The dependence structure between Carbon Emission Allowances and Financial Markets - A Copula Analysis

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Abstract

This paper investigates the relationship between CO_2 emission allowance prices and those of various other financial variables and commodities. Different copulas are applied in order to model the complex dependence structure between the return series of carbon emission allowance prices and those of various commodities as well as other financial series. As suggested in the literature, the use of correlation as the only measure of dependence can lead to an underestimation of the risk of joint extreme price movements. What is more, copula models represent a more flexible method for deriving the nature of dependence and provide an appropriate fit also for the tails of multivariate distributions. The findings suggest that the relationship between EU emission allowance (EUA) future returns and those of the other commodities - in particular gas and oil markets - is relatively weak. However, we find some dependence between EUA futures and equity or energy index returns. These results at least somehow contradict earlier studies that report no statistically significant or even negative correlations between returns of emission allowances and other financial variables. Regarding the nature of dependence, we also find some evidence of weak symmetric tail dependence for most of the considered series. Our findings generally suggest that EUAs can be characterized as an asset appropriate for portfolio optimization and diversification in equity and commodity markets.

Key words: CO_2 Emission Trading, Commodity Markets, Copula Models, Dependence Structure

JEL Classification: Q28, G13, C19

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1 Introduction

Under the Kyoto Protocol the EU has committed to reducing greenhouse gas (GHG) emissions by 8% compared to the 1990 level by the years 2008-2012. The intention is to give a price to carbon emissions and to incentivize the reduction of the respective GHG, an EU-wide CO_2 emissions trading system (EU-ETS) has been set up and the right to emit a particular amount of CO_2 has become a tradable commodity. The new market not only requires regulated emitters to run an adequate risk management, it also provides new business development opportunities for market intermediaries and service providers like brokers or marketeers. However, it is essential for carbon market players to learn about price dynamics in order to realize trading strategies, risk strategies and investment decisions.

In particular with the end of the first trading period in 2007 empirically analyzing emission allowance prices receives growing attention in the literature. Papers such as Paolella and Taschini (2008), Benz and Trück (2009) and Gronwald and Ketterer (2009) epitomize these concerted research efforts. While these papers employ univariate techniques, applications of multivariate approaches become considerably popular, see e.g. Alberola et al. (2009) and Nazifi and Milunovich (2009).

In this paper we contribute to the literature in the following way. The first goal is to provide an analysis of the dependence structure between EUA spot and futures returns and those of other financial and commodity markets. It could be argued that as a factor of production changes in emission allowance prices might be related to the dynamics of other commodity markets. Further, since EUA prices are not primarily determined by financial markets but rather by policy measures and regulatory changes, they could potentially be used for portfolio diversification. To our best knowledge this is a pioneer study on applying and testing different copula models in emission allowance markets. Finally, we provide a risk analysis comparing a standard variance-covariance approach to the estimated copula models with respect to the quantification of the risk.

The remainder of the paper is organized as follows. Section 2 provides a brief description of market mechanism for CO_2 emission allowances, a classification of the assets as well as price drivers of the market. Section 3 provides a review of different copula models with respect to estimation, model testing, modeling the dependence structure and risk analysis. Section 4 describes the considered data and provides the empirical results of our study. Section 5 concludes and gives suggestions for future work.

2 The European Emission Trading Scheme

2.1 Regulatory Setting

Combustion installations exceeding 20 MW are affected by the trading scheme including different kinds of industries like metal, cement, paper, glass etc. as well as refineries or coke ovens. In total, the EU-ETS includes some 12300 installations, representing approximately 42% of EU's GHG emissions. From 2013 onwards the system will cover more GHG emissions, as PFCs and N_2O. After an initial pilot trading period (2005-2007), new allocation plans have been issued in 2008 for the first Kyoto commitment trading period from 2008-2012. From 2013 the third trading period will commence that lasts until 2020.The caps for these years and the allocation mechanisms are already set. Allocation is currently assessed by the European Commission in so-called National Allocation Plans (NAP). In the third period these will be replaced by unified rules applying to all member states. Generally, allowances may either be allocated free of charge, auctioned off or sold at a fixed price while according to the European Commission the importance of auctioning will further increase over time. However, it is important to note that the annual quantity of allocated emission allowances is limited and already specified by the EU-Directive for all trading periods already.

Some regulatory settings are particularly important, as they shape compliance behavior. Under the current system banking and borrowing, hence the storage of unused certificates and the use of future allowances in earlier years, give more leeway for complying parties and smoothes prices. A detailed analysis of banking and borrowing rules is provided by Alberola and Chevallier (2009). Another particularity of current framework is a period of allocation overlap: allowances for the new compliance year are obtained in February, certificates due for the previous year have to be handed back in April.
Generally, the lack of allowances requires a company to either invest in some plant-specific or process improvements or the purchase of additional allowances and emission credits from CDM or JI projects, the Flexible Kyoto Mechanisms. Failure to submit a sufficient amount of allowances results in sanction payments of 100 Euro per missing ton of CO_2 allowances. In addition, companies have to surrender the missing allowances in the following year. As a consequence, participating companies face several risks specific to emissions trading. In particular, price risk (of fluctuating allowance prices) and volume risk (due to unexpected fluctuations in energy demand the emitters do not know ex ante their exact demand for EUAs) have to be considered. Naturally, market generic risks – like counterparty, operational, reputational, etc. – are also present. For a discussion see e.g. Bokenkamp et al. (2005).

EUAs are different than more traditional commodities. What is actually sold is a lack or absence of the gas in question. Therefore, emissions become either an asset or a liability for the obligation to deliver allowances that cover those emissions (PointCarbon, 2004). Benz and Trück (2009) point out the differences between emission allowances and classical stocks. While the demand and the value of a stock is based on profit expectations of the underlying firm, the CO_2 allowance price is determined directly by the expected market scarcity induced by the current demand and supply at the carbon market. Notably, firms by themselves are able to influence market scarcity and hence the market price by their CO_2 abatement decisions. It is important to note that the annual quantity of allocated emission allowances is limited and already specified by the EU-Directive for all trading periods.
A more appropriate approach in specifying CO_2 emission allowances is their consideration as a factor of production (Fichtner, 2004). The shortage of emission allowances by reducing the emissions cap for the commitment periods classifies the assets as 'normal' factors of production. They can be 'exhausted' for the production of CO_2 and after their redemption or at the end of the commitment period when they expire, they are removed from the market. Accordingly, it seems more adequate to compare the right to emit CO_2 with other operating materials or commodities than with a traditional equity share and hence to adopt rather commodity than stock pricing models.

In order to build a commodity pricing model, it is of great importance to identify the key price determinants of the CO_2 emission allowances. According to the investigation of SO_2 permit prices by Burtraw (1996), we categorize the principle driving factors of CO_2 allowance prices into (i) policy and regulatory issues and (ii) market fundamentals that directly concern the production of CO_2 and thus demand and supply of CO_2 allowances.
Regulatory settings, as in part (i), are likely to shape long-term development of prices. For our pricing model we are interested in the determinants of short-term price behavior. Policy changes may lead to sudden price changes in the case of decisions concerning the National Allocation Plans (NAPs) or a change of the European commitment to reduce 30% instead of 20% until 2020. Hence, the consequences of changes in such regulatory or policy issues may be sudden price jumps and phases of extreme volatility (Gronwald and Ketterer (2009), Sanin and Violante (2009)). Chevallier et al. (2009a) specifically investigate the EUA price drop in April 2007 and show that the market perception of risk changed substantially.
Incorporating part (ii), allowance prices may also show phases of specific price behavior due to fluctuations in production levels. In general, CO_2 production depends on a number of factors, such as weather data (temperature, rain fall and wind speed), fuel prices and economic growth. Some compre-

hensive research on determinants has been conducted by considering the determinants of European carbon prices Alberola et al. (2008) or Chesney and Taschini (2008). Especially unexpected (environmental) events and changes in fuel spreads shock the demand and supply side of CO_2 allowances and consequently market prices. A short term measure for the power and heat sector to invest in CO_2 abatement projects are the relative costs of coal and cleaner fossil fuels such as oil and natural gas. The price spread between these fuels have a large influence on the demand for emission reduction certificates. For example, for an electricity producer switching from 'cheap-but-dirty' coal to 'expensive-but-cleaner' gas can significantly reduce emissions per MWh of produced electricity. Therefore, fuel-switching from coal to gas implies less emissions to be covered with permits what might make the price of EUAs dependent on prices of gas and coal, see e.g. Fehr and Hinz (2006).

Since the beginning of the spot market trading in 2005, a number of studies are analyzing the behavior of emission certificate prices. Benz and Trück (2009), Seifert et al. (2008) as well as Paolella and Taschini (2008) provide an econometric analysis of the behavior of allowance prices and investigate different models for the dynamics of short-term spot prices. More recently there have also been studies investigating derivative products in EUA markets like convenience yields and the term structure of futures prices (Trück et al., 2006) as well as the effects of options trading on market volatility (Chevallier et al., 2009b)). Böhringer and Lange (2005) and Schleich et al. (2006) conduct simulation studies on CO_2 market prices with respect to changes in different market design parameters. Maeda (2001) gives a rather theoretical analysis on banking impacts and forward pricing on the market.

Finally, only few studies investigate the dependence between returns of emission reduction certificates and those of other financial or commodity markets. As suggested by the literature one could expect a significant impact of commodity prices on the prices of emission allowances. For example, for an electricity producer switching from 'cheap-but-dirty' coal to 'expensive-but-cleaner' gas can significantly reduce emissions per MWh of produced electricity. Therefore, fuel-switching from coal to gas implies less emissions to be covered with permits what might make the price of EUAs dependent on prices of gas and coal. Further, rising carbon prices as a factor of production could be related to additional costs and uncertainties for producers and consumers and might have an adverse effect on equity markets in general or equities of certain industries in particular.

Kosobud et al. (2005) find no statistically significant correlations between monthly returns of SO2 emission allowance prices in the US market and returns from various financial investments. On the other hand, Daskalakis et al. (2009) find negative correlations of EUA futures with equity market returns what may offer significant diversification opportunities to European equity investors. They argue that the factors determining stock and bond prices are substantially different from those affecting emission permit ones. Kara et al. (2008) examine the impacts of EU CO_2 emissions trading on electricity markets and consumers in Finland but do not consider daily or weekly returns of the series. Also the results of the influence of carbon prices on other commodity prices are varied: so far there seems to be no common agreement whether energy prices are yet significantly influenced by the price of carbon emission allowances. Bunn and Fezzi (2007) investigate the economic impact of the EU-ETS for carbon on wholesale electricity and gas prices in the UK. Using a structural co-integrated VAR model, they conclude that the prices of carbon and gas jointly influence the equilibrium price of electricity and estimate the transmission of shocks between gas, carbon and power prices. Nazifi and Milunovich (2009) find contrary evidence when they apply a restricted VAR model in first differences to test for existence of causal relationship and long-run links between the price of carbon and the prices of energy fuels and electricity. They apply Granger-causality tests and generalized impulse response analysis and their results suggest that the dynamics of energy prices are rather independent from the price of carbon emissions permits for the considered time period. However, they find weak evidence of Granger causal relation from carbon futures prices to natural gas prices. Reinaud (2007) investigates the interaction between the CO_2 allowance and electricity prices and the impact on the industry's electricity purchasing strategies in Europe. While the author concludes that there is no universal answer on how the EU ETS has affected electricity prices, at least some evidence for the CO_2 pass-through into electricity prices was provided during the abrupt fall of the CO_2 price in May 2006. The fall by ten Euros per tonne of CO_2 was immediately followed by a drop in wholesale electricity prices by five to ten Euros per MWh in several markets. Reinaud further argues that this electricity price adjustment can be directly attributable to the CO_2 price fall, since it was not connected to other energy market movements that could also affect electricity prices.

To our best knowledge, so far there has been no empirical study concentrating mainly on the dependence structure between EUA returns and those of other financial variables or commodity markets. Next to standard approaches investigating linear dependence by correlation analysis in our study we also apply different copulas to model the complex dependence structure between the return series of carbon emission allowance prices, commodity and equity markets.

3 Copula Models

Recently, there has been some criticism towards the assumptions of multivariate normality for the joint distribution of asset returns and the use of a covariance matrix as the natural measure of dependence between financial assets. As shown in various studies, see e.g. Jondeau and Rockinger (2006a), Junker et al. (2006), Luciano and Marena (2003) or McNeil et al. (2005), the use of correlation does not appropriately describe the dependence structure between financial assets and could lead to inadequate measurement of the risk. The authors suggest the application of copula methods for modelling the dependence structure of the asset returns in order to overcome this problem. For an excellent overview on copula methods in finance, see Cherubini et al. (2004), where the range of applications of copula methods includes various topics such as portfolio analysis, derivative pricing, interest rates or credit risk analysis. With respect to analysing the dependence structure between different financial assets, the methodology of copulas as alternative to the multivariate normal model has the advantage that it does not require the assumptions of joint normality for the distributions. Instead it allows joining arbitrary marginal distributions into their one dimensional multivariate distribution allowing for a wide range of dependence structures by using different copulas. So the multivariate joint distribution can be decomposed into marginal distributions and an appropriate functional form for the dependence between the asset returns.

3.1 Copula Functions

This section provides a brief review on the estimation and goodness-of-fit tests for copulas that will be used in the empirical analysis. Since this can be considered as a pioneer study on applying and testing different copula models to emission allowance markets, we also briefly illustrate some basic concepts of copula families and the dependence measure Kendall's tau. A copula is a function that combines marginal distributions to form a joint multivariate distribution. The concept was initially introduced by Sklar (1959), but has only gained high popularity in modelling financial or economic variables in the last decade. For an introduction to copulas see e.g. Nelsen (1999) or Joe (1997), for applications to various issues in financial economics and econometrics, see e.g. Cherubini et al. (2004), McNeil et al. (2005), Frey and McNeil (2003) and Hull and White (2004) to name a few. As shown by Cherubini and Luciano (2001), Jondeau and Rockinger (2006), Junker et al (2006) or Luciano and Marena (2003), the use of correlation usually does not appropriately describe the dependence structure between financial assets and could lead to inadequate measurement of the risk. Longin and Solnik (2001) empirically show that asset returns are more highly correlated during volatile markets and during market downturns. Dowd (2004) shows the strength of the copula comes from its feature that it does not have any assumptions on the joint distributions among the financial assets in a portfolio. Overall, the use of copulas offers the advantage that the nature of dependence can be modeled in a more general setting than using only linear dependence that is explained by correlation. It also provides a technique to decompose a multivariate joint distribution into marginal distributions and an appropriate functional form for the dependence between the asset returns.

A copula is the distribution function of a random vector in $\mathbb{R}^n$ with standard uniform marginals. Let $X = (X_1, \ldots, X_n)'$ be a random vector of real-valued random variables whose dependence structure is completely described by the joint distribution function

$$F(x_1, \ldots, x_n) = P(X_1 < x_1, \ldots, X_n < x_n). \tag{1}$$

Each random variable X_i has a marginal distribution of F_i that is assumed to be continuous for simplicity. The transformation of a continuous random variable X with its own distribution function F results in a random variable $F(X)$ which is standardly uniformly distributed. Thus transforming equation (1) component-wise yields

$$\begin{aligned} F(x_1, \ldots, x_n) &= P(X_1 < x_1, \ldots, X_n < x_n) \\ &= P[F_1(X_1) < F_1(x_1), \ldots, F_n(X_n) < F_n(x_n)] \\ &= C(F_1(x_1), \ldots, F_n(x_n)), \end{aligned} \tag{2}$$

where the function C can be identified as a joint distribution function with standard uniform marginals — the copula of the random vector X. Equation (2) illustrates how the copula combines the marginals to the joint distribution. The copula framework can be generalized for any collection of marginal distributions and joint distributions. In our application we will only consider the bivariate case with a function $C(u, v)$ such that,

$$C(u, v) = C[F(x), G(y)]. \tag{3}$$

Then the function $C(u, v)$ is defined as a copula function which relates the marginal distribution functions $F(x)$ and $G(y)$ into their joint probability distribution. Moreover, if marginal distributions $F(x)$ and $G(y)$ are continuous, the copula function $C(u, v)$ is unique, see e.g. Sklar (1959). In the following we will describe four of the most commonly applied copulas: the Gaussian, Student-t, Clayton and Gumbel copula.

3.2 Examples of copulas

The literature reports a wide range of different copulas, see e.g. Joe (1997) or Nelsen (1999) for an overview of the most common parametric families of copulas. In the following we will limit ourselves to a description of a number of copula families that will be used later on in the empirical analysis. In particular we will briefly describe the Gaussian copula, the Student t-copula as well as the Clayton and Gumbel copula.

We will start with the multivariate Gaussian and Student t-copula that belong to the class of elliptical copulas. The probably most intensively used copula in financial applications is the Gaussian copula. It is constructed from the multivariate normal distribution and can be denoted by

$$C_\rho^N(u_1, \ldots, u_d) = \Phi_\Sigma^d(\Phi^{-1}(u_1), \ldots, \Phi^{-1}(u_d)) \tag{4}$$

Hereby, Φ denotes the the standard normal cumulative distribution function, Φ^{-1} the inverse of the standard normal cumulative distribution function and Φ_Σ^d the standard multivariate Normal distribution with correlation matrix Σ. Applying C_ρ^N to two univariate standard normally distributed random variables results in a standard bivariate normal distribution with correlation coefficient ρ. Further note that, since the copula and the marginals can be arbitrarily combined, this (and any other) copula can be applied to any set of univariate random variables. The outcome will then surely not be multivariate normal, but the resulting multivariate distribution has inherited the dependence structure from the multivariate normal distribution. The multivariate normal copula correlates the random variables rather near the mean and, therefore, fails to incorporate dependence in the tail. Alternatively, to also capture tail dependence, we can use the Student t-copula which is denoted by:

$$T_{\Sigma,v}(u_1, u_2, \ldots, u_d) = t_{\Sigma,v}(t_v^{-1}(u_1), t_v^{-1}(u_1), \ldots, t_v^{-1}(u_d)) \tag{5}$$

where $t_{\Sigma,v}$ is the multivariate Student t distribution with v degrees of freedom and correlation matrix Σ. Depending on the degrees of freedom parameter, the Student t copula can also determine the strength of the tail dependence. Generally, low values of the parameter v indicate strong tail dependence.

It is a common occurrence for economic and financial variables to exhibit tail-dependence in only one of the tails, either the upper right or lower left of the data. For example, tail-dependence in the lower left tail indicates that the two variables show simultaneous extreme negative returns while high positive returns in one of the variables may not affect the other variable that much. To model asymmetric tail dependence, so-called Archimedean copulas can be used, see e.g. Cherubini et al. (2004). Two of the most prominent members of the family of Archimedean copulas are the Clayton and Gumbel copula that will be briefly described in the following. The Clayton copula is an asymmetric Archimedean copula, exhibiting greater dependence in the negative lower tail than in the positive upper one. The multivariate Clayton copula can be denoted by:

$$C_\theta^{Cl}(u_1, ..., u_d) = \left[\sum_{i=1}^{d} u_i^{-\theta} - d + 1\right]^{1/\theta}, \tag{6}$$

For the Clayton copula, the parameter $\theta > 0$ is used to measure the degree of dependence. The greater θ, the stronger is the dependence between the considered variables, in particular in the lower left tail. The Gumbel copula, on the other hand, exhibits greater dependence in the upper right tail and is denoted by:

$$C_\phi^{Gu}(u_1, ..., u_d) = \exp\left[-\left\{\sum_{i=1}^{d}(-ln(u_i)^{\phi}\right\}^{1/\phi}\right], \tag{7}$$

where $\phi > 1$ indicates the dependence between the random variables $X_1, ...X_d$. In the next section we will illustrate how the dependence parameters of the elliptical and Archimedean copulas can be related to measures of association or dependence like Kendall's tau. For further properties and examples of Archimedean copulas and on the construction of such copulas by using generator functions, we refer to Nelsen (1999) and Cherubini et al. (2004).

For selecting the most appropriate among a set of copulas, the literature usually suggests goodness-of-fit tests investigating the distance between the estimated and the so-called empirical copula, see e.g. Genest et al. (2006, 2009). In order to determine the empirical copula, usually the empirical margins are used. Let $(X_{1i}, ..., X_{ni})$ be n observations of the random variable X_i. Then the empirical marginal cdf for a random variable X_i is:

$$\hat{F}_i(x) = \frac{1}{n+1}\sum_{j=1}^{n} I(X_{ji} \leq x) \qquad i = 1, .., d \tag{8}$$

where $I(.)$ denotes the indicator function returning the value of 1 if $X_{ji} \leq x$ and 0 otherwise. Further, in the denominator $n + 1$ is used to keep the empirical cdf to be smaller than 1. Note that the empirical marginal distribution converges towards the actual distribution function for n approaching infinity. Defining the empirical probability integral transforms $u_{ji} = \hat{F}_i(x_{ji})$ for $i = 1, .., d; \quad j = 1, .., n$, for the vector $u = (u_1, .., u_d)$, using the marginal cdf's, the empirical copula is given by

$$C^{emp}(u) = \frac{1}{n+1}\sum_{j=1}^{n} I(\hat{F}_1(x_{j1}) \leq u_1), ..., \hat{F}_d(x_{jd}) \leq u_d)) \tag{9}$$

$$= \frac{1}{n+1}\sum_{j=1}^{n} I(U_1 \leq u_1, ..., U_d \leq u_d) \tag{10}$$

Note that the empirical copula is not really a copula according to the definition by Deheuvels (1979), but rather the observed frequency of $P(U_1 \leq u_1, ..., U_d \leq u_d)$.

3.3 Measuring the Dependence

Kendall's tau is often used to measure the dependence structure when employing Archimedean (Clayton and Gumbel) and elliptical Gaussian and Student t copulas. Kendall's tau τ is a rank-based measure of dependence that provides consistent estimation of the true underlying copula as it is shown for example in Deheuvels (1979). The use of Kendall's tau is probably best motivated for the bivariate case. Assume that we have observations of two variables (X_i, Y_i), $i = 1, ..., n$, for example the return series of two prices or financial assets (stocks prices, equity and bond indices). We then consider pairs of vectors of the original observations (X_s, Y_s) and (X_t, Y_t). A pair of vectors is said to be concordant if $X_s > X_t$ when $Y_s > Y_t$ or if $X_s < X_t$ when $Y_s < Y_t$. On the other hand a pair is said to be discordant if $X_s > X_t$ when $Y_s < Y_t$ or if $X_s < X_t$ when $Y_s > Y_t$. Note that adjustments might be necessary if the slope is 0 or infinite, but this should not occur when the data are continuous and measured with precision. This process is repeated for all choices of distinct pairs (u_s, v_s) and (u_t, v_t). Overall, there are $m = n(n-1)/2$ such choices. Kendall's τ is then simply the sum of all concordant minus discordant pairs or the sum of +1s and -1s, divided by m. Obviously, values of τ range from -1 to $+1$, while in the case of independence τ will be 0, see e.g. Nelsen (1999). In some applications as an alternative to Kendall's tau also Spearman's rank correlation coefficient rho is used. For comparison of these two measures that emphasize different aspects of the dependence, see e.g. Caperaa and Genest (1993). In the bivariate case, based on the estimated value of τ the dependence parameter for the chosen copula can be calculated as a function of τ. For the Gaussian, Student t, Clayton and Gumbel copula this is straightforward and as pointed out by Genest and Rémillard (2008) under weak regularity conditions on the copula family, this yields a consistent estimator of the dependence parameter.

Figure 1 shows scatter plots for four different copula functions based on the

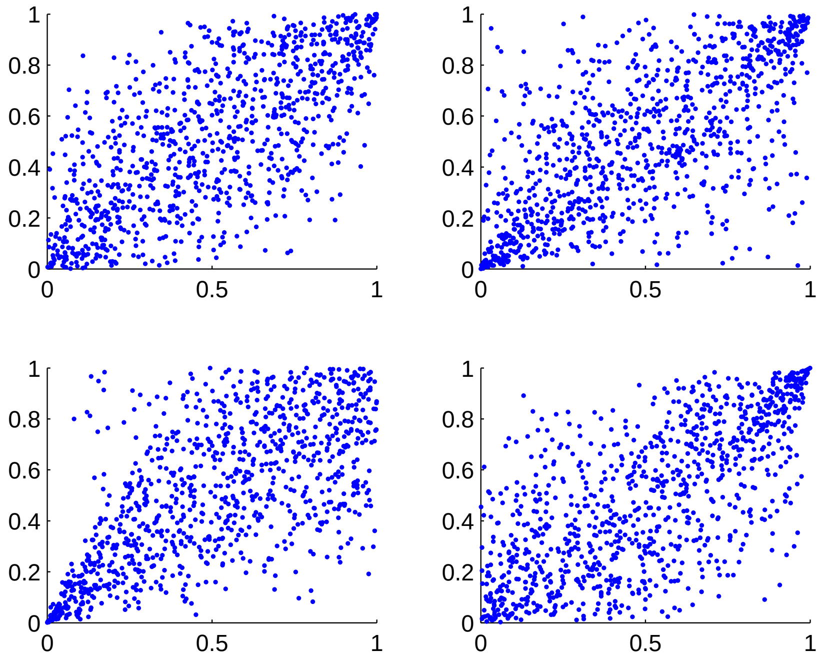

Fig. 1. Scatter plot of simulated dependence structure of ranks for different copulas with the same Kendal's tau $\tau = 0.5$. The graph illustrates the dependence between the ranks for the Gaussian (upper left panel), the Student t (upper right panel), Clayton (lower left panel) and Gumbel copula (lower right panel).

same Kendall's tau $\tau = 0.5$. The graph illustrates the symmetric dependence structure for the Gaussian and Student t copula, while the Student t copula exhibits more tail dependence in the lower left and upper right tail in comparison to the Gaussian one. Further the asymmetric Clayton copula exhibits greater dependence in the negative lower tail, while the Gumbel copula exhibits greater dependence in the positive upper tail as illustrated by the graph.

3.4 Goodness-of-fit Tests

One of the challenges is deciding on which copula provides the best fit to the actual dependence structure of the data. Berg and Bakken (2006) note that information criteria such as e.g. Akaike's Information Criterion (AIC) are generally not able to provide any understanding about the power of the decision rule employed. Instead, goodness-of-fit (GOF) approaches are more powerful in deciding whether to reject or accept parametric copulas, making them the preferred choice in empirical applications, see e.g. Genest et al (2006, 2009).

We provide a brief overview on how such tests can be conducted. Hereby, we concentrate on so-called 'blanket tests', where the implementation does not require an arbitrary categorization of the data or any strategic choice of smoothing parameters, weight functions or kernels. Genest et al. (2009) provide various options for such tests by conducting a large Monte Carlo experiment and report particularly good results for the blanket tests using ranks and the Rosenblatt transform. With respect to the chosen distance measure, the authors recommend the so-called Cramér-Von Mises statistic. Based on these results, we only describe tests based on ranks that use the Cramér-Von Mises for measuring the difference between the estimates and the empirical copula. For various alternative tests, we refer to Berg and Bakken (2006) or Genest et al. (2009). For the suggested approach the test procedure for investigating whether the dependence structure of a multivariate distribution is well-represented by a specific parametric family of copulas can be roughly summarized as follows:

1. Based on the vectors of rank observations and the estimated Kendall's tau for the empirical data, the corresponding dependence parameters for the copula families can be determined. Then the values $\hat{C}^{emp}(U_i)$ and $C_\theta(U_i)$ for the empirical and the estimated family of copulas can be calculated.

2. Using the Cramér-Von Mises statistic, the distance between the empirical and estimated copula is calculated by

$$S_n = \sum_{i=1}^{n} [\hat{C}^{emp}(U_i) - C_\theta(U_i)]^2$$

3. Then for some large integer N, the following steps are repeated: (a) Generate a random sample from C_θ and compute the associated rank vectors $(U_1^*, ..., U_n^*)$ as well as the empirical copula $\hat{C}^{emp*}(u)$. (b) Estimate Kendall's tau τ^* for the generated random sample and estimate the parametric copula C_θ^*. (c) Determine $S_n^* = \sum_{i=1}^{n} [\hat{C}^{emp*}(U_i) - C_\theta^*(U_i)]^2$ for the generated sample.

4. From the N bootstrap samples, an approximate p-value, measuring the goodness-of-fit of the copula, can be calculated as the fraction of simulations where $S_n^* > S_n$. If the considered copula provides a good fit to the actual dependence structure of the data, we should expect to get high p-values, while for a copula providing a bad fit to the actual data, we will expect the p-value to be low. In this case, depending on the level of confidence, the hypothesis that the dependence structure of a multivariate distribution is well-represented by a specific parametric family of copulas will be rejected.

4 Empirical Analysis

4.1 The Data

In this section we will investigate the dependence structure between returns from traded emission allowance contracts and various other financial variables during the time period January 2, 2009 to December 24, 2009. We are particularly interested in the dependence structure between returns from EUA spot and 2010 futures contracts and those of CER spot and futures, commodity prices, a European stock market index, indices for investment in energy companies, renewable energy companies and returns from oil, electricity and gas futures contracts. Data on EUA and CER prices is obtained from the London-based ECX. As for commodities, electricity spot and futures (Phelix baseload future) are taken from EEX in Leipzig. The gas and oil futures are downloaded from the ICE. Regarding the stock market indices, the analysis includes the Eurostoxx 50, the more energy specific DJ Europe Energy Stock Index (E1ENE) and the European Renewable Energy Index (ERIXP). For our analysis we consider log-returns that are calculated as $r_t = ln(P_{t+1}/P_t)$ from the original price series.

4.2 Estimation of the copula functions

As described in Section 2, a possible way to estimate the dependence between two random variables via a copula is to model the dependence between the rank transforms. This has the advantage that the possibly unknown marginal distribution is not required, since the empirical marginal cdf can be used. Figure 2 provides bivariate scatter plots of the rank transforms for returns of CER 2010 futures, Gas 2010 futures, Oil 2010 futures and E1ENE spot returns versus EUA 2010 futures returns. The figure illustrates that in particular between EUA and CER 2010 futures returns there is a strong dependence, while returns of between EUA futures and the considered commodity futures and financial variables exhibit rather low correlation or dependence.

In a next step, for each of the considered series, based on the rank transformations we calculate Kendall's tau. Further using the relationships between Kendall's tau and the copula parameters described in the previous section, the dependence parameters θ for the Clayton, ϕ for the Gumbel and the coefficient of correlation ρ for the Gaussian and Student t copula are calculated. The results are displayed in Table 1. We find that Kendall's tau ranges from approximately 0.05 to 0.77 for the different series what corresponds to a correlation coefficient ranging from 0.08 to 0.94. Obviously, the highest value for

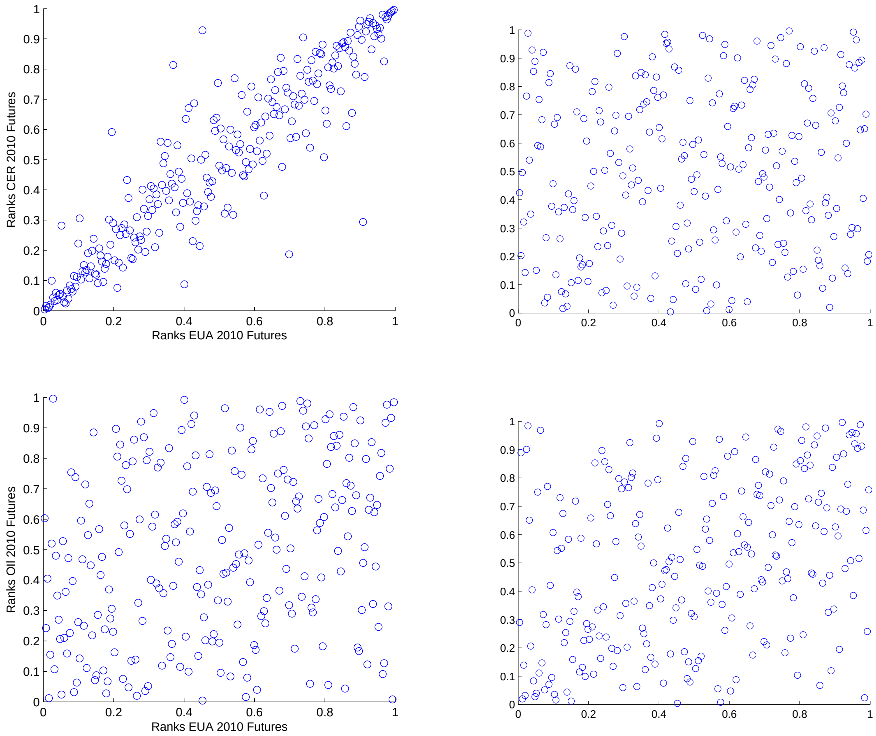

Fig. 2. Scatter plot of ranks for daily CER 2010 Futures (upper left panel), 2010 Gas Futures (upper right panel), 2010 Oil Futures (lower left panel) and E1ENE spot returns (lower right panel) returns versus ranks of EUA 2010 Futures returns.

Kendall's tau can be observed for the returns of the CER 2010 futures contracts while we observe the lowest rank dependence and correlation between the ranks of the 2010 EUA and gas futures contracts. Regarding the returns of CER and EUA futures contracts, these results could have been expected. As indicated by earlier studies, prices of these two certificates show strong correlations and similar price movements, since they express an almost identical asset. On the other hand, it is surprising that the returns of stock market indices like Eurostoxx 50 and the energy specific DJ Europe Energy Stock Index (E1ENE) and the European Renewable Energy Index (ERIXP) seem to exhibit a higher degree of dependence with EUA futures returns than any of the considered commodities except electricity. Our results also partly contradict earlier studies by Kosobud et al. (2005) and Daskalakis et al. (2009). The former found no statistically significant correlations between returns of SO_2 emission allowances and returns from other financial variables while the latter observed that EUA futures returns were negatively correlated with equity market returns during the pilot trading period.

Based on the estimated parameters, we then fit the different copulas to the bivariate series. Note that for the Student t copula also the degree of freedom parameter v needs to be estimated. We determine v such that the distance

Asset	τ	θ	ϕ	ρ
CER 2010 Futures	0.7704	6.7104	4.3552	0.9357
Gas 2010 Futures	0.0528	0.1115	1.0558	0.0829
Oil 2010 Futures	0.1839	0.4506	1.2253	0.2848
Coal 2010 Futures	0.2367	0.6203	1.3101	0.3633
EEX 2010 Futures	0.3902	1.2798	1.6399	0.5753
Eurostoxx 50 Spot	0.2686	0.7344	1.3672	0.4095
E1ENE Spot	0.2437	0.6446	1.3223	0.3736
ERIXP Spot	0.2580	0.6954	1.3477	0.3943

Table 1
Kendall's τ and corresponding dependence parameters θ for the Clayton copula, ϕ for the Gumbel copula and coefficient of correlation ρ for the Gaussian and Student t copula for returns in 2009.

between the empirical and estimated Student t copula is minimized. In order to investigate which of the copulas describes best the dependence structure between the refined and crude oil returns, we use the Cramér-Von Mises statistic

$$S_n = \sum_{i=1}^{n} [\hat{C}^{emp}(U_i) - C_\theta(U_i)]^2$$

to measure the distance between the empirical and estimated copulas. Figure 3 provides a plot of returns for daily EUA 2010 Futures versus CER 2010 Futures, ranks for daily EUA 2010 Futures returns versus CER 2010 Futures returns, a 3d histogram of ranks for daily EUA 2010 Futures returns versus CER 2010 Futures returns and the fit of the Student t copula to the ranks. The same graphs are also provided for the series daily EUA 2010 Futures versus Oil 2010 Futures in figure 4.

The results are presented in 2 and generally support the Student t copula providing the best fit for the dependence structure. For each of the considered series, except the Eurostoxx 50, it yields the smallest distance between the estimated and the actually observed empirical copula. We conclude that most of the series the daily returns exhibit at least some degree of tail dependence both in the lower left and upper right tail. Only for the relationship between EUA 2010 futures and Eurostoxx 50 returns, the Gumbel copula that gives more tail dependence in the upper right tail yields the best fit to the dependence structure between the returns. Surprisingly, for most of the series also the Gaussian copula outperforms both the Clayton and Gumbel copula that

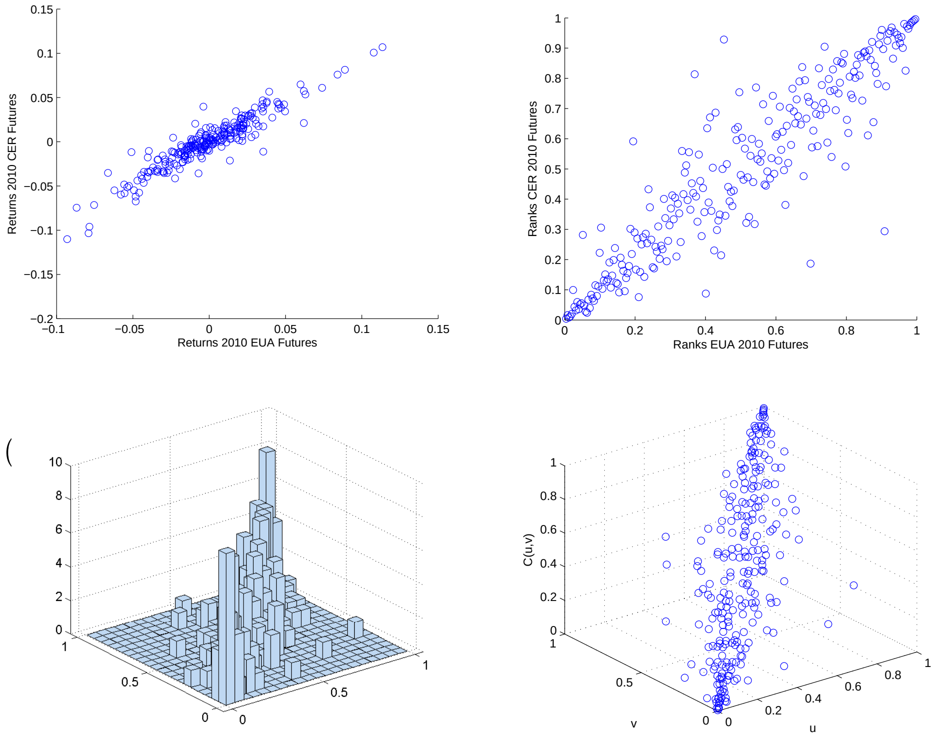

Fig. 3. Plot of returns for daily EUA 2010 Futures versus CER 2010 Futures (upper left panel), ranks for daily EUA 2010 Futures returns versus CER 2010 Futures returns (upper right panel), 3d histogram of ranks for daily EUA 2010 Futures returns versus CER 2010 Futures returns (lower left panel) and fit of the Student t copula to the ranks (lower right panel).

exhibit dependence either only in the lower left or upper right tail. Generally, the Clayton copula yields the greatest distance from the empirical copula while the fit of the Gumbel is significantly better, but still worse than the Gaussian and Student t copula. Only for the dependence structure between EUA and Gas 2010 futures contracts, all of the considered copulas provide a similar fit to the data. However, as it is indicated by Kendall's τ between the returns of these two variables no significant dependence could be detected. Overall, the results suggest that symmetric copulas seem to be more appropriate to capture the dependence structure between EUA returns and the returns of commodity futures and European equity indices. Further, we observe that there is some tail dependence between the returns, but it is generally not only exhibited in the lower left or upper right tail but rather in a symmetric way.

Since the distance between the estimated and empirical copula alone is not sufficient to determine whether any of the models really provides a good fit to the data, following Genest et al. (2009) also goodness-of-fit tests are conducted. Recall that for the goodness-of-fit tests, the null hypothesis is that the examined copula provides an appropriate fit to the data. Following the

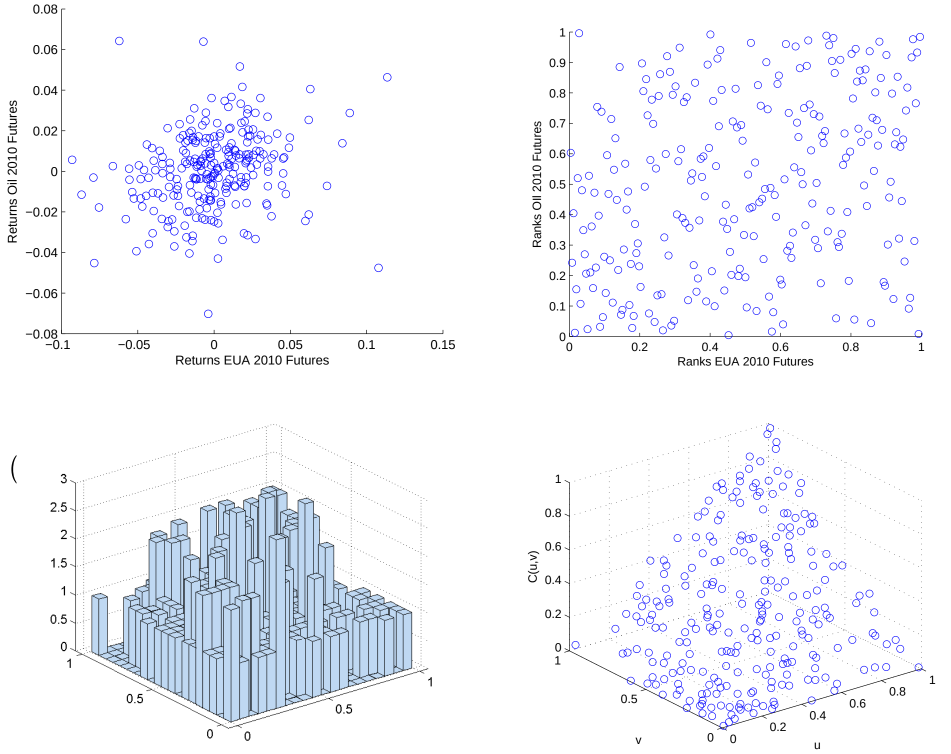

Fig. 4. Plot of returns for daily EUA 2010 Futures versus Oil 2010 Futures (upper left panel), ranks for daily EUA 2010 Futures returns versus Oil 2010 Futures returns (upper right panel), 3d histogram of ranks for daily EUA 2010 Futures returns versus Oil 2010 Futures returns (lower left panel) and fit of the Student t copula to the ranks (lower right panel).

Asset	Clayton	Gumbel	Gaussian	Student t
CER 2010 Futures	0.0542	0.0171	0.0074	0.0060 ($v = 3$)
Gas 2010 Futures	0.0108	0.0113	0.0108	0.0102 ($v = 10$)
Oil 2010 Futures	0.0274	0.0159	0.0117	0.0117 ($v = 24$)
Coal 2010 Futures	0.0305	0.0402	0.0261	0.0234 ($v = 4$)
EEX 2010 Futures	0.0634	0.0147	0.0084	0.0078 ($v = 10$)
Eurostoxx 50 Spot	0.0671	0.0094	0.0153	0.0152 ($v = 22$)
E1ENE Spot	0.0363	0.0238	0.0165	0.0142 ($v = 5$)
ERIXP Spot	0.0373	0.0171	0.0112	0.0100 ($v = 6$)

Table 2
Distance between estimated and empirical copula for the considered series. Consistently, the Student t copula yields the lowest distance according to Cramér-Von Mises statistic.

Asset	Clayton	Gumbel	Gaussian	Student t
CER 2010 Futures	0.000	0.052	0.795	0.960 ($v = 3$)
Gas 2010 Futures	0.926	0.564	0.918	0.962 ($v = 10$)
Oil 2010 Futures	0.091	0.306	0.882	0.846 ($v = 24$)
Coal 2010 Futures	0.052	0.004	0.102	0.174 ($v = 4$)
EEX 2010 Futures	0.001	0.620	0.979	0.995 ($v = 10$)
Eurostoxx 50 Spot	0.001	0.954	0.565	0.573 ($v = 22$)
E1ENE Spot	0.185	0.023	0.475	0.687 ($v = 5$)
ERIXP Spot	0.466	0.212	0.862	0.941 ($v = 6$)

Table 3
Results as p-value for bootstrap goodness-of-fit test according to Genest et al (2009).

test procedure described in the previous section, for each of the copula families, based on the distance between the empirical and estimated copulas for our bootstrap samples, p-values with respect to the null hypothesis can be calculated. The p-value provides the level of significance at which the null hypothesis would be rejected and therefore a measure of how much evidence we have against the null hypothesis of a good fit of the suggested copula. Table 3 lists the p-values for the different copula families.

Also for these tests the Student t and Gaussian copula perform best and generally outperform the Clayton and Gumbel copula. Still, based on the conducted goodness-of-fit tests, it is difficult to reject the adequacy of the Gumbel and Clayton copula. This confirms results by Genest et al. (2009) who state that the power of goodness-of-fit tests for copulas is often small when the dependence between the variables is low and only a small number of observations can be considered. Once again, for the dependence between EUA 2010 futures and Eurostoxx 50 returns, the Gumbel copula seems to be the best suited and yields the highest p-value. However, for most series the Gaussian and Student t copula seem to be most appropriate and for none of the considered series the null hypothesis of an appropriate fit can be rejected for these copulas. On the other hand, for the Clayton copula, the null hypothesis of an appropriate fit is rejected for the dependence between EUA 2010 futures and CER 2010 futures, EEX 2010 futures and Eurostoxx 50 returns. For the Gumbel copula, the null hypothesis is rejected at the 10% level for the CER 2010 futures and the DJ Europe Energy Stock Index. Overall, we conclude that there is superior fir of the elliptical Gaussian and Student t copula. Generally, the smallest distance between estimated and empirical copula is usually given by the Student t copula indicating some symmetric tail dependence.

In the following we extend the analysis to a risk management perspective and consider an exemplary portfolio with weights of 25% in EUA 2010 Futures contracts, 25% in Oil 2010 Futures contracts, 25% in Eurostoxx 50 Index and 25% in DJ Europe Energy Index. In order to determine the distribution of the portfolio returns, we will both consider the standard variance-covariance approach and an approach modeling the dependence structure between the returns using the Student t copula. Of course, also other copula models could be considered. However, as illustrated in the previous section for most of the series, the Student t copula model provided the best results and seemed to be most appropiate to describe the dependence structure between the return series.

In a first step we investigate the marginal distributions of the two series. A fit of the normal distribution to the return series of EUA 2010 futures contracts yields $\mu = -0.0009$ and $\sigma = 0.0307$, while the corresponding parameter estimates are $\mu = 0.0007$ and $\sigma = 0.0191$ for 2010 Oil futures returns, $\mu = 0.0006$ and $\sigma = 0.0176$ for Eurostoxx 50 returns and $\mu = 0.0007$ and $\sigma = 0.0210$ for the E1ENE returns. We further apply a Kolmogorov-Smirnov goodness-of-fit test in order to examine whether the normal distribution provides an appropriate fit to the return series. The tests yields a test statistic of $d = 0.0542$ and a p-value of $p = 0.4359$ for the EUA 2010 futures returns while the corresponding statistics are $d = 0.0526$ and $p = 0.4743$ for 2010 Oil Futures returns, $d = 0.0418$ and $p = 0.0.7576$ for Eurostoxx 50 returns and $d = 0.0354$ and $p = 0.9011$ for E1ENE returns. We conclude that there is no significant evidence against the null hypothesis of an appropriate fit of the normal distribution to the marginal return series.

Once the marginal distributions have been specified, they can be used for determining the distribution of portfolio returns. In our analysis we compare approaches using a Student t copula model to the standard multivariate normal or variance-covariance approach that is generally applied in portfolio management. For the variance-covariance approach we simply need to estimate the variance-covariance matrix Σ for the return series. Then using portfolio theory, based on the mean of the marginal return series, the portfolio weights and the estimated variance-covariance matrix, we can calculate the distribution of the portfolio return. We obtain $\mu_p = 0.0003$ and $\sigma_p = 0.0167$. Note that due to diversification, the return distribution of the portfolio has a smaller standard deviation than each of the individual return series. Based on the determined distribution it is then straightforward to also determine the 95%, 99% and 99.9% Value-at-Risk (VaR) figures that are reported in Table 4.

For determining the return distribution using the Student t copula models, we

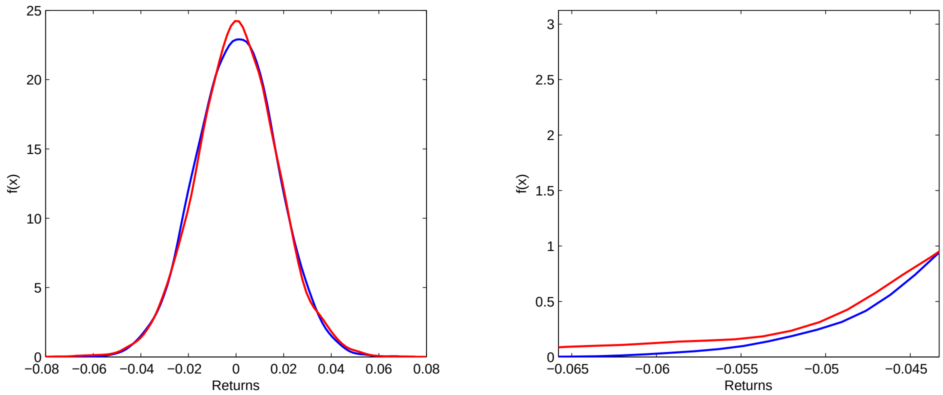

Fig. 5. Plot of simulated return distribution (left panel) and tail of simulated return distribution (right panel) for the considered portfolio. For both plots the blue line is the probability density for the multivariate normal approach, while the red line provides the simulated density for a model using the Student t copula to model the dependence structure between the rank transforms.

first estimate the multivariate student t copula, hence the correlation matrix C and degrees of freedom parameter v for the rank transforms of the return series. The estimation yields approximately $v = 6.54$. Then we generate samples of dependent ranks using the multivariate Student t copula with correlation matrix C and degrees of freedom parameter v. Thus, we simulate 10000 pairs of dependent uniformly distributed random variables (u_1, u_2, u_3, u_4). In a next step, the inverse of the estimated normal distributions for the margins are used to calculate the simulated dependent returns for the series. Finally, using the portfolio weights we can then determine a simulated return distribution based on a dependence structure modelled by the Student t copula and Gaussian margins. The results on standard deviation, skewness, kurtosis and corresponding VaR figures are reported in Table 4. Further, a plot of the simulated return distribution using the Student t copula model in comparison to the standard variance-covariance approach is provided in 5.

Our results indicate that the standard variance-covariance approach underestimates the risk in particular in the extreme tail of the distribution. The standard deviation and in particular the kurtosis of the distribution are higher for the model using the Student t copula for the dependence structure. Further, while for 95% and 99% VaR, the copula approach only yields a VaR that is approximately 3% and 6% higher for the 99.9% VaR there is a significant difference between the two approaches. The 99.9% VaR is underestimated by approximately 16% when the multivariate normal approach is used. These results could be important for risk management or hedging purposes, but also for the purpose of portfolio optimisation, in particular when not only the mean and variance but also higher moments of the portfolio return distribu-

Approach	Std	Skew	Kurtosis	$VaR_{0.95}$	$VaR_{0.99}$	$VaR_{0.999}$
Variance/Covariance	0.0167	0.000	3.0000	-0.0272	-0.0386	-0.0513
Student Copula	0.0173	0.0130	3.5282	-0.0279	-0.0409	-0.0595

Table 4
Value-at-Risk for a portfolio with weights of 25% in EUA 2010 Futures contracts, 25% in Oil 2010 Futures contracts, 25% in Eurostoxx 50 Index and 25% in DJ Europe Energy Index. We consider $VaR_{0.95}$, $VaR_{0.99}$, $VaR_{0.999}$ for both a standard multivariate normal (variance-covariance) approach and the estimated dependence structure according to the Student t copula with margins from the normal distribution.

tion are considered or when risk-adjusted measures are used, see e.g. Jondeau and Rockinger (2006b); Jorion (2001); Keating and Shadwick (2002). Note that our results with respect to an underestimation of the risk were also robust when alternative portfolio weights or different combination of assets were considered.

4.4 Time-Varying Copulas

To investigate the nature of the dependence through time we further apply a time-varying estimation of the copula parameters for the different bivariate series. Hereby, we decide to estimate the different copula parameters using a rolling window approach as it is applied e.g. in Giacomini et al. (2009); Grégoire et al. (2008). Note that more advanced approaches on the estimation of time-varying copulas have been suggested e.g. by Patton (2006); Rodriguez (2007); Giacomini et al. (2009) but our aim in this section is to provide a preliminary and rather descriptive analysis of the dependence structure through time. The length of the window was chosen to be 126 trading days what corresponds to approximately six months. Figure 6 shows a plot of the estimated copula parameters for the Clayton, Gumbel and Gaussian / Student t copula for a six month rolling window period. Thus, the first six month period considers returns from January 5, 2009 to June 30, 2009 while the last window data from July 1, 2009 to December 24, 2009.

For the dependence structure between EUA and CER 2010 future returns, we find that the estimated correlation coefficient is rather constant while there is some time-variation in the estimated dependence parameter for the Gumbel and Clayton copula. Further, we find that overall the dependence between EUA 2010 futures returns and 2010 Gas futures, 2010 Electricity future and Eurostoxx 50 spot returns is slightly decreasing over time. For each of the considered copulas the dependence parameters indicate a marginal lower degree of dependence for the later six month periods in 2009. However, to conclude that there is a structural break or a significant change in the dependence structure

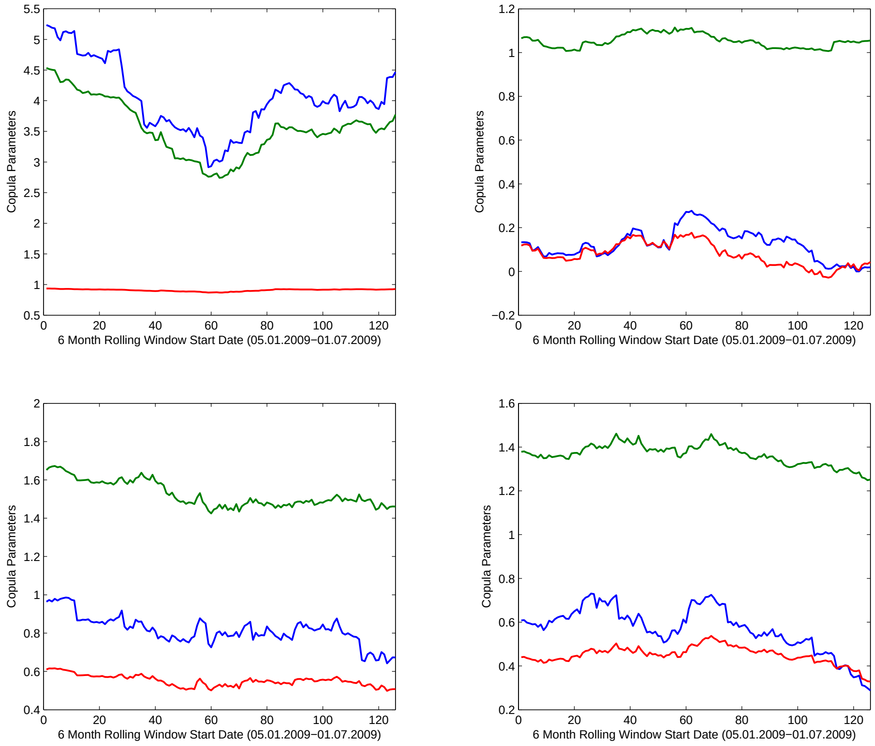

Fig. 6. Plot of estimated copula parameters for Clayton (blue), Gumbel (green) and Gaussian and Student t (red) copula for a six month rolling window period with start dates from January 5, 2009 up to July 1, 2009. The graphs show the results for dependence structure between returns for daily EUA 2010 Futures and CER 2010 Futures (upper left panel), 2010 Gas Futures (upper right panel), 2010 EEX Electricity Futures (lower left panel) and Eurostoxx 50 Spot contracts (lower right panel).

during the considered period further statistical tests as suggested by Patton (2006) or Giacomini et al. (2009) would be required, what is left to future work.

5 Conclusions

This paper applies different copulas in order to investigate the dependence structure between EUA future returns and those of other financial assets and commodities during the Kyoto commitment period. The results suggest that the dependence between EUA and oil and gas futures returns is relatively weak. On the other hand, a more significant dependence structure is found between EUA and electricity futures returns as well as between EUA futures and equity and energy index spot returns. These results at least somehow contradict earlier studies by Kosobud et al. (2005) and Daskalakis et al. (2009).

The former found no statistically significant correlations between returns of SO_2 emission allowances and returns from other financial variables while the latter observed that EUA futures returns were negatively correlated with equity market returns during the pilot trading period. Not surprisingly the relationship between EUA and CER future returns is characterized by a very strong dependence.

Regarding the nature of dependence, we find some evidence of symmetric tail dependence for most of the return series. Surprisingly, for most of the series also the Gaussian copula outperforms both the Clayton and Gumbel copula that exhibit dependence either only in the lower left or upper right tail. Generally, the Clayton copula yields the worst fit while the fit of the Gumbel is significantly better, but still worse than the Gaussian and Student t copula. We also conduct a risk analysis for a portfolio consisting of equal weights in emission allowance futures, oil futures and two equity indices. We find that applying the standard variance-covariance approach might underestimate the kurtosis and in particular tail risk of the portfolio return distribution. Further, investigating Kendall's tau and the dependence parameters through time we find that overall the dependence between EUA 2010 futures returns and 2010 Gas futures, 2010 Electricity future and Eurostoxx 50 spot returns is slightly decreasing during the considered period.

The obvious conclusion that can be drawn from this study is that EUAs are useful for diversifying asset portfolios, in particular those that focus on energy commodities and energy assets. In addition, a number of extensions to the current study are conceivable. Firstly, it might be worthwhile to investigate whether the dependence structures between EUA returns and other financial assets and commodities is different to the dependence structure between CER contracts. Maybe such an analysis could also contribute to the literature that is concerned with the EUA-CER spread. Secondly, extending the analysis using data from different years would allow conclusions about the stability of the dependence structure between the variables through time. This would yield interesting insights in the general development of this newly established market and its relationship to other financial markets.

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