta^{\beta}(1-\beta)^{(1-\beta)}YG^{-\beta}. \tag{5.11}$$

This specification is identical to the specification of the social welfare used in Fujita et al. (1999, p. 48) (see section 6).

5.1.2 Demand and supply

Expected demand for location j equals

$$S_j = n_j s_j = N s_j x_j. \tag{5.12}$$

Alternatively, (5.12) could be derived by applying Roy's Identity to (5.11) (Feenstra, 1995, p. 636). The individual demand s_j can be derived directly as the standard Marshallian demand according to the maximisation problem (5.2):

$$s_j = \beta \frac{y}{p_j}. \tag{5.13}$$

Combining (5.12) and (5.13) results in

$$S_j = n_j s_j = N\beta \frac{y}{p_j} x_j = \beta \frac{Y}{p_j} x_j. \tag{5.14}$$

Going back to the demand (5.12), it will be assumed that the supply is fixed: $S_j \equiv A_j$, as in section 4.2. For convenience, it will also be assumed that the amount of available land is the same for every district: $A_j \equiv A \quad \forall j$ (see (4.8)). As a result, from (5.14) it follows that

$$AR_j = \beta Y x_j. \tag{5.15}$$

This expression maintains that the fraction x_j of the part β of the aggregate income, $Y = Ny$, spent on land, is spent at location j. It also implies that

$$p_j \sim x_j. \tag{5.16}$$

This is the simplest specification possible for *endogenous prices*[12]. Substitution of (5.15) in (5.4) results in

$$x_j = \frac{\left(\frac{x_j^{\gamma}}{p_j^{\beta}}\right)^{\frac{1}{\mu}}}{\sum_k \left(\frac{x_k^{\gamma}}{p_k^{\beta}}\right)^{\frac{1}{\mu}}}. \tag{5.17}$$

The solution of (5.17) conforms to the Nash equilibrium in Brock and Durlauf (2003). In this simple case (5.17) can be solved analytically, by elimination of (5.7) in (5.4) (in the denominator) and (5.7). Finally, substitution in (5.10) yields the following solution for the equilibrium local population density:

[12]Eq. (5.15) specification implies a fixed output level for each 'producer' (i.e., owner of a location), excluding the strategic behaviour among producers (price, quantity of quality setting).

$$x_j^* = \frac{q_j^{\frac{\gamma}{\beta+\mu}}}{\sum_{k=1}^{M} q_k^{\frac{\gamma}{\beta+\mu}}}. \tag{5.18}$$

Equation (5.18) establishes the relation with the model of Alonso (1964) (Fujita and Thisse, 2002), as the population density is increasing in quality level[13].

Substitution of (5.18) in (5.15) results in (cf. (4.13))

$$R_j^* = \frac{\beta Y}{A} \frac{q_j^{\frac{\gamma}{\beta+\mu}}}{\sum_{k=1}^{M} q_k^{\frac{\gamma}{\beta+\mu}}}, \tag{5.19}$$

which accounts for the interpretation of a Nash equilibrium in bid rents. Finally, substitution of (5.19) in (5.11) results in an analytical solution for social welfare:

$$V = A^{\beta} \left[(1-\beta)Y\right]^{1-\beta} \left(\sum_{k=1}^{M} q_k^{\frac{\gamma}{\beta+\mu}} \right)^{\beta+\mu}. \tag{5.20}$$

It follows that social welfare with endogenous prices is only function of the quality of all sites and the number of sites, M. Assuming that the population relocates itself over the initial supply, M can be kept fixed and the impact of local changes in quality on social welfare can be examined.

5.2 GE-WTP and quality-adjusted expenditure

The advantage of the translation of the individual logit specification to the social welfare function of eq. (5.11) for the entire population, is best illustrated by the expression for the aggregated GE-WTP (see also Smith and Von Haefen (1997) for a regular WTP). First of all,

$$N \cdot (y - WTP_{GE})\, \hat{G^*}^{-\beta} = N \cdot y {G^*}^{-\beta}, \tag{5.21}$$

[13]Fujita and Thisse (2002) show how Alonso's model in continuous space is able to account for several stylised facts. One is that population density is decreasing as distance to the CBD is increasing.

so that

$$N \cdot WTP_{GE} = Y\left[1 - \left(\frac{\hat{G}^*}{G^*}\right)^{\beta}\right]. \tag{5.22}$$

Here,

$$\hat{G}^* \equiv \left[\sum_{j=1}^{M}\left(\frac{\hat{p}_j^*}{\hat{q}_j^{\frac{\gamma}{\beta}}}\right)^{-\frac{\beta}{\mu}}\right]^{-\frac{\mu}{\beta}}. \tag{5.23}$$

The implication of (5.23) is that the new equilibrium land price, $\hat{p}_j^*$, together with the changed characteristic, $\hat{q}_j$, is captured for all M locations in one variable $\hat{G}^*$. This $\hat{G}^*$ can be read as the expenditure on land at *quality-adjusted* prices $\left(\hat{p}_j^*/\hat{q}_j^{\frac{\gamma}{\beta}}\right)$ (Feenstra, 1995). Therefore, the GE-WTP only depends on the ratio of the quality-adjusted expenditures on land. The interpretation of the 'quality adjustment' in this expression will be detailed further in section 6.

Finally, based on a corresponding nested CES specification for the direct utility that leads to the same indirect utility (5.11), the following maximisation problem for a *representative consumer* can be considered as equivalent with the maximisation problem for a population above:

$$\max_{Z,Q} U = Q^{\beta}Z^{1-\beta} \qquad s.t. \qquad Z + GQ = Y. \tag{5.24}$$

Here, utility has become a function of the aggregated numéraire, Z, and *quality-adjusted* space (land), Q, with

$$Q \equiv \left[\sum_{j=1}^{M}\left(S_j q_j^{\frac{\gamma}{\beta}}\right)^{\rho}\right]^{\frac{1}{\rho}}, \tag{5.25}$$

with $\rho \equiv \frac{\beta}{\beta+\mu}$.

The following relation holds: $GQ = \beta Y = \sum_{j=1}^{M} p_j^* A$. This relation still holds in the new equilibrium—with adjusted quality level and corresponding prices—because for a Cobb-Douglas utility function the expenditure is strictly divided between the amount of $(1-\beta)Y$ spent on the numéraire and βY spent on the other good. For the model developed here, this can be verified by the observation that due to (5.15) and $\sum_{j=1}^{M} x_j = 1$. It follows that $GQ - \hat{G}\hat{Q} = 0$. This means for the interpretation of $N \cdot WTP_{GE}$ as a compensation in expenditure according to (5.21), that the difference in expenditure must be entirely accounted for in the numéraire. The interpretation is therefore similar as for the marginal quality change in (3.6), but here also a *non-marginal* quality change would be accounted for in a equivalent 'real money' change in expenditure, E, by $\Delta E = -(1-\beta)N \cdot WTP_{GE}$. Using (5.23), it follows that the expenditure on the numéraire good that is needed to maintain the initial level of welfare is defined by

$$\widetilde{Z} = (1-\beta)\left(Y - N \cdot WTP_{GE}\right) = (1-\beta)Y\left(\frac{\hat{G}^*}{G^*}\right)^{\beta}. \tag{5.26}$$

This expression forms the final contribution to the alternative interpretation of a GE-WTP and the relation with quality-adjusted expenditure.

6 Interpretation and discussion

The interpretation of the model developed in the previous section is conducted along two lines. First, the relation with recent developments in spatial economics is explored. This concerns imperfect competition, indivisibility and interactions. These modelling aspects are followed by an assessment of the quality-adjusted prices in terms of price indices.

6.1 Product differentiation and interactions

Since the advance of the New Economic Geography (NEG) (Krugman, 1991; Fujita et al., 1999) in *spatial economics* and its use of models of imperfect competition—above all the Dixit-Stiglitz model (Dixit and Stiglitz, 1977)—for assessing spatially explicit economic issues, geography is incorporated in many subdisciplines of economics, along various lines of existing traditions. As already suggested by Koopmans (1957), *indivisibility* or *product differentiation*[14] is the key to spatial modelling. The NEG itself also introduced a location choice model (Krugman, 1991). The so-called *core-periphery model* is a two-region, general equilibrium model in which a mobile labour force (as the only production factor) chooses a region. The utility in this model is derived from a nested, Dixit-Stiglitz, constant elasticity of substitution (CES) utility function, expressing a preference for diversified products for a representative consumer. Anderson et al. (1992) show that for certain specifications there exists a relation between an individual discrete choice random utility model (RUM) and a constant elasticity of substitution (CES) model for a representative consumer. In the model developed in this paper, space (land) can be thought of as a consumption good differentiated by local quality, while at the same time a multinomial logit (McFadden, 1984) is employed for modelling location choices.

The driving force behind agglomeration in the core-periphery model is the consumption of locally manufactured goods vs. costly imported goods. Transport costs for imported goods connect the two regions, by affecting the real wage in each region. The NEG adopts the same selection mechanism from evolutionary game theory as in section 4—the

[14]Indivisible goods, increasing returns, specialisation and product differentiation are basically equivalent when contrasted with the assumptions of the Arrow-Debreu framework, (see Koopmans (1957, p. 151)).

replicator dynamics (4.10)—, based on the fractions of a population choosing a region when the utility ('fitness' in a evolutionary context, real wage in the core-periphery model) is above average. In section 5, locations were connected directly through strategic interaction in the bid rent, resulting in a Nash equilibrium for a multinomial logit with interactions, in the sense of the Brock and Durlauf (2003) framework for *discrete choice with social interactions*. This framework also forms the basis for the locational sorting framework by Bayer et al. (2002) (Bayer and Timmins, 2002; Timmins, 2003).

The social welfare function in section 5 is, nevertheless, essentially the same as in the NEG. An indirect utility function of a representative consumer reflects the social welfare level for the entire population, consisting of N individual consumers making discrete location choices. The interdependent equilibrium land prices are grouped, together with quality levels, in one aggregate price, G^*.

6.2 Quality adjustment and indices

Feenstra (1995) uses the CES-logit translation in Anderson et al. (1992) for deriving bounds for *hedonic price indices*. He (Feenstra, 1995, p. 636) points out that the aggregate demand function as in (5.12) can be derived from a representative consumer's indirect utility function, but states that this function is a *monotonic transformation* of "... *the sum over individuals of the expected value of maximised utility...*" (ibid). It means that in that case the aggregate welfare measure would need to be transformed first, before it can be interpreted as a utilitarian social welfare function. In section 5 it is shown that if the RUM specification for the individual consumer follows Timmins (2003), the representative consumer's indirect utility function is simply the sum of the individual utility functions. Therefore, if the monotonic transformation is applied at the individual level, the indirect utility function of the representative consumer can be interpreted directly as the utilitarian social welfare function, i.e., without further transformation.

While the NEG adopts *monopolistic competition* as a simplified specification of the supply side, Feenstra explores various forms of *oligopolistic competition*, identifying the market equilibrium with a Nash equilibrium for producers. Because in section 5 supply was kept fixed—in accordance with models in urban economics—the Nash equilibrium in bid rents for consumers arises. Nevertheless, the relation with a hedonic price index still holds. If a change in the public good level can affect the demand for a market good, the consumer must be able to substitute quantity for quality, and vice versa. Feenstra (1995, p. 636) points out that an expression like (5.3) is an example of the *repackaging model*, usually ascribed to Fisher and Shell (1972). Their canonical example (Fisher and Shell, 1972, p. 26) concerns a box that initially contains ten widgets. If the same box—sold at the same price—would contain twenty widgets, the 'quality improvement' of a box would equal half the price of the initial packaging.

Alternatively, for expressing the impact of a non-marginal quality change on expenditure, the quality change could be translated to an equivalent price change. This would conform to a price-quality rate of substitution and only applies in the case of *weak complementarity*. The exploitation of weak complementarity in the valuation of public goods involves various limitations and technical complications (Willig, 1978; Small and Rosen, 1981). When a public good is consumed only together with a market good, the value of the public good can—in principle—be derived from empirical data on the consumption of the market good. This amounts to reconstructing a demand curve for the public good on the basis of an observed demand curve for the private good. One condition is that the public good should be a weak complement to the private good. The 'weakness' refers to the possibility to refrain from consumption if the price for the private good becomes too high, i.e., the private good must be *non-essential*. Otherwise, behavioural data would contain no information. Examples are found in the recreational use of the environment. A trip to the forest, e.g., involves the consumption of gas for the car, among other expenses. A family can decide to stay at home, thereby spending no money on gasoline for the forest trip. The family will be forced to stay at home, if the expenses for the trip will reach a certain limit (the *choke price*). However, there is a difference between the demand for the public good and the demand for the private good. The latter reflects uncompensated (Marshallian) demand, while the valuation of a quality change is intrinsically based on income compensated demand and compensated demand cannot be observed in reality. Marshallian demand is observed in reality, but methods for relating it to a Hicksian welfare measure are elaborate and subject to additional conditions (Bockstael and McConnell, 1993).

From a conceptual point of view it seems appealing to consider hedonic prices for land or housing in terms of weak complementarity. The environmental quality of a parcel of land could be interpreted as a locally available public good and as a weak complement to the house, which is—as a private good—traded on a market. This would allow for a direct implementation of a price index proposed by Willig (1978) as a *green price index* (Banzhaf, 2005). This paper adopts a different approach. A single house might be considered non-essential, but the location choice model developed in section 5 has its basis in the formulation (3.1) for the utility function, implying that land itself is an *essential* good. This is consistent with the *repackaging model*, mentioned above. The problem of translating a Marshallian demand for the market good to a Hicksian demand for the public good can thereby be circumvented. This is, in turn, consistent with the claim in section 3.3, that a GE-WTP might be interpreted as being defined purposefully on the basis of a Marshallian demand.

As a consequence of repackaging in combination with land as an essential good, the change of quality is effectively fully capitalised in the rent. Whereas a green price index seeks to define an expenditure that includes marginal virtual prices for public goods, the expenditure captured by (5.23) contains quality-adjusted prices for land. A hedonic price index—or rather, an *ideal cost of living index*—, I, is the ratio of expenditures (at two points in time) needed to maintain the same level of utility. Referring to (5.23) and (5.26), this index is given by:

$$I = \frac{\Delta E}{E} + 1 = \left(\frac{\hat{G}^*}{G^*}\right)^{\beta}. \tag{6.1}$$

This expression reconfirms the relation between a GE-WTP and a *correction in quality-adjusted expenditure* in spatial equilibrium. The relation with a hedonic price index suggests a possibility for measuring spatial welfare using hedonic regressions.

7 Conclusions

In this paper a unifying framework has been developed, aiming at capturing the essential elements for defining a measure of spatial welfare. This was done, inspired by the definition of a general equilibrium willingness to pay (GE-WTP) in locational sorting models that are currently employed in valuation methodology in environmental economics. Locational sorting models commonly define a GE-WTP as a regular, Hicksian WTP adjusted for endogenous prices. While maintaining a close relation with hedonic pricing, it is shown that the GE-WTP could also be interpreted as valuing the level of spatial welfare. Spatial welfare is defined here as the level of welfare that corresponds to the same definition as in urban economic models. It is argued that drawing this analogy can be justified on the basis of a fixed supply. The fixed supply in turn arises in locational sorting models, due to the assumption that the current population resorts over the existing housing stock, in a hypothetical equilibrium.

The hybrid locational equilibrium model developed here offers three benefits:

1. The original econometric frameworks for applications in environmental economics are are supported by an theoretical interpretation that connects them to a well-established literature in urban economics.

2. Using a relation with the New Economic Economics, expenditure in a simplified locational sorting framework can be interpreted as an aggregate *quality-adjusted* expenditure on land.

3. Following recent combinations of the New Economic Geography with *New Growth Theory*, eventually a relation between optimal land use and sustainable economic growth might be established.

In this paper, only the first two benefits were explored, focusing on the simultaneous consumption of space and quality. The current notion of *spatial welfare* addresses the *capitalisation* of local quality—as public good—in the price of land. Understanding the welfare aspects of capitalisation is crucial in land use planning. Given its position as a social planner, a regional government is faced with the simultaneous task of supplying (or preserving) public goods and securing the competitiveness of the land market. The alternative interpretation of a GE-WTP might contribute to a better understanding of the reactions of the land market to changes in local public goods. Elaboration of the third benefit would, above all, imply that the 'consumable space'—as output—needs to specified as produced with *land* as *capital* input, drawing an analogy between development and investment. It could extend the concept of spatial welfare with issues of urban use vs. habitat protection. This is left for future research.

Finally, again based on the existing locational sorting frameworks, a suggestion is put forward how a GE-WTP-like concept might be operationalised as a regional indicator. Using a multinomial logit model—which use is well-established in econometric implementations—a measure of regional expenditure is derived, in which the total expenditure on land is adjusted for the quality level. The GE-WTP can therefore also be translated to a difference in two quality-adjusted expenditures. This might serve as the basis for developing a *spatial hedonic price index*, if indeed the relation with the bid rent concept in urban economics could be sustained in hedonic regressions.

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