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Imperfect competition and congestion in the City

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Abstract

This paper presents a model to study the interplay of imperfect competition and congestion. Residents live in the city center while they shop and work in subcenters (shopping centers, airports, etc.). Each subcenter offers one differentiated product and one differentiated workplace. Shopping and commuting from the city center to the subcenter requires the use of transport infrastructure that can be congested. We derive the Nash equilibrium in prices and in wages and analyze the welfare impacts of congestion charging and infrastructure policies. We generalize the literature on imperfect competition with (spatially) differentiated products in the presence of (un) priced congestion.

JEL classification: D43; L13; R3; R41

Keywords: Road pricing; Product differentiation; Location theory; New economic geography; Congestion

1. Introduction

This paper presents a model to study imperfect competition with congestion. A city with fixed population is served by a number of subcenters that offer variants of the same product. The location of the subcenters and of the population is fixed and the access to each of the subcenters can be congested by shoppers, workers and trucks. These subcenters can stand for different types of products: they can represent specialized shops selling one product (cars or bikes) or they can represent shopping centers selling a fixed bundle. In the case of metropolitan areas they can also represent larger facilities like airports. Each subcenters produces only one variant of the product.

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The subcenters compete in prices for customers coming from the city but also compete in wages for their employees as each subcenter is a differentiated workplace. Both types of competition are linked since selling more product requires a larger work force. In the short term, the number of subcenters is fixed and we consider a (monopolistic competition) Nash equilibrium in prices and wages. In the long run, free entry and exit can change the number of subcenters. We study the properties of the short and long run equilibrium and examine the effects of congestion pricing, road capacity expansion and other policies that affect directly the number of subcenters.

Our model can be compared to four strands of the literature: the imperfect competition literature, the literature on congestion pricing with imperfect competition, the urban economics literature and the literature on the endogenous location of shopping centers. Our model uses the logit model to represent differentiated goods. Compared to the traditional models of imperfect competition (surveyed in [8]), our model offers two additional features. First, it examines imperfect competition in a general equilibrium (yet simple) framework as the labor market and the delivery of intermediate goods are explicitly modeled. Second, our model introduces congestion. Both elements will be shown to have an important effect on the equilibrium outcome. Introducing a general equilibrium framework and a differentiated job market offers more complexity as firms compete on two markets rather than one. The equilibrium mark-up and the equilibrium number of firms are shown to be increasing in the product and job heterogeneity parameters. Congestion adds another component to the equilibrium mark-up because congestion acts as a disincentive to cut prices. The welfare economics of the number of firms also changes as we now have two market imperfections that interact. Congestion (and market power) can be relieved by having congestion pricing, by having more subcenters but also by having larger road infrastructure. The three strategies are to some extent substitutes.

The interplay between congestion and imperfect competition has already been covered in the case of homogeneous goods for a monopoly by [7], and for a duopoly by [6]. They show that congestion can lead to higher mark ups if the level of congestion is indeed a function of the total sales of the monopolist. We generalize this literature in three ways. First, we use a general equilibrium framework with shopping, commuting and delivery traffic where the three types of traffic are influenced by the strategy of the firms. Second, we study the case of differentiated rather than homogeneous goods and finally we allow for any number of competitors on the market.

The urban economics literature takes a more global approach to the problem of congestion and imperfect competition by including endogenous location of production and residence and having therefore an endogenous urban form (see [11]). In our paper, locations of subcenters and residences of population are fixed although the number of subcenters is endogenized. Since we consider only symmetric equilibria, all subcenters are at the same distance from the center and we can therefore make the natural assumption that the price of land is identical for all subcenters and can therefore be omitted from the analysis. This would not be true if firms were competing for space within each subcenter, a case that is disregarded in this paper.[1] Given the many differences in the type of forces at work (relocation and agglomeration or present in the urban economy models but not in our model), our results are not directly comparable to the results obtained with endogenous location models.

Fujita and Thisse [5, p. 221] survey shopping center models. These models study the endogenous location of shops and employment centers as well as consumers in a linear or homogeneous space. Shopping centers may exist because of search costs or when they offer sufficiently dif-

[1] We discuss the case of several firms per subcenter briefly in the conclusions.

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ferentiated products. Our model has a different focus: the location of consumers is given (they reside in the city center), the potential locations of subcenters are given ex-ante and every subcenter has only one producer that offers a given variety of the good. This means that we do not aim to study the origin, location or composition of subcenters, instead we limit ourselves to the study of the properties of the competition between different subcenters.

In the model interpretation we follow in this paper, we have residents that live in the city center but shop at and commute to subcenters. This is not necessarily the most common urban structure (see [1]). Our generic model allows an alternative interpretation. In this alternative interpretation, households choose a subcenter to reside in (they "shop" for a residence) and they work in the city and in another subcenter. In the interest of clarity we do not emphasize this alternative interpretation in this paper.

In Sections 2 and 3 we develop the model structure. In Sections 4 and 5 we study the equilibrium and the optimum without congestion. The main result in this first section is the generalization of the imperfect competition setting from competition in prices only to competition in prices and wages. In Section 6 we add congestion and study the effect of congestion on the Nash equilibrium in the short and the medium term. In Section 7 we discuss the potential of three types of policies: road congestion charging, limiting the number of subcenters, and extending the access capacity to the different subcenters. Section 8 concludes.

2. The model setting

We consider a center, and n subcenters. Residents are located in the center and consume a differentiated good and a homogeneous good. They supply differentiated labor as well as homogeneous labor. Each resident is active and provides the same amount of work. The homogeneous good is produced competitively in the center using homogeneous labor and requires no transport costs. We focus our attention on the production and the consumption of the differentiated good. There are n differentiated goods with subcenter i producing the quantity D_i such that $D = \sum_{i=1,\dots,n} D_i$. In each subcenter, one producer offers one variety of the good (e.g., due to increasing returns to scale), hires heterogeneous labor, uses the homogeneous good as intermediary input and sells his product at the factory gates. We denote by t_i the travel time per trip between the center and subcenter i (distance divided by speed). Households commute to subcenter i to supply labor with a travel time of $\alpha^w t_i$, where α^w denotes the number of trips per unit of labor. Households also make shopping trips to subcenter j with a travel time $\alpha^d t_j$, per unit of differentiated good, where α^d denotes the number of shopping trips per unit of consumption, for $i, j = 1, \dots, n$. These two trips are treated as independent (trip chaining is not considered here). The intermediary (homogeneous) goods needed in the subcenters are transported from the center to the subcenter with a travel time per unit of intermediary good of $\alpha^h t_i$, where α^h denotes the number of freight trips per unit of production. We first neglect congestion; in this case transportation cost t_i is independent of the number of drivers using the road. From Section 6 onwards, we treat congestion by recognizing that the transportation cost increases with the number of cars and trucks and decreases with road capacity.

2.1. The production possibilities

There are N households who all work and each household supplies a fixed amount, $(1+\theta)$ units of time, devoted to production and transportation. The production of one unit of the differentiated good requires one unit of labor time. The remaining labor time of the household θ is

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devoted to the production of the homogeneous good. Each household consumes one unit of the differentiated good, the rest of his income is spent on the homogeneous good.

We assume linear production technologies. The homogeneous good is produced using labor in a one-to-one ratio (one unit of the homogeneous good is produced during one unit of time). The homogeneous good is either consumed directly or used as input for the differentiated good and for the transport services (fixed and variable input). The production of the differentiated good in subcenter i requires a fixed set-up cost F (in the form of inputs of the homogeneous good) per subcenter and an intermediate input equal to c^1 units of the homogeneous good per unit of the differentiated good. Moreover, each subcenter requires some road infrastructure. The production of this road infrastructure requires K units of the homogeneous good. The total consumption of the homogeneous good is denoted by G.

We can present the total production possibilities of the economy by comparing the net inputs and the total uses of the homogeneous good. We have the following identity for the supply and the demand for labor:

$$(1+\theta)N = D + c^1 D + nF + \left(\alpha^w + \alpha^d + \alpha^h\right) \sum_{i=1,\dots,n} t_i D_i + nK + G, \tag{1}$$

where the LHS represents the total supply of labor. The first term in the RHS represents the direct use of labor in the production of the differentiated good (D with $D = N$) while the remaining terms represent the use of the homogeneous good as input into the production of the differentiated good ($c^1 D + nF$), to pay for the transportation costs $(\alpha^w + \alpha^d + \alpha^h) \sum_{i=1,\dots,n} t_i D_i$ and to pay for the infrastructure cost nK. The remaining production of the homogeneous good (G) is used as the final consumption good by the household.

The total consumption of the homogeneous good, G, is endogenous. It is computed as a residual and is given by the resources available when variable costs of heterogeneous goods production, time costs and fixed production and infrastructure costs are accounted for

$$G = \theta N - c^1 D - \left(\alpha^w + \alpha^d + \alpha^h\right) \sum_{i=1,\dots,n} t_i D_i - n(F+K).$$

As expected, the production of the homogeneous good, decreases with transport costs. In Section 6, congestion is taken into account and transport costs are themselves endogenous. In the symmetric case (considered in most of this paper), $t_i = t,\ i = 1,\dots,n$ and the average individual consumption of homogeneous good $g(n)$ where there are n subcenters, is given by

$$g(n) = \theta - c^1 - \left(\alpha^w + \alpha^d + \alpha^h\right)t - n(F+K)/N. \tag{2}$$

Note that at least one center (center 1) is sustainable provided that

$$g(1) = \theta - c^1 - \left(\alpha^w + \alpha^d + \alpha^h\right)t - (F+K)/N > 0.$$

2.2. *Market structures and taxes*

The homogeneous good is produced competitively in the center. The wage in this industry is normalized to one. As the market is competitive and one needs one unit of homogeneous labor per unit of homogeneous product, we normalize the price and wage in the homogeneous industry

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to one (in this case, the transport cost is not incurred by the producers of the homogeneous good). As a consequence, the value of time is one[2] and the transport cost equals the travel time, t_i.

The (relative) price of the differentiated good i is denoted by p_i and the (relative) wage offered by firm i producing the differentiated good i is denoted by w_i, $i = 1, \dots, n$. The government finances the public infrastructure input by imposing a head-tax T and a fixed levy on the firms S: $nK = NT + nS$.

2.3. Household preferences

Each household consumes a variable amount of the homogeneous good (at the city center) and one unit of the differentiated good in one of the n subcenters. The choice of the differentiated good corresponds to a standard discrete choice model. For the preference foundations of this model we refer the reader to (Anderson et al. [8, Ch. 2]). It is assumed that each household must supply θ units of labor in the city center for the production of the homogeneous good and exactly one unit of labor in one subcenter for the production of the differentiated good. The choice of the differentiated workplace (subcenter) is again a standard discrete choice problem. As both labor supply and the quantity of the differentiated good are fixed, the consumption of the homogeneous good is the residual. We consider that each household chooses a single place of employment (besides the city center) and a single shopping destination (besides the city center). Therefore the only choice of interest for the household is the choice of the employment subcenter (where the differentiated good is produced) and the choice of the type of differentiated good to consume (where to shop).

The direct utility function of a household who supplies one unit of labor to the differentiated industry i and buys one unit of the differentiated good of type k is[3]

$$V_{ik} = g_{ik} + \tilde{h}_k - \tilde{\beta}_i - \beta\theta, \tag{3}$$

where g_{ik} represents the consumption of the homogeneous good (whose marginal utility is one), $\tilde{h}_k$ is the direct utility of the consumption of one unit of the differentiated good k, $\tilde{\beta}_i$ is the disutility of labor in subcenter i and β is the disutility of labor in the center.

We assume that households have an equal share of the total profit, $\sum_{l=1,\dots,n} \pi_l$ and that the profit share is small. As a consequence, consumers take the profits as given and the owner of a differentiated firm does not take into account the impact of his pricing policy on his utility as a consumer or as a worker.[4] The household budget constraint is

$$\left(w_i - \alpha^w t_i\right) + \theta + \frac{1}{N} \sum_{l=1,\dots,n} \pi_l = \left(p_k + \alpha^d t_k\right) + g_{ik} + T. \tag{4}$$

According to identity (4), the revenue from supplying labor to subcenter i minus the commuting cost plus the revenue from supplying labor to the center, plus the share in total profits is equal to the cost of consumption, including shopping cost, plus the cost of the homogeneous good plus

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[2] One unit of time allows the production of one unit of the homogeneous good, which has a price equal to one, so the opportunity cost of one unit of time spent on the road is one.

[3] Remember that the utility function is only defined when exactly one unit of the differentiated good is consumed and exactly one unit of differentiated labor is supplied. As this saves on notation, we assume this troughout this text.

[4] This way we avoid one of the major problems in general equilibrium with imperfect competition. For a survey see [3].

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the head tax. By substitution of the budget constraint in (3), we get the indirect utility function (recall that all prices and wages are normalized by the price of the homogeneous good):

$$U_{ik} = \left(w_i - \alpha^w t_i\right) - \tilde{\beta}_i + \theta(1-\beta) + \tilde{h}_k - \left(p_k + \alpha^d t_k\right) + \frac{1}{N}\sum_{l=1,\dots,n} \pi_l - T. \tag{5}$$

To recognize the fact that the jobs in the differentiated industry are heterogeneous, we model the disutility of labor, $\tilde{\beta}_i$ as a random variable

$$\tilde{\beta}_i = \beta_i - \mu^w \varepsilon_i, \tag{6}$$

where $\mu^w > 0$ is a scale parameter that measures employment heterogeneity and ε_i are i.i.d. double exponentially distributed.[5] The idiosyncratic terms ε_i express the match values between the employments and the workers.

Similarly, the goods produced in the subcenter are differentiated from the shoppers perspectives. We assume that

$$\tilde{h}_k = h_k + \mu^d \varepsilon_k, \tag{7}$$

where $\mu^d > 0$ is a scale parameter and ε_i are i.i.d. double exponentially distributed.[6]

We consider the symmetric case: $\beta_i = \beta,\ i = 1,\dots,n$ (the centers are on average equally attractive from the worker perspective), $h_k = h,\ k = 1,\dots,n$ (all differentiated goods have the same gross benefit), and $t_i = t,\ i = 1,\dots,n$ (all subcenters are equally far away). In this case, the conditional indirect utility (5) reduces to

$$U_{ik} = \Omega + w_i - p_k + \mu^w \varepsilon_i + \mu^d \varepsilon_k, \tag{8}$$

where

$$\Omega = -\left(\alpha^w t + \beta\right) + \theta(1-\beta) + h - \alpha^d t + (1/N)\sum_{i=1,\dots,n} \pi_l - T. \tag{9}$$

Note that this model requires information on the distribution of the match values (ε_i and ε_k). The precise value of the match value of a given household is unknown. In other words, the individuals are statistically independent and nothing changes in the model at the aggregate level if the match values were to change. As a consequence, the households are allowed to modify their employment choice and the shopping choices provided that this will not change the expected demand addressed to each firm and the expected number of workers hired by each firm.

2.4. *Profits of firms*

Recall that D_i denotes the demand addressed to Firm i (with $\sum_{i=1}^n D_i = N$), w_i the wage offered by Firm i and p_i the price charged by Firm i for one unit of the differentiated good. In the symmetric case, the marginal cost of intermediate inputs is $c = c^1 + \alpha^h t,\ i = 1,\dots,n$ (in the nonsymmetric case, it is $c_i = c^1 + \alpha^h t_i,\ i = 1,\dots,n$), and the marginal production cost is $c + w_i$. The profit of Firm i is

$$\pi_i(w,p) = (p_i - w_i - c)D_i - (F+S), \tag{10}$$

[5] The c.d.f. of the double exponential is $F(x) = \exp[-\exp(-x)]$.

[6] For symmetric distributions (such as for normal), this formulation is the same as $\tilde{h}_k = h_k - \mu^d \varepsilon_k$. Later on, we use double exponential distribution which lead to the Logit model with the specification (7). The specification $\tilde{h}_k = h_k - \mu^d \varepsilon_k$, with double exponential distribution leads to the reverse Logit, which is substantially less tractable (see [9]), and therefore it is not considered here.

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where $w = (w_1, \dots, w_n)$ and $p = (p_1, \dots, p_n)$ denote the wage and the price vectors.

3. Household choices

3.1. The labor market choices

Given the choice of subcenter k for shopping, the utility of working in i becomes (see [8]):

$$U_{i|k} = \Omega_k + w_i + \mu^w \varepsilon_i,$$

where $\Omega_k = \Omega - p_k + \mu^d \varepsilon_k$.

The probability that a worker chooses to commute to subcenter i is, given the choice of subcenter k for shopping, $P^w_{i|k} = \text{Prob}\{U_{i|k} \geqslant U_{j|k},\ j = 1, \dots, n\}$. Note that this choice probability is independent of k and therefore shall be written as P^w_i, with

$$P^w_i = \text{Prob}\bigl\{w_i + \mu^w \varepsilon_i \geqslant w_j + \mu^w \varepsilon_j,\ j = 1, \dots, n\bigr\}.$$

Using the fact that ε_i are double exponentially distributed

$$P^w_i = \frac{\exp\bigl(\frac{w_i}{\mu^w}\bigr)}{\sum_{j=1,\dots,n} \exp\bigl(\frac{w_j}{\mu^w}\bigr)}, \quad i = 1, \dots, n. \tag{11}$$

Therefore, the choice probabilities for the labor market have a logit type. Note that all the workers will select the job which offers the largest wage if the heterogeneity parameter μ^w is zero. Otherwise, a worker may accept a reduced wage in order to work for a firm which best fits his preferences. The average expected number of workers in subcenter i is NP^w_i.

3.2. Consumer choices

When a household is choosing in which subcenter k to shop, all the terms $(w_i + \mu^w \varepsilon_i)$ connected with the choice of employment are identical and therefore do not affect their choice. In this case, we can rewrite the conditional utility of shopping in k given the choice of workplace i as (see Eq. (8))

$$U_{k|i} = \Omega_i - p_k + \mu^d \varepsilon_k,$$

where $\Omega_i = \Omega + w_i + \mu^w \varepsilon_i$. The probability that a household located in the center patronizes subcenter k is $P^d_{k|i} = \text{Prob}\{U_{k|i} \geqslant U_{l|i},\ l = 1, \dots, n\}$. As before, the choice probability $P^d_{k|i}$ is independent of the choice i and denoted by P^d_k. We have $P^d_k = \text{Prob}\{-p_k + \mu^d \varepsilon_k \geqslant -p_l + \mu^d \varepsilon_l,\ l = 1, \dots, n\}$. With the double exponential distribution, we get:

$$P^d_k = \frac{\exp\bigl(\frac{-p_k}{\mu^d}\bigr)}{\sum_{l=1,\dots,n} \exp\bigl(\frac{-p_l}{\mu^d}\bigr)}, \quad k = 1, \dots, n. \tag{12}$$

3.3. Market clearing conditions

Recall that every household consumes one unit of the differentiated good and that the production of every unit of the differentiated good requires one unit of labor (provided by one household). Assuming that the labor market clears (wages are flexible), the fraction of workers which decides to work at subcenter i must be equal to the fraction of shoppers which patronize

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subcenter i, whatever the wages and the prices. Thus $P_i^w = P_i^d$, where P_i^w is given by (11) and P_i^d is given by (12). We get a relation between the price p_i and the wage w_i set by Firm i

$$\frac{\exp\left(\frac{w_i}{\mu^w}\right)}{\sum_{j=1,\dots,n}\exp\left(\frac{w_j}{\mu^w}\right)} = \frac{\exp\left(\frac{-p_i}{\mu^d}\right)}{\sum_{j=1,\dots,n}\exp\left(\frac{-p_j}{\mu^d}\right)}. \tag{13}$$

Therefore, the demand for the differentiated product sold in subcenter i is $D_i = NP_i^d = NP_i^w$.

4. Equilibrium without congestion

4.1. The profit function

We look for a symmetric Nash equilibrium in prices and wages between firms (or subcenters). The strategic variables of subcenter i are w_i and p_i. Given the market clearing condition (13), the choice of w_i determines the choice of p_i and vice versa.

Consider subcenter i which takes all other wages and prices as given. Since the LHS of (13) is strictly increasing in w_i and the RHS of this equation is strictly decreasing in p_i, there is a one-to-one relation between w_i and p_i, the other prices and wages being fixed. Let $p_i = f_i(w_i)$. Note that $f_i(w_i) = f(w_i, w_{-i}, p_{-i})$ where w_{-i} and p_{-i} are the vectors w and p with the ith component missing. We shall use the following result:

$$\frac{\mathrm{d}f_i(w_i)}{\mathrm{d}w_i} = -\frac{\frac{P_i^w(1-P_i^w)}{\mu^w}}{\frac{P_i^d(1-P_i^d)}{\mu^d}} = -\frac{\mu^d}{\mu^w} < 0. \tag{14}$$

This expression is negative since when a firm raises its wage, it increases the number of workers hired. In order to be able to sell the additional production, a firm needs to reduce its prices. The price reduction needs to be larger when μ^d is larger because then the consumers are more loyal to their ideal product. Conversely, the price reduction is smaller when μ^w is larger, since in this case the workers are more loyal to their preferred workplace and less amenable to changing jobs for a wage increase.

Given the relation between price and wage of Firm i, the profit of subcenter i only depends on a single strategic variable (we select the wage as the strategic variable). In this case $\pi_i(w_i, w_{-i}, f_i(w_i), p_{-i}) = \tilde{\pi}_i(w_i, w_{-i}, p_{-i})$ with

$$\tilde{\pi}_i(w_i, w_{-i}, p_{-i}) = \left[f_i(w_i) - w_i - c\right]NP_i^w - (F + S), \tag{15}$$

where we use the identity $D_i = NP_i^w$, and where $c = c^1 + \alpha^h t$.

4.2. Short-run equilibrium

In the short-run equilibrium, we keep the number of firms fixed. The road size is kept fixed too but this is irrelevant here since by assumption there is no congestion.

Subcenters are competing in wages and prices in a noncooperative Nash game. We wish to find the candidate symmetric equilibrium in prices and wages denoted by (p^e, w^e). As shown above there is a market clearing condition that links the product and the labor markets so the subcenters compete in either wage or price. We consider here that the strategic variable is the wage, w_i.

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The best reply of subcenter i to the wages and prices set by the other subcenters is

$$\frac{\mathrm{d}\tilde{\pi}_i(w_i, w_{-i}, p_{-i})}{\mathrm{d}w_i} = \left\{-1 + \big(f_i(w_i) - w_i - c\big)\frac{(1-P_i^w)}{\mu^w}\right\} N P_i^w + \frac{\mathrm{d}f_i(w_i)}{\mathrm{d}w_i} N P_i^w = 0. \tag{16}$$

The first term in this expression corresponds to the standard term in oligopoly partial equilibrium models, while the second term ($\mathrm{d}f_i(w_i)/\mathrm{d}w_i \; N P_i^w$, which is negative) is specific to the interaction between labor and the product markets. Note that at the symmetric candidate equilibrium $P_i^w = P_i^d = 1/n$ and recall that $\mathrm{d}f_i(w_i)/\mathrm{d}w_i = -\mu^d/\mu^w$. Therefore (16), set at the symmetric candidate equilibrium, leads to

$$-\left(\frac{\mu^d}{\mu^w} + 1\right) + \big(p^e - w^e - c\big)\frac{(n-1)}{n\mu^w} = 0.$$

We prove in Appendix A that the candidate equilibrium is a Nash equilibrium. Therefore

Proposition 1. *In the absence of congestion, there exists a unique symmetric Nash equilibrium in prices and wages given by*

$$p^e = c + w^e + \big(\mu^d + \mu^w\big)\frac{n}{(n-1)}. \tag{17}$$

Interestingly, the equilibrium markup $p^e - (c + w^e)$ is increasing with product heterogeneity but also with job heterogeneity as a consequence of the interplay between labor and product markets. The role of product heterogeneity is well known (see Anderson et al. [8]) while the role of job heterogeneity is new: more job heterogeneity means that workers are also interested in other dimensions than the wage they earn (such as the proximity of the gym facility or the charms of the boss) so that wage differences become less important and this increases the profit margin. Interestingly, both types of heterogeneity work in the same direction and are additive. Each firm has market power, which decreases when the number of competitors increases. The markup remains bounded away from zero as $n \to \infty$, since each firm keeps a monopoly power in the product and in the labor markets. The equilibrium markup in the symmetric monopolistic competition models à la Chamberlin is strictly positive and given by $\lim_{n\to\infty}[p^e - (c + w^e)] = \mu^d + \mu^w > 0$.

4.3. Long-run equilibrium

In the long-run equilibrium, we allow the number of subcenters to vary by free entry and exit. The long-run equilibrium is such that the profit of each subcenter is zero (we neglect integer problems).

The equilibrium profit (see (15)) at the symmetric equilibrium is

$$\pi^e = \big[p^e - w^e - c\big]\frac{N}{n} - (F + S)$$

or after substitution of the equilibrium price levels (see Proposition 1)

$$\pi^e = \big(\mu^d + \mu^w\big)\frac{N}{(n-1)} - (F + S).$$

The profit is a decreasing function of the number of subcenters: further entry drives profits to zero.

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The long-run n^f number of subcenters is

$$n^f = 1 + \left(\mu^d + \mu^w\right)\frac{N}{F+S} > 1. \tag{18}$$

At the free entry equilibrium, the consumption g^f of the homogeneous good is

$$g^f = g(1) - \left(\mu^d + \mu^w\right)\frac{(F+K)}{(F+S)},$$

where $g(1)$ is given by Eq. (2). Less homogeneous good is consumed when the product differentiation and/or the job heterogeneity increases since both factors increase profit margins and with free entry, also the number of firms. In this case, a larger number of firms increases the resource cost needed to produce the differentiated good, $n(F+K)$, and therefore decreases the amount of residual consumption of the homogeneous good.

5. Optimum without congestion

5.1. The welfare function

In the first-best, all quantities can be chosen freely and the only constraints are the production possibilities. Using the definition of the utility function (3), (6), and (7), and the production possibility constraint (2), we obtain, for the per capita welfare to be optimized, denoted by $W(n)$:

$$\begin{aligned} W(n) &= \left[g(n) - \beta\theta\right] + E\left[\max(\tilde{h}_k - \tilde{\beta}_k)\right] \\ &= \left[\theta - c^1 - \left(\alpha^w + \alpha^d + \alpha^h\right)t - n(F+K)/N - \beta\theta\right] \\ &\quad + \left\{(h-\beta) + E\left[\max_{i,k}\left(\mu^w\varepsilon_i\right)\right] + E\left[\max_{i,k}\left(\mu^d\varepsilon_k\right)\right]\right\}. \end{aligned}$$

Using the expression for the expected maximum if i.i.d. random variable (recall that with the double exponential distribution: $E[\max_i \varepsilon_i] = \ln(n)$ (see Anderson et al. [8]), we obtain

$$W(n) = \Psi - \frac{n}{N}(F+K) + \left(\mu^d + \mu^w\right)\log(n), \tag{19}$$

where Ψ is given by

$$\Psi = -\beta + \theta(1-\beta) + h - c^1 - \left(\alpha^h + \alpha^d + \alpha^w\right)t. \tag{20}$$

The first-best optimum in the short run (exogenous number of subcenters) and in the long run (endogenous number of subcenters) is characterized by

Proposition 2. *In the absence of congestion, the short-run first-best optimum welfare function is given by*

$$W(n) = \Psi - \frac{n}{N}(F+K) + \left(\mu^d + \mu^w\right)\log(n), \tag{21}$$

where Ψ is given by (20)*. The long-run first-best optimum number of subcenters is*

$$n^{\mathrm{o}} = \left(\mu^d + \mu^w\right)\frac{N}{(F+K)}. \tag{22}$$

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Proof. It remains to determine the long run optimum. The function $W(n)$ is concave in n. The optimal number of subcenters, n^{o} given by (22) is obtained by differentiation of $W(n)$ where n is treated as a real number. □

At the optimum, the consumption of the homogeneous good is $g^{\mathrm{o}} = g(n^{\mathrm{o}})$ or

$$g^{\mathrm{o}} = \theta - c^1 - \left(\alpha^w + \alpha^d + \alpha^h\right)t - \left(\mu^d + \mu^w\right) = g(0) - \left(\mu^d + \mu^w\right).$$

Where $g(n)$ is given by (2). Note that this expression is independent of the fixed costs F and K. The comparative statics on the first-best number of subcenters and on the consumption of the goods are left to the reader.

5.2. *Equilibrium versus optimum number of subcenters*

We can now compare the equilibrium and the optimum numbers of subcenters. Note that

$$(p - w)^{\mathrm{e}} = (p - w)^{\mathrm{o}} + \left(\mu^d + \mu^w\right)\frac{n}{n-1},$$

that is to say firms charge a price (net of wage) above the socially optimal level (c). However the excessive price level will not induce distortions in the economy in the short run since the demand for the differentiated goods and the differentiated labor supply are inelastic. We show below that the market power of firms induces excessive entry in the long run.

If firms pay the total cost of the road infrastructure to their subcenter ($S = K$), the equilibrium number of subcenters is larger than the optimum one ($n^f > n^{\mathrm{o}}$), where n^f is given by (18), and n^0 is given by (22): $n^f = 1 + (\mu^d + \mu^w)\frac{N}{F+K}$ and

$$n^f = 1 + n^{\mathrm{o}}. \tag{23}$$

Anderson et al. [8] showed that monopolistic competition in a product market with a logit model always generates an overentry of exactly one firm.[7] We generalized this result to the general equilibrium context with heterogeneous product and labor markets envisaged in this paper. The intuition for this result is that the introduction of an heterogeneous job market corresponds to an additional source of heterogeneity. However, since the labor and product market are related, the total degree of product heterogeneity stays about the same and the number of firms is too large by exactly one unit as in the case where there is only product differentiation.

Assume that $1 \leqslant n^{\mathrm{o}}$, that is $(F + K)/N \leqslant (\mu^d + \mu^w)$.[8] In this case, since there is excessive entry, there exists a level of tax which can decentralize the social optimum. The optimal tax is given by

$$S = \frac{n^{\mathrm{o}}K + F}{(n^{\mathrm{o}} - 1)}. \tag{24}$$

Therefore the firm should optimally be charged more than the price of the infrastructures (since $S > K$). We have

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[7] The excess entry is the norm for discrete choice models with log-concave error term. The one firm result is specific to the Logit. For the pure oligopolistic model (one market), the upper limit, attained on the boundary of the cone of log-concave function is about 12% excessive entry (see [2]).

[8] The minimum number of firms at the optimum is equal to one, since each consumer has to buy a product.

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Proposition 3. *In the absence of congestion, in a free-entry Nash equilibrium, the first-best optimum can be decentralized by a levy per firm larger than the infrastructure cost per subcenter. If the levy covers exactly the infrastructure cost per subcenter, at equilibrium there is one subcenter too many.*

6. Equilibrium with congestion

6.1. Model setting

We have assumed till now that the travel time on road i, t_i, is constant. From now on, we drop this assumption and explicitly recognize that congestion may occur. In this case, travel time cost on road i is an increasing function of the number of vehicles on this road. Each road is occupied by shoppers and by commuters as well as by trucks (that deliver the intermediate input from the center to the subcenters). If they travel at the same time, the usage on road i, expressed in car equivalent, is

$$\rho_i = N\left[\left(\kappa\alpha^h + \alpha^d\right)P_i^d + \alpha^w P_i^w\right], \tag{25}$$

where κ represents the car equivalent of a truck. Alternatively, if trucks are traveling off peak and without congestion, then $\rho_i = N[\alpha^d P_i^d + \alpha^w P_i^w]$. If only shopping cars experience congestion, then: $\rho_i = N\alpha^d P_i^d$, etc. To fix ideas, we retain here expression (25). The other cases are straightforward to analyze.

The relation between travel cost, t_i, and total activity on the road i, ρ_i, is given by

$$t_i = t + \delta\frac{\rho_i}{s}. \tag{26}$$

The first term t represents the transport time in the absence of any congestion. The second term in (26) represents the variable travel cost, where s is the exogenous capacity of the road measured in car equivalent, and where δ is a coefficient which depends on schedule delay costs parameters for early and late arrivals. This expression is the reduced form of the bottleneck equilibrium cost (see Arnott et al. [4]), where road users decide on their trip timing.[9] To ease the exposition, we consider here the simplest version of the bottleneck model that involves only one type of users that have all the same values of time, the same schedule delay parameters and the same desired arrival times.[10]

Recall the market clearing condition (13): $P_i^d = P_i^w$. Equation (26) reduces to

$$t_i = t + \delta\frac{N}{s}\alpha P_i^d = t + \delta\frac{N}{s}\alpha P_i^w, \tag{27}$$

[9] In a dynamic model, users select departure time and route choice, and wish at the same time to reduce travel time and early/or late arrival at destination. At equilibrium, if users are equal, all users incur the same cost, which depends on the parameter values of the problem: demand over capacity ratio (ρ_i/s) and demand parameters: values of queuing time, of early and late schedule delays φ and $\varkappa$, respectively. Here only a combination δ of these parameters enters in the cost function with: $\delta = \varphi\varkappa/(\varphi + \varkappa)$. Note that the reduced form of the equilibrium cost $\delta\rho_i/s$ can be found directly without computing the equilibrium solution by only using the conservation law of the number of drivers and the equilibrium condition which implies that all users (and in particular the first and the last one) incur the same cost. The same formula is valid with different classes of users who wish all to arrive at the same time and with proportional parameter values.

[10] One can allow for some degree of heterogeneity. Users can be differentiated with respect to their desired arrival time, and their values of δ, provided that there are discrete homogeneous classes (such as shoppers, workers, freight transportation) with sufficiently differentiated arrival times so that the classes of vehicles do not interact. In this case, the cost function can be written as follows: $(\sum \delta^k \rho_i^k)/s$. It could be assumed, also, that some users do not experience congestion: this is the case if the distribution of desired arrival time is sufficiently spread over time.

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where $\alpha \equiv \kappa\alpha^h + \alpha^d + \alpha^w$. In the symmetric case, $P_i^w = P_i^d = 1/n$ and the travel cost, denoted by t^e, is the same on all routes

$$t^e = t + \frac{\delta}{n}\frac{N}{s}\alpha. \tag{28}$$

6.2. Demand for goods and supply of labor

With congestion, the indirect utility of a consumer working at i and consuming at k is $U_{ik} = \Omega_{ik} + w_i - p_k + \mu^w\varepsilon_i + \mu^d\varepsilon_k$, where

$$\Omega_{ik} = \left(-\alpha^w t_k - \beta\right) + \theta(1-\beta) + h - \alpha_i^d t_i + (1/N)\sum_{i=1,\dots,n}\pi_l - T.$$

Using the same notation as in the noncongested case, this expression can we written as

$$U_{ik} = \Omega - \Lambda^w P_i^w - \Lambda^d P_k^d + w_i - p_k + \mu^w\varepsilon_i + \mu^d\varepsilon_k, \tag{29}$$

where Ω is given by Eq. (9), $\Lambda^w = \alpha^w\delta\frac{N}{s}\alpha$ and $\Lambda^d = \alpha^d\delta\frac{N}{s}\alpha$.

As in the noncongestion case, we need to compute the derivative $\mathrm{d}g(w_i)/\mathrm{d}w_i$, where $p_i = g_i(w_i)$ (see (14) in the noncongestion case). With congestion, the probability that a consumer purchases good i is

$$P_k^d = \frac{\exp\left(\frac{-p_k - \Lambda^d P_k^d}{\mu^d}\right)}{\sum_{l=1,\dots,n}\exp\left(\frac{-p_l - \Lambda^d P_l^d}{\mu^d}\right)}. \tag{30}$$

This equation reduces to (12), when the variable travel time is zero or in the symmetric case ($P_k^d = 1/n$). This is an implicit equation since the travel time on route k depends on the total traffic on route k, which is an increasing function of P_k^d (see Eq. (27)).

Since the travel costs depend on congestion, they cannot be assumed to be symmetric. Indeed, when a firm deviates from a symmetric candidate equilibrium, it will affect road use and travel costs. For example, a price cut in subcenter i will increase the level of demand, labor supply and intermediate inputs and therefore the level of congestion and the travel cost t_i.

Using the implicit function theorem, we get

$$\frac{\mathrm{d}P_k^d}{\mathrm{d}p_k} = -\frac{\frac{1}{\mu^d}P_k^d(1-P_k^d)}{1 + \frac{\Lambda^d}{\mu^d}P_i^d(1-P_i^d)}.$$

Therefore, in the symmetric case $P_i^d = 1/n$

$$\left.\frac{\mathrm{d}P_i^d}{\mathrm{d}p_i}\right|_{\mathrm{Sym}} = -\frac{\frac{1}{\mu^d}\frac{1}{n}\left(\frac{n-1}{n}\right)}{1 + \frac{\Lambda^d}{\mu^d}\frac{1}{n}\left(\frac{n-1}{n}\right)} < 0. \tag{31}$$

Note that the price sensitivity in the symmetric case decreases as the impact of congestion measured by Λ^d (that contains α^d and α) gets larger. Congestion decreases the incentive to cut prices, since a lower price implies more customers, more workers and more intermediate deliveries and therefore more congestion, which both reduce the benefit of the initial price cut. In fact the initial price cut is compensated partially by congestion so that the firm is exchanging a lower profit margin for more time losses rather than for more customers. With an extremely high level of congestion ($\Lambda^d \to \infty$) the demand for one specific variety is inelastic.

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Similarly, for the labor market we have

$$P_i^w = \frac{\exp\left(\frac{w_i - \Lambda^w P_i^w}{\mu^w}\right)}{\sum_{j=1,\dots,n} \exp\left(\frac{w_j - \Lambda^w P_j^w}{\mu^w}\right)} > 0. \tag{32}$$

This expression reduces to (11) when the variable travel time is zero: when there is no congestion. At a symmetric situation

$$\left.\frac{\mathrm{d}P_i^w}{\mathrm{d}w_i}\right|_{\mathrm{Sym}} = \frac{\frac{1}{\mu^w}\frac{1}{n}\left(\frac{n-1}{n}\right)}{1 + \frac{\Lambda^w}{\mu^w}\frac{1}{n}\left(\frac{n-1}{n}\right)}. \tag{33}$$

The market clearing condition $P_i^w - P_i^d = 0$ (see (13)) has a unique solution $p_i = g_i(w_i)$ given that $\mathrm{d}P_i^w/\mathrm{d}w_i > 0$ and $\mathrm{d}P_i^d/\mathrm{d}p_i < 0$. We have

$$\left.\frac{\mathrm{d}g_i(w_i)}{\mathrm{d}w_i}\right|_{\mathrm{Sym}} = -\frac{\mu^d}{\mu^w}\frac{1 + \frac{\Lambda^d}{\mu^d}\frac{1}{n}\left(\frac{n-1}{n}\right)}{1 + \frac{\Lambda^w}{\mu^w}\frac{1}{n}\left(\frac{n-1}{n}\right)}. \tag{34}$$

There are two limiting cases of interest. First, without congestion, this expression reduces to Eq. (14). This case can also be obtained in the limit where the product and the labor market diversities are very large compared to congestion ($\mu^d \gg \Lambda^d$ and $\mu^w \gg \Lambda^w$). Second, when congestion costs are present and very high compared to the product and labor market diversities ($\Lambda^d \gg \mu^d$ and $\Lambda^w \gg \mu^w$), then

$$\left.\frac{\mathrm{d}p_i}{\mathrm{d}w_i}\right|_{\mathrm{Sym}} = -\frac{\Lambda^d}{\Lambda^w} - \frac{\alpha^d}{\alpha^w}.$$

In this case the wages and the prices are solely driven by the level of congestion, since the workers and the shoppers select their destination only as a function of variable travel times.

6.3. Short-run equilibrium

We study first the equilibria in the absence of government interventions: no congestion pricing, no limit on the number of centers and an exogenous road capacity. As before we assume that in the short run, the number of subcenters is given.

We know that the marginal cost is $c_i = c^1 + \alpha^h t_i$, where $t_i = t + \delta\frac{\rho_i}{s}$, and road usage ρ_i is given by (25). Since the travel time t_i is variable, the marginal cost becomes variable and endogenous. We have

$$c_i = c^1 + \alpha^h\left(t + \delta\frac{\rho_i}{s}\right) = c + \Lambda^h P_i^w,$$

where $\Lambda^h = \alpha^h\delta\frac{N}{s}\alpha$ (using Eq. (27)), and where we have defined $c = c^1 + \alpha^h t$. This means that the firm bears directly, via the intermediate delivery cost, part of the congestion costs it creates. Using the market clearing condition, the profit of Firm i is

$$\tilde{\pi}_i(w_i, w_{-i}, p) = \left[g_i(w_i) - w_i - c - \Lambda^h P_i^w\right] N P_i^w - (F + S). \tag{35}$$

The first-order condition for optimal wage (and price) setting is: $\mathrm{d}\tilde{\pi}_i/\mathrm{d}w_i = 0$ or

$$\left[\frac{\mathrm{d}g_i(w_i)}{\mathrm{d}w_i} - 1\right] P_i^w + \left[g_i(w_i) - w_i - c - 2\Lambda^h P_i^w\right]\frac{\mathrm{d}P_i^w}{\mathrm{d}w_i} = 0.$$

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Substituting the expressions (34) and (33), the first-order condition, at the symmetric candidate equilibrium reduces to

$$\left[\frac{\mu^d}{\mu^w}\left(1+\frac{\Lambda^d}{\mu^d}\frac{1}{n}\left(\frac{n-1}{n}\right)\right)+1+\frac{\Lambda^w}{\mu^w}\frac{1}{n}\left(\frac{n-1}{n}\right)\right]$$
$$-\left(p^e-w^e-c-\Lambda^h\frac{2}{n}\right)\frac{1}{\mu^w}\frac{n-1}{n}=0. \tag{36}$$

Therefore, the candidate equilibrium price is given by the solution of (36)

$$p^e=c+\frac{\Lambda^h}{n}+w^e+\left(\mu^d+\mu^w\right)\frac{n}{n-1}+\frac{\delta}{n}\frac{N}{s}\hat{\alpha}^2,$$

where $\hat{\alpha}=\sqrt{\alpha(\alpha^h+\alpha^d+a^w)}$ and where $\Lambda^h=\alpha^h\delta\frac{N}{s}\alpha$. Note that when $\kappa=1$, $\hat{\alpha}=\alpha$. The markup $(p^e-c-\Lambda^h/n-w^e)$ now has two components. The first one is the product/wage heterogeneity term (proportional to $(\mu^d+\mu^w)$), as in the noncongested case. The second term, represents the externality due to congestion, which is equal to the variable user cost.[11] One easy way to understand the role of congestion, is to introduce optimal time dependent road pricing[12]: with such pricing, the variable transport cost is halved and therefore equilibrium price is given by (38).[13]

Proposition 4. *With congestion, there exists a unique symmetric Nash equilibrium in prices and wages given by*

$$p^e=c+\frac{\Lambda^h}{n}+w^e+\left(\mu^d+\mu^w\right)\frac{n}{n-1}+\frac{\delta}{n}\frac{N}{s}\hat{\alpha}^2. \tag{37}$$

With optimal congestion pricing, there exists a unique symmetric Nash equilibrium in prices and wages given by

$$p^e=c+\frac{\Lambda^h}{n}+w^e+\left(\mu^d+\mu^w\right)\frac{n}{n-1}+\frac{\delta}{2n}\frac{N}{s}\hat{\alpha}^2. \tag{38}$$

Proof. See Appendix B. □

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[11] The total cost is $TC=\alpha N t^e=\alpha N(t+\delta\alpha\frac{N}{sn})$. Therefore the externality, which is the difference between the marginal cost and the average cost is equal to: $\delta\alpha^2\frac{N}{sn}=\alpha(t^e-t)$.

[12] We only discuss fine and step tolls. A fine toll evolves continuously over time. A one-step toll means that the relevant period can be subdivided into two periods: one period with a fixed toll and one period without a toll. This is a much simpler but also a socially less performant instrument than a fine toll. We could consider other charging instruments (cordon tolls or parking levies that are not time differentiated) but these can in our simple model be reduced to head taxes per consumer or to a levy per firm. Fixed levies are not able to change the distribution over time of trips and are therefore not efficient in reducing congestion. They can only affect the total level of demand for the differentiated good which is fixed in this paper.

With the bottleneck congestion model (Arnott et al. [4]), the total variable travel cost per individual $(\hat{\alpha})^2\frac{\delta}{n}\frac{N}{s}$ can be reduced by a factor 2 when an optimal fine toll is used and by a factor 4/3 when an optimal one-step toll is used. With an optimal fine toll, there are only schedule delay costs left as queuing is eliminated. The average congestion charge that corresponds to the fine toll equilibrium will be equal to the average schedule delay cost. With an optimal coarse toll, queuing is not completely eliminated.

[13] The average consumer price including toll is $p^e_{cp}+\alpha^d\frac{\delta}{2n}\frac{N}{s}\alpha$, while the average net wage after deduction of the toll is $w^e_{cp}-\alpha^w\frac{\delta}{2n}\frac{N}{s}\alpha$.

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The existence proof, relegated in Appendix B, is quite complex, due to the fact that the two markets (product and labor) are interdependent. Note that without congestion, the equilibrium price reduces to Eq. (17). Equation (37) implies that the equilibrium price and profit margins increase as congestion builds up (by example through exogenous reduction of the road capacity). With congestion, there are two additional positive terms in the RHS. First the marginal production cost is now $c + \Lambda^h/n + w^e$ and contains a congestion term translating the increased cost of intermediate deliveries. The second term is related to the congestion created by shopping, commuting and intermediate delivery traffic and represents the increased market power effect.

As discussed, congestion reduces the incentive to cut prices, and therefore, increases equilibrium prices. This may explain why shops often lobby against policy measures which aim to improve traffic conditions although a firm individually will be in favor of local improvements of traffic, i.e. measures which improve the accessibility to workers and consumers.

The short-run equilibrium profit (see Eqs. (35) and (37)) is

$$\pi^e(s) = \left(\mu^d + \mu^w\right)\frac{N}{(n-1)} + \frac{\delta}{s}\left(\frac{\hat{\alpha}N}{n}\right)^2 - (F+S) \tag{39}$$

which is an increasing function of the congestion level. Note that the profit is lower with road pricing: it decreases by $\delta/2s(\hat{\alpha}N/n)^2$ (see Eq. (38)).

6.4. Long-run equilibria

For a fixed level of road capacity s, the free entry equilibrium with congestion denoted by $n^f(s)$ solves $\pi^e = 0$. In order to study the free entry equilibrium, we need to specify the fixed levy per firm S. As the default value, we use $S = K$ so that every firm pays the public infrastructure that is specific to the firm. This leads to a cubic equation, and its solution is not too illuminating. The profit $\pi^e(s)$ is a decreasing function of the number of subcenters n. Given that the equilibrium profit with congestion is larger than without, the free entry equilibrium with congestion involves more firms than in the absence of congestion: $n^f(s) > n^f$.

We can find a lower bound ($n^o(s) < n^f(s)$) and an upper bound for the solution of (39). As lower bound, we use $n^o(s)$ that is the solution of the following equation:

$$\left(\mu^d + \mu^w\right)\frac{N}{n^o(s)} + \frac{\delta}{s}\left(\frac{\hat{\alpha}N}{n^o(s)}\right)^2 - (F+K) = 0. \tag{40}$$

We will show in the next section that $n^o(s)$ is the optimal number of firms for given road capacity and in the absence of congestion charging. Observe that $\pi^e(n^o) > 0$, $n^o(s) < n^f(s)$, so that Eq. (40) has a unique positive root given by

$$n^o(s) = n^o + \frac{n^o}{2}\left(\sqrt{\left(\frac{\hat{\alpha}N}{n^o}\right)^2\frac{4\delta}{s(F+S)} + 1} - 1\right), \tag{41}$$

where $n^o = (\mu^d + \mu^w)N/(F+S)$ represents the optimum number of subcenters without congestion (see Eq. (22)) provided that the firm pays the road infrastructure cost ($S = K$). Note that, as expected, the number of subcenters increases when the level of monetary cost associated to congestion increases, that is, when the value of parameter δ increases.

As upper bound for $n^f(s)$, we use $n^o(s) + 1$. We have

$$\pi^e\left(n^o(s) + 1\right) = \left(\mu^d + \mu^w\right)\frac{N}{n^o(s)} + \frac{\delta}{s}\left(\frac{\hat{\alpha}N}{n^o(s)+1}\right)^2 - (F+K).$$

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Subtracting (40) from this equation, we get

$$\pi^e\left(n^{\mathrm{o}}+1\right)=\frac{\delta}{s}(\hat{\alpha}N)^2\left(\frac{1}{(n^{\mathrm{o}}+1)^2}-\frac{1}{(n^{\mathrm{o}})^2}\right)<0.$$

As a consequence, $n^f(s)<n^{\mathrm{o}}(s)+1$.

Summarizing, for given road capacity, in the absence of road pricing and for an infrastructure charge on firms $S=K$, we have an upper and lower bound for the equilibrium number of subcenters where $n^{\mathrm{o}}(s)$ denotes the optimal number of subcenters

$$n^{\mathrm{o}}(s)<n^f(s)<n^{\mathrm{o}}(s)+1; \qquad n^f<n^f(s). \tag{42}$$

Therefore, given that the optimum number of subcenters $n^{\mathrm{o}}(s)$ increases with congestion costs, so does the equilibrium number of subcenters $n^f(s)$. From the equilibrium point of view, congestion decreases competition and therefore increases market opportunities and encourages entry. From the social point of view, additional congestion increases the negative externalities and therefore additional subcenters are beneficial.

The discussion concerned with the impact of congestion costs on dispersion has no place in this paper since subcenters are located at the same exogenous distance from the city-center. We refer the reader to Anas and Kim [11], who analyze (in the setting of perfect competition) the joint decisions of households of where to work, where to shop and where to live. Clearly, this approach and ours are complementary.

7. Optimum with congestion

7.1. *First-best optimum*

In the first-best, we control all consumption and production decisions. More specifically, we control the number of subcenters and the departure times of shoppers, workers and trucks as well as the size of the roads. The welfare per capita without congestion is given by (19). With congestion, we need to add the symmetric variable transport cost, which is defined in (28). The individual variable transport cost equals

$$\left(\alpha^h+\alpha^d+a^w\right)\left(\frac{\delta}{n}\frac{N}{s}\alpha\right).$$

Using the definition $\hat{\alpha}=\sqrt{\alpha(\alpha^h+\alpha^d+a^w)}$, and assuming that the road infrastructure cost is linear in capacity ($K=\xi^2 s$), the welfare function is

$$W(n,s)=\Psi-\frac{n}{N}\left(F+\xi^2 s\right)+\left(\mu^d+\mu^w\right)\log(n)-\frac{\delta}{n}\frac{N}{s}\hat{\alpha}^2. \tag{43}$$

When, in the bottleneck model, the departure times are set optimally for all road users, the variable transport costs are halved so that the welfare per capita (see Arnott et al. [4]) becomes

$$W(n,s)=\Psi-\frac{n}{N}\left(F+\xi^2 s\right)+\left(\mu^d+\mu^w\right)\log(n)-\frac{\delta}{2n}\frac{N}{s}\hat{\alpha}^2.$$

Note that, alternatively, the welfare function depends on the total road capacity (ns) and on the number of subcenters, n. We maximize this expression with respect to the number of subcenters n and the capacity of the road s to obtain (for interior solutions)

$$\frac{\partial W(n,s)}{\partial s}=\frac{1}{s}\left(-\frac{(ns)}{N}\xi^2+\frac{\delta}{2}\frac{N}{(ns)}\hat{\alpha}^2\right)=0, \tag{44}$$

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$$\frac{\partial W(n,s)}{\partial n} = -\frac{1}{N}\left(F+\xi^2 s\right) + \frac{(\mu^d+\mu^w)}{n} + \frac{\delta}{2n^2}\frac{N}{s}\hat{\alpha}^2 = 0. \tag{45}$$

The first-best number of subcenters is

$$n^{fb} = \frac{N}{F}\left(\mu^d+\mu^w\right).$$

It increases with product/labor heterogeneity, and decreases with the fixed production cost per firm.[14] Since only the total road capacity ns matters, as expected n^{fb} is independent of the transport parameters: δ, $\hat{\alpha}$ and ξ when there are constant returns to scale in road production.

The first-best capacity of the road increases with the user cost (δ) and decreases with transport cost (ξ). We obtain

$$s^{fb} = \sqrt{\frac{\delta}{2}}\frac{\hat{\alpha}N}{\xi n^{fb}} = \sqrt{\frac{\delta}{2}}\frac{\hat{\alpha}F}{\xi(\mu^d+\mu^w)}.$$

Comparative static results go along intuition and are left to the reader. In particular, note that when heterogeneity increases, there are more subcenters and narrower roads. However, total road capacity per individual in the urban system: $(n^{fb}s^{fb})/N = (\sqrt{\delta/2}\hat{\alpha})/\xi$ only depends on the transport parameters: on the user side δ and $\hat{\alpha}$ and on the road supply side ξ.

We can decentralize the first-best results by combining the following policies: time-dependent tolls which optimize departure times so as to eliminate all queuing levy on the firm which determines the number of subcenters (this levy is positive since overentry occurs at equilibrium). The optimal road size can be left to transport operators if they have no set-up costs. In this case, the free entry equilibrium for the road transport operators involves the equalization of toll revenues and construction costs. This is the self-financing property which leads to optimal road capacities when returns to scale are constant and the user cost function is homogeneous of degree 0 in usage and capacity [12].

7.2. Short-run second best policies

In the short run the number of subcenters n is fixed (and therefore heterogeneity is fixed). This implies that the efficiency gains have to come from lower transport costs. The second best policies available in the short run are either to introduce congestion pricing or to adapt the capacity of the roads (or both).

With no congestion pricing, congestion is largest and as expected, optimal road capacity is larger. More precisely, the optimal road size without road pricing $s^{sb}(n)$ satisfies: $s^{sb}(n) = \sqrt{2}s^{sb}_{cp}(n)$.[15]

With optimal congestion pricing in a bottleneck model, we know that total transport costs are divided by a factor 2. In this case, the optimal road capacities are

$$s^{sb}_{cp}(n) = \sqrt{\frac{\delta}{2}}\frac{\hat{\alpha}N}{\xi n},$$

[14] Note that a fixed road construction cost per firm, Υ would decrease the optimal number of subcenters: $n^{fb} = \frac{N}{(F+\Upsilon)}(\mu^d+\mu^w)$.

[15] Recall that the profit of firms is redistributed to users. As a consequence, in the short run, the infrastructure can be paid equivalently by consumers or firms.

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and therefore, the total road capacity (capacity per road times the width of the road) is the same as in the first-best. In order words, if the number of subcenters is twice the optimal number, the second best road capacity will be half of the width in the first-best optimum.

We know that in the bottleneck model, the total toll revenue is equal to the total construction cost $n\,\xi^2 s_{cp}^{sb}(n)$, since the construction cost is linear and the user cost is homogeneous of degree 0 in N and s (see [12]). Moreover, it can be verified that the total construction cost is equal to the total optimal user cost: $(\delta/2n)(N^2/s)\hat{\alpha}^2$, when $s = s_{cp}^{sb}(n)$.

We summarize our discussion in the following proposition.

Proposition 5. *When the number of subcenters is fixed, and with linear construction technology, congestion pricing halves the total transport costs, the total congestion cost is equal to the total construction cost for optimal road capacities. With optimal road pricing, total congestion cost equals total toll revenue.*

7.3. Long-run second-best policies

In the long run, the policy maker can select the number of subcenters by using an appropriate fixed levy per firm. The second-best optimal number of subcenters (without road pricing and for given road capacity) is given by: $\partial W(n,s)/\partial n = 0$ (see (45) where $\xi^2 s = K$ and where the absence of road pricing doubles the variable transport cost). This gives a unique maximum denoted by $n^{\mathrm{o}}(s)$ which solves

$$\left(\mu^d + \mu^w\right)\frac{N}{n^{\mathrm{o}}(s)} + \frac{\delta}{s}\left(\frac{\hat{\alpha}N}{n^{\mathrm{o}}(s)}\right)^2 - (F+K) = 0.$$

This is the same equation as (40). Using expression (42), we are able to compare the value of $n^{\mathrm{o}}(s)$ with the long-run free entry equilibrium with congestion and with n^{o}, the optimal number of subcenters in the absence of congestion. As a consequence, the second-best optimal number of subcenters is the lower bound proposed for the equilibrium number of subcenters (given that $K = S$) and again excess entry prevails: $n^{\mathrm{o}}(s) < n^f(s)$. The optimal number of subcenters increases with the level of congestion and $n^{\mathrm{o}} < n^{\mathrm{o}}(s)$, where n^{o} is the optimal number of subcenters in the absence of congestion.

Next proposition shows that, at the long-run free entry equilibrium and at the optimum, congestion induces more (and smaller) subcenters. Excessive entry remains the norm but there is at most one subcenter too many.[16]

Proposition 6. *Assume fixed road capacity and no congestion charging. In the long run, congestion increases the equilibrium and the optimal number of subcenters. If the fixed levy on firms exactly covers the infrastructure cost per subcenter, the equilibrium number of firms is larger than the optimal number of firms, but there is at most one subcenter too many.*

8. Summary and conclusions

We start by summarizing the results obtained so far. Because the total demand for the differentiated good is fixed, only two parameters matter for the welfare analysis: the number of firms and

[16] Of course, as in the noncongested case, there exist an optimal level of tax S which decentralizes the social optimum.

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Table 1
Long-run optimum and equilibrium number of firms in the symmetric case under different congestion and policy assumptions

	Capacity given LR equilibrium	Capacity given LR optimum	Optimum capacity LR optimum
No cong. no toll	$n^f = 1 + A$	$n^o = A$	$n^o = A$
Cong. no toll	$n^f < n^f(s)$ $n(s) \leqslant n^f(s)$ $n^f(s) \leqslant n^o(s) + 1$	$n^o < n(s)$	$n^o = \bar{A}$ $s(n) = \frac{\hat{\alpha}N}{n\xi}\sqrt{\delta}$
Cong. fine toll	$n^f_{cp}(s) < n^f(s)$ $n_{cp}(s) < n(s)$	$n^o_{cp} < n_{cp}(s)$ $s(n) = \frac{\hat{\alpha}N}{n\xi}\frac{\sqrt{\delta}}{\sqrt{2}}$	$n = A'$
With	$A = (\mu^d + \mu^w)\frac{N}{F+K}$ $\bar{A} = (\mu^d + \mu^w)\frac{N}{F}$		

the total transport costs. The total number of firms depends on the profit margin of the firms in the Nash equilibrium. When there is no congestion, in the equilibrium there is always one subcenter too many (see first line in Table 1). The equilibrium and optimum numbers of subcenters are always (increasing) linear functions of the same parameters A and $\bar{A}$: $A = (\mu^d + \mu^w)N(F + K)$ and $\bar{A} = (\mu^d + \mu^w)N/F$. More heterogeneity (on the product or labor market) leads to a higher optimal number of subcenters. Higher fixed production costs, lead to a lower optimum number of subcenters. When the road size cannot be optimized, the public infrastructure cost also points to a lower optimal number of subcenters.

When capacity is not infinite and congestion may occur, we need to distinguish the case with or without road capacity optimization and with or without optimal road tolling. We discuss first the case with given road capacity (columns 1 and 2 in Table 1). Without tolling and given road capacity, the short-run profit margin is always larger in the presence of congestion so that the free-entry equilibrium always entails more subcenters than in the situation without congestion (see second line in Table 1). The free entry equilibrium with congestion has at most one subcenter too many. Optimum congestion pricing can reduce but not eliminate the additional profit margins due to congestion. This explains that in equilibrium and with road capacity given, the equilibrium number of firms is highest if there is no congestion pricing (see first column in Table 1).

Any number of subcenters can be implemented by choosing the right fixed levy per firm. For the free-entry equilibrium computed in Table 1, we have assumed that the fixed levy equals the infrastructure costs per firm (firms are then responsible for the construction of the infrastructure). As can be seen in Table 1, we need a fixed levy per firm higher than the infrastructure costs to obtain the optimum number of subcenters. When the planner can optimally choose the road capacity, she compares the welfare cost of congestion with the marginal cost of capacity expansion. Without congestion tolling, the benefit of road expansion will be larger than with road pricing. Indeed, in the case of fine tolls, the optimum road capacity will be smaller by a factor $1/\sqrt{2}$.

We have studied so far the symmetric model which allows us to derive analytical results. It is straightforward to write down the nonsymmetrical version where costs, quality and transport costs differ among subcenters. In this case it is necessary to resort to numerical approaches based on variational inequalities in order to analyze the properties of the solutions (see de Palma et al. [10]).

One can also study trip chaining of shopping and commuting trips. In this case the individual that shops and works at the same subcenter can economize transport costs. The proposed framework allows to study the positive and negative impacts of trip chaining on the market conduct.

It is assumed in this paper that there is one firm per subcenter. Alternatively it can be assumed that the heterogeneity parameter is associated to the subcenter so that two or more firms cannot locate in the same subcenter since Bertrand competition with homogeneous goods would occur within this subcenter and firms would not cover their fixed costs (of course this additional firm would have no social value). The proposed framework can be extended to accommodate more than one firm per subcenter using a nested (logit) structure.

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Appendix A. Proof of Proposition 1

It suffices to show that the profit function is quasi-concave. Since there exists a candidate equilibrium, quasi-concavity is sufficient to guarantee that this candidate equilibrium is Nash. We prove below that at any extremum, the function is concave.

At any extremum, the first-order condition is satisfied

$$\frac{1}{N}\frac{\mathrm{d}\tilde{\pi}_i}{\mathrm{d}w_i} = \left[-\left(\frac{\mu^d}{\mu^w}+1\right) + \big(f_i(w_i) - w_i - c\big)\frac{(1-P_i^w)}{\mu^w}\right]P_i^w.$$

The corresponding second-order condition is

$$\frac{1}{N}\frac{\mu^w}{P_i^w(1-P_i^w)}\frac{\mathrm{d}^2\tilde{\pi}_i}{\mathrm{d}w_i^2}\bigg|_{\frac{\mathrm{d}\tilde{\pi}_i}{\mathrm{d}w_i}=0} = -2\left(\frac{\mu^d}{\mu^w}+1\right) + \big(f_i(w_i) - w_i - c\big)\big(1-2P_i^w\big)\frac{1}{\mu^w}.$$

But, using the fist-order condition, we get

$$\begin{aligned}\frac{1}{N}\frac{\mu^w}{P_i^w(1-P_i^w)}\frac{\mathrm{d}^2\tilde{\pi}_i}{\mathrm{d}w_i^2}\bigg|_{\frac{\mathrm{d}\tilde{\pi}_i}{\mathrm{d}w_i}=0} &= -2\left(\frac{\mu^d}{\mu^w}+1\right) + \frac{\left(\frac{\mu^d}{\mu^w}+1\right)}{(1-P_i^w)}\big(1-2P_i^w\big) \\ &= \left(\frac{\mu^d}{\mu^w}+1\right)\left(-2+\frac{(1-2P_i^w)}{(1-P_i^w)}\right),\end{aligned}$$

or

$$\frac{1}{N}\frac{\mu^w}{P_i^w}\frac{\mathrm{d}^2\tilde{\pi}_i}{\mathrm{d}w_i^2}\bigg|_{\frac{\mathrm{d}\tilde{\pi}_i}{\mathrm{d}w_i}=0} = -\left(\frac{\mu^d}{\mu^w}+1\right) < 0.$$

Therefore, any turning point, where $\mathrm{d}\tilde{\pi}_i/\mathrm{d}w_i = 0$ is such that it is a maximum. As a consequence, the profit function is quasi-concave, and the symmetric candidate equilibrium is a Nash equilibrium. □

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Appendix B. Proof of Proposition 4

The profit function is (where N is normalized to one, w.l.o.g.)

$$\tilde{\pi}_i(w_i, w_{-i}, p) = \left[g_i(w_i) - w_i - c - \Lambda^h P_i^w\right] N P_i^w - (F + S).$$

The first-order condition is

$$\frac{\partial \tilde{\pi}_i(w_i, w_{-i}, p)}{\partial w_i} = \left(\frac{\mathrm{d}g_i(w_i)}{\mathrm{d}w_i} - 1\right) P_i^w + \left[g_i(w_i) - w_i - c - 2\Lambda^h P_i^w\right] \frac{\mathrm{d}P_i^w}{\mathrm{d}w_i} = 0.$$

Moreover, we have

$$\frac{\partial^2 \tilde{\pi}_i(w_i, w_{-i}, p)}{\partial w_i^2} = \left(\frac{\mathrm{d}^2 g_i(w_i)}{\mathrm{d}w_i^2}\right) P_i^w + 2\left(\frac{\mathrm{d}g_i(w_i)}{\mathrm{d}w_i} - 1\right) \frac{\mathrm{d}P_i^w}{\mathrm{d}w_i} - 2\Lambda^h \left(\frac{\mathrm{d}P_i^w}{\mathrm{d}w_i}\right)^2 + \left[g_i(w_i) - w_i - c - 2\Lambda^h P_i^w\right] \frac{\mathrm{d}^2 P_i^w}{\mathrm{d}w_i^2}.$$

We wish to show that any turning point is a maximum

$$\left.\frac{\partial^2 \tilde{\pi}_i(w_i, w_{-i}, p)}{\partial w_i^2}\right|_{\mathrm{FOC}} < 0.$$

If this condition is satisfied everywhere, the profit function $\tilde{\pi}_i(w_i, w_{-i}, p)$ is quasi-concave, and the candidate symmetric equilibrium is Nash.

Note that, the first-order condition equation can be rewritten as

$$g_i(w_i) - w_i - c - 2\Lambda^h P_i^w = -\frac{\left(\frac{\mathrm{d}g_i(w_i)}{\mathrm{d}w_i} - 1\right) P_i^w}{\frac{\mathrm{d}P_i^w}{\mathrm{d}w_i}}.$$

Using this expression, we obtain after simplifications

$$\Omega \equiv \frac{\mathrm{d}P_i^w}{\mathrm{d}w_i} \left.\frac{\partial^2 \tilde{\pi}_i(w_i, w_{-i}, p)}{\partial w_i^2}\right|_{\mathrm{FOC}} = \left(-2\left(\frac{\mathrm{d}P_i^w}{\mathrm{d}w_i}\right)^2 + P_i^w \frac{\mathrm{d}^2 P_i^w}{\mathrm{d}w_i^2}\right) \times \left(1 - \frac{\mathrm{d}g_i(w_i)}{\mathrm{d}w_i}\right) + \frac{\mathrm{d}P_i^w}{\mathrm{d}w_i}\left(\frac{\mathrm{d}^2 g_i(w_i)}{\mathrm{d}w_i^2} P_i^w - 2\Lambda^h \left(\frac{\mathrm{d}P_i^w}{\mathrm{d}w_i}\right)^2\right).$$

We show that this expression Ω is negative given that $\mathrm{d}P_i^w/\mathrm{d}w_i > 0$. To to that, when there is no ambiguity, in order to simplify expressions, we use the following notations:

$$\begin{cases} P \equiv P_i^w = P_i^d, \\ P' \equiv \frac{\mathrm{d}P_i^w}{\mathrm{d}w_i}, \\ P'' \equiv \frac{\mathrm{d}^2 P_i^w}{\mathrm{d}w_i^2}, \\ g' \equiv \frac{\mathrm{d}g_i(w_i)}{\mathrm{d}w_i}, \\ g'' \equiv \frac{\mathrm{d}^2 g_i(w_i)}{\mathrm{d}w_i^2}. \end{cases}$$

Using these notations, we have equivalently

$$\Omega = \left(-2(P')^2 + PP''\right)(1 - g') + P'\left(g''P - 2\Lambda^h (P')^2\right). \tag{B.1}$$

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We now need to compute P', P'', g' and g'' at any point (i.e. not only at the symmetric candidate equilibrium).

First let compute P' and P''. Recall that

$$P = P_i^w = \frac{\exp\left(\frac{w_i - \Lambda^w P_i^w}{\mu^w}\right)}{\sum_{j=1,\ldots,n} \exp\left(\frac{w_j - \Lambda^w P_j^w}{\mu^w}\right)}.$$

We have, using again the implicit function theorem

$$P' = \frac{\frac{\partial P}{\partial w_i}}{1 - \frac{\partial P}{\partial P_i^w}} = \frac{\frac{1}{\mu^w} P(1-P)}{1 + \frac{\Lambda^w}{\mu^w} P(1-P)}.$$

Note that

$$\left[P(1-P)\right]' = (1-2P)P'.$$

Therefore, after simplifications, we get

$$P'' = \frac{\left(\frac{1}{\mu^w}\right)^2 P(1-P)(1-2P)}{\left[1 + \frac{\Lambda^w}{\mu^w} P(1-P)\right]^3}.$$

Second, we compute g' and g''. Recall that the solution of the equation $P_i^w = P_i^d$ is unique and denoted by $p_i = g_i(w_i)$. We have

$$P_k^d = \frac{\exp\left(\frac{-p_k - \Lambda^d P_k^d}{\mu^d}\right)}{\sum_{l=1,\ldots,n} \exp\left(\frac{-p_l - \Lambda^d P_l^d}{\mu^d}\right)},$$

so that, using the same reasoning as above

$$\frac{\mathrm{d}P_i^d}{\mathrm{d}p_i} = -\frac{\frac{1}{\mu^d} P_i^d (1-P_i^d)}{1 + \frac{\Lambda^d}{\mu^d} P_i^d (1-P_i^d)}.$$

Differentiation of the expression $P_i^w - P_i^d = 0$, as a function of w_i leads to

$$\frac{\mathrm{d}P_i^w}{\mathrm{d}w_i} - \frac{\mathrm{d}P_i^d}{\mathrm{d}p_i}\frac{\mathrm{d}p_i}{\mathrm{d}w_i} = 0,$$

thus (using again the condition $P_i^w = P_i^d$)

$$g' = \frac{\frac{\mathrm{d}P_i^w}{\mathrm{d}w_i}}{\frac{\mathrm{d}P_i^d}{\mathrm{d}p_i}} = -\frac{\mu^d}{\mu^w}\frac{1 + \frac{\Lambda^d}{\mu^d} P_i^d(1-P_i^d)}{1 + \frac{\Lambda^w}{\mu^w} P_i^w(1-P_i^w)} < 0. \tag{B.2}$$

Therefore

$$g'' = -\frac{\mu^d}{\mu^w}\frac{\Phi}{\left[1 + \frac{\Lambda^w}{\mu^w} P_i^w(1-P_i^w)\right]^2},$$

with

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$$\Phi = \frac{\Lambda^d}{\mu^d}\left(1-2P_i^d\right)\frac{\mathrm{d}P_i^d}{\mathrm{d}p_i}g' \times \left[1+\frac{\Lambda^w}{\mu^w}P_i^w\left(1-P_i^w\right)\right]$$
$$-\frac{\Lambda^w}{\mu^w}\left(1-2P_i^w\right)\frac{\mathrm{d}P_i^w}{\mathrm{d}w_i} \times \left[1+\frac{\Lambda^d}{\mu^d}P_i^d\left(1-P_i^d\right)\right].$$

After simplification, we get

$$\Phi = -\frac{1}{\mu^w}\left(\frac{\Lambda^d}{\mu^d}-\frac{\Lambda^w}{\mu^w}\right)\frac{(1-2P)P(1-P)}{\left[1+\frac{\Lambda^w}{\mu^w}P(1-P)\right]}.$$

Hence, using the expression above, we obtain

$$g'' = -\frac{\mu^d}{(\mu^w)^2}\left(\frac{\Lambda^d}{\mu^d}-\frac{\Lambda^w}{\mu^w}\right)\frac{(1-2P)P(1-P)}{\left[1+\frac{\Lambda^w}{\mu^w}P(1-P)\right]^3}.$$

The sign of g'' is ambiguous (and note that without congestion $g''=0$). We are now ready to sign the expression (B.1).

We first compute the expression $(-2(P')^2+PP'')$. We have

$$\left(-2(P')^2+PP''\right) = -\frac{P^2(1-P)}{(\mu^w)^2\left[1+\frac{\Lambda^w}{\mu^w}P(1-P)\right]^3}\left[1+2\frac{\Lambda^w}{\mu^w}P(1-P)^2\right] < 0.$$

Furthermore, replacing the expression for g' and after simplifications, we get

$$(1-g') = \frac{\left(1+\frac{\mu^d}{\mu^w}\right)+\frac{(\Lambda^w+\Lambda^d)}{\mu^w}P(1-P)}{1+\frac{\Lambda^w}{\mu^w}P(1-P)} > 0.$$

A combination of the last two expressions leads to

$$\left(-2(P')^2+PP''\right)(1-g') = -\frac{P^2(1-P)}{(\mu^w)^2\left[1+\frac{\Lambda^w}{\mu^w}P(1-P)\right]^4}\left[1+2\frac{\Lambda^w}{\mu^w}P(1-P)^2\right]$$
$$\times\left[\left(1+\frac{\mu^d}{\mu^w}\right)+\frac{(\Lambda^w+\Lambda^d)}{\mu^w}P(1-P)\right].$$

We are ready to compute the second term of (B.1). After substitution, we obtain

$$P'g''P = -\frac{\mu^d}{(\mu^w)^3}\left(\frac{\Lambda^d}{\mu^d}-\frac{\Lambda^w}{\mu^w}\right)\frac{(1-2P)P^3(1-P)^2}{\left[1+\frac{\Lambda^w}{\mu^w}P(1-P)\right]^4}.$$

Note that $\Omega = \Omega^1 - 2\Lambda^h(P')^3 < \Omega^1$ (since $P'>0$), with

$$\Omega^1 = \left(-2(P')^2+PP''\right)(1-g') + P'g''P.$$

We show that $\Omega^2 < 0$ with

$$\Omega^1 = \frac{P^2(1-P)}{(\mu^w)^2\left[1+\frac{\Lambda^w}{\mu^w}P(1-P)\right]^4}\Omega^2.$$

Using the two expressions derived above, we get

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$$\Omega^2 = \left[1 + 2\frac{\Lambda^w}{\mu^w}P(1-P)^2\right]\left[\left(1+\frac{\mu^d}{\mu^w}\right) + \frac{(\Lambda^w+\Lambda^d)}{\mu^w}P(1-P)\right] - \frac{\mu^d}{\mu^w}\left(\frac{\Lambda^d}{\mu^d} - \frac{\Lambda^w}{\mu^w}\right)(1-2P)P(1-P).$$

We can expand and regroup the terms to get

$$\Omega^2 = -2\frac{\Lambda^d}{\mu^w}P(1-P)^2 - \frac{\mu^d\Lambda^w}{(\mu^w)^2}P(1-P) - \left(1+\frac{\mu^d}{\mu^w}\right) - \frac{\Lambda^w}{\mu^w}P(1-P)\big[1+2(1-P)\big] - 2\frac{\Lambda^w(\Lambda^w+\Lambda^d)}{(\mu^w)^2}P^2(1-P)^3.$$

This shows, as required, that $\Omega^2 < 0$ and therefore Ω^1 and $\Omega < 0$. As a consequence

$$\left.\frac{\partial^2\tilde{\pi}_i(w_i, w_{-i}, p)}{\partial w_i^2}\right|_{\mathrm{FOC}} < 0. \qquad \square$$

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